



NOTE

LIFETIMES AND POLARIZABILITIES OF LOW LYING S, P AND D STATES OF FRANCIUM

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Abstract—The lifetimes as well as the scalar and tensor polarizabilities are computed for over 60 low lying S, P and D states of francium using a Coulomb approximation. The result for the $7P_{3/2}$ state lifetime agrees very closely with a recent measurement while the scalar polarizability of the ground state is estimated to be substantially lower than previously predicted. No other lifetime or polarizability determinations were found in the literature. This data will facilitate future studies of francium spectroscopy. © 1999 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Considerable attention has recently been devoted to trapping francium.¹⁻³ This atom is the heaviest alkali atom and can therefore be modelled relatively easily since it has only a single valence electron. Francium is predicted to have a parity violation effect that is 16 times larger than that found in cesium and is also of interest for searches for a permanent electric dipole moment.⁴ Substantial progress in francium spectroscopy has occurred primarily during the last decade because the longest lived isotope of francium has a half life of only 22 min. Hence, experiments must use either a radioactive source or take place at an accelerator.

A number of different groups have studied francium Rydberg states,⁵ isotope shifts and hyperfine structure.^{6,7} A great deal of theoretical and experimental work has also been devoted to determining the energies of various francium states. Dzuba et al⁸ have estimated the energies of a number of low lying S, P, D, F and G states using many body perturbation theory. Laser spectroscopy has been used by Arnold et al⁹ to measure the $(10 - 22)S_{1/2}$ and $(8 - 20)D_{3/2,5/2}$ state energies with an uncertainty of less than 0.01 cm^{-1} . Similar techniques were used by Bauche et al¹⁰ and Duong et al¹¹ to study the $7P_{1/2,3/2}$ and $8P_{1/2,3/2}$ states respectively. Simsarian et al¹² employed two photon spectroscopy to study the $7S_{1/2} \rightarrow 9S_{1/2}$ transition.

In contrast, progress to determine other atomic properties has been limited. The first radiative lifetime measurement of a francium state was reported this year¹³ and only one paper was found that estimated a state's polarizability.¹⁴ Polarizabilities have been used to determine Stark shifts which have been precisely measured in other alkali elements and used to stringently test theory.¹⁵⁻¹⁷ This paper presents the lifetimes and polarizabilities of over 60 of the lowest lying francium states. The calculation was done using a Coulomb approximation which has been shown to yield lifetimes and polarizabilities of the other alkali atoms that are within a few percent of the measured values.¹⁸⁻²⁰ This method developed by Bates and Damgaard,^{21,22} approximates the potential experienced by the valence electron by a Coulomb potential that is generated by the nucleus and inner electron core.

2. COMPUTATION OF POLARIZABILITIES AND RADIATIVE LIFETIMES

The Hamiltonian describing the effect of an electric field E on an atom is given by²³

$$H = - \left\{ \alpha_0 + \alpha_2 \frac{3J_z^2 - \mathbf{J}^2}{J(2J-1)} \right\} \frac{E^2}{2}, \quad (1)$$

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Table 1. Lifetimes and polarizabilities of low lying S, P and D francium states ($(a_0)^3$ is the Bohr radius cubed).

State	n^*	Lifetime (nsec)	Polarizabilities (a_0) ³	
			α_0	α_2
7S _{1/2}	1.828		3.050E2	
8S _{1/2}	2.893	6.306E1	4.600E3	
9S _{1/2}	3.910	1.243E2	2.700E4	
10S _{1/2}	4.917	2.304E2	1.059E5	
11S _{1/2}	5.921	3.935E2	3.196E5	
12S _{1/2}	6.923	6.244E2	8.152E5	
13S _{1/2}	7.925	9.355E2	1.840E6	
14S _{1/2}	8.926	1.339E3	3.737E6	
15S _{1/2}	9.927	1.847E3	7.007E6	
16S _{1/2}	10.927	2.472E3	1.205E7	
17S _{1/2}	11.927	3.228E3	2.009E7	
18S _{1/2}	12.928	4.123E3	3.314E7	
19S _{1/2}	13.928	5.169E3	5.145E7	
7P _{1/2}	2.307	2.844E1	1.106E3	
8P _{1/2}	3.357	1.025E2	2.249E4	
9P _{1/2}	4.357	2.334E2	1.489E5	
10P _{1/2}	5.357	4.243E2	6.360E5	
11P _{1/2}	6.357	6.912E2	2.061E6	
12P _{1/2}	7.357	1.029E3	5.671E6	
13P _{1/2}	8.357	1.482E3	1.339E7	
14P _{1/2}	9.357	2.007E3	2.936E7	
15P _{1/2}	10.357	2.691E3	5.796E7	
16P _{1/2}	11.357	3.432E3	1.108E8	
17P _{1/2}	12.357	4.270E3	2.019E8	
18P _{1/2}	13.357	5.374E3	3.388E8	
7P _{3/2}	2.408	2.090E1	2.102E3	- 4.027E2
8P _{3/2}	3.455	6.113E1	4.293E4	- 6.463E3
9P _{3/2}	4.455	1.307E2	2.859E5	- 3.891E4
10P _{3/2}	5.455	2.313E2	1.250E6	- 1.607E5
11P _{3/2}	6.455	3.748E2	4.045E6	- 5.017E5
12P _{3/2}	7.455	5.624E2	1.102E7	- 1.334E6
13P _{3/2}	8.455	7.999E2	2.642E7	- 3.137E6
14P _{3/2}	9.455	1.093E3	5.728E7	- 6.698E6
15P _{3/2}	10.455	1.432E3	1.172E8	- 1.359E7
16P _{3/2}	11.455	1.839E3	2.225E8	- 2.561E7
17P _{3/2}	12.455	2.363E3	3.863E8	- 4.390E7
18P _{3/2}	13.455	2.919E3	6.650E8	- 7.509E7
6D _{3/2}	2.573	5.592E2	- 3.302E2	2.542E2
7D _{3/2}	3.573	7.590E1	- 2.695E2	4.016E3
8D _{3/2}	4.573	9.987E1	- 1.256E4	2.894E4
9D _{3/2}	5.574	1.449E2	- 9.717E4	1.302E5
10D _{3/2}	6.574	2.084E2	- 4.004E5	4.456E5
11D _{3/2}	7.575	2.917E2	- 1.298E6	1.286E6
12D _{3/2}	8.575	3.967E2	- 3.411E6	3.103E6
13D _{3/2}	9.575	5.256E2	- 8.214E6	7.036E6
14D _{3/2}	10.576	6.805E2	- 1.746E7	1.383E7
15D _{3/2}	11.576	8.639E2	- 3.556E7	2.711E7
16D _{3/2}	12.576	1.077E3	- 6.822E7	5.216E7
17D _{3/2}	13.576	1.324E3	- 1.197E8	8.711E7
6D _{5/2}	2.592	1.915E3	2.687E2	- 4.761E1
7D _{5/2}	3.592	7.029E1	- 7.367E3	1.838E4
8D _{5/2}	4.592	9.589E1	- 5.969E4	1.281E5
9D _{5/2}	5.594	1.417E2	- 3.083E5	5.789E5
10D _{5/2}	6.595	2.058E2	- 1.075E6	1.907E6
11D _{5/2}	7.595	2.898E2	- 3.102E6	5.266E6
12D _{5/2}	8.596	3.956E2	- 7.787E6	1.276E7
13D _{5/2}	9.596	5.255E2	- 1.753E7	2.791E7
14D _{5/2}	10.596	6.816E2	- 3.742E7	5.763E7
15D _{5/2}	11.596	8.662E2	- 7.283E7	1.099E8
16D _{5/2}	12.597	1.082E3	- 1.289E8	1.921E8
17D _{5/2}	13.597	1.331E3	- 2.278E8	3.328E8

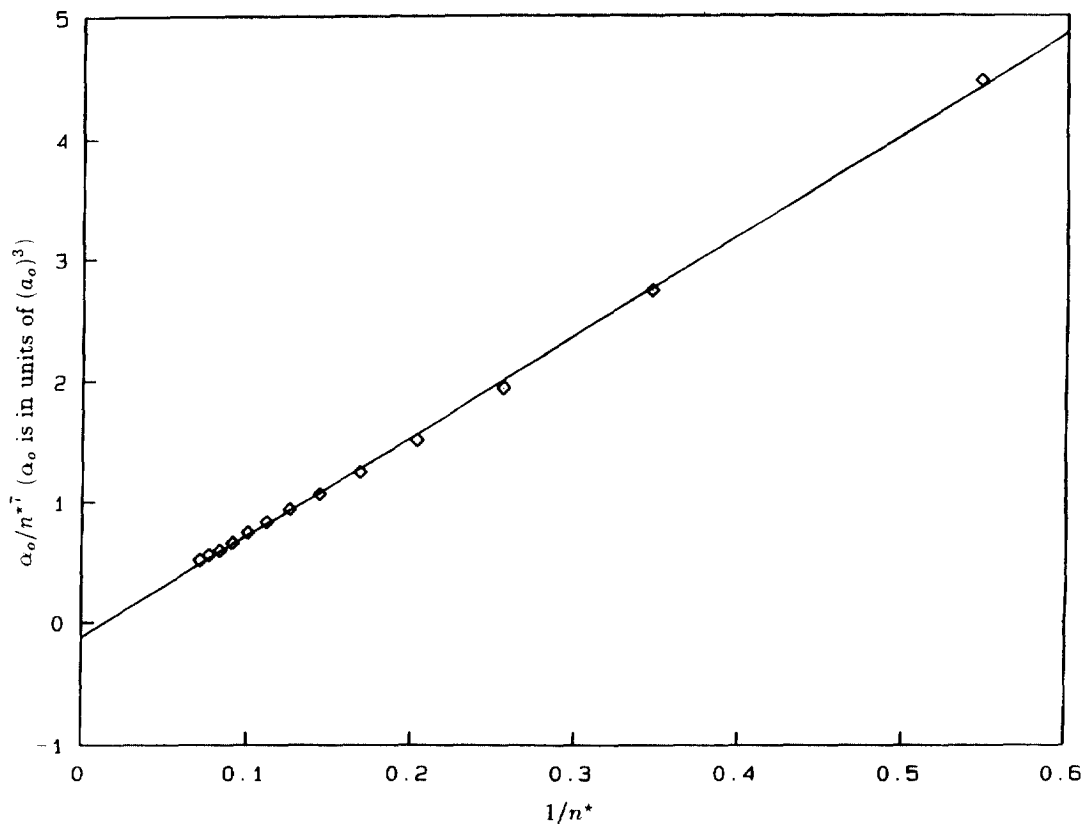


Fig. 1. Dependence of the Scalar Polarizabilities for the (7 - 19) $S_{1,2}$ States on Effective Principal Quantum Number n^* . The data were fitted to a function as is discussed in the text.

where \mathbf{J} is the electronic angular momentum and the quantization axis z is parallel to the field direction. The second term vanishes when $J < 1$. α_0 and α_2 are the scalar and tensor polarizabilities, respectively, and are defined by

$$\alpha_0 = \frac{r_0}{4\pi^2} \sum_{J'} \lambda_{JJ'}^2 f_{JJ'} \quad (2)$$

$$\alpha_2 = \frac{r_0}{8\pi^2} \frac{1}{(2J+3)(J+1)} \sum_{J'} \lambda_{JJ'}^2 f_{JJ'} [8J(J+1) - 3X(X+1)], \quad (3)$$

where $X = J'(J'+1) - 2 - J(J+1)$. r_0 is the classical electron radius, $\lambda_{JJ'}$ is the wavelength for a transition between states J and J' and $f_{JJ'}$ is the transition oscillator strength.

The state energies were used to calculate the effective principal quantum number n^* using

$$n^* = \left\{ \frac{E_{\text{ion}} - E}{R} \right\}^{-1/2}, \quad (4)$$

where R is the Rydberg energy, E_{ion} is the ionization energy which for francium⁹ is 32848.87 cm^{-1} and E is the energy of the state in question. The energies of all S, P, D and F states having energies more than 400 cm^{-1} below the ionization energy were considered. In the case where the energy of a state had not been measured, n^* was estimated using the energy of a nearby state. For example, n^* of the $9P_{3/2}$ state was found by adding 1 to the value of n^* of the $8P_{3/2}$ state. The Schrodinger equation was then solved and the oscillator strengths were found. The resulting radiative lifetimes and polarizabilities are listed in Table 1.

Table 2. Scalar polarizabilities of alkali ground states (units of $(a_0)^3$)

Atom	Khan et al ¹⁴	Coulomb approximation	Experiment
Li	132, 134, 154	162 (this work)	164.0 ± 3.34^{25}
Na	160, 162	162^{18}	159.2 ± 3.4^{25} 163.01 ± 0.56^{26}
K	311, 328	278^{18}	292.8 ± 6.1^{25}
Rb	350, 363	314^{20}	319.2 ± 6.1^{25}
Cs	446, 478	394^{19}	402.2 ± 8.1^{25}
Fr	349, 402	305 (this work)	

3. DISCUSSION OF RESULTS AND CONCLUSIONS

The dependence of the scalar polarizabilities of the $(7 - 19) S_{1/2}$ states on n^* is illustrated in Fig. 1. The polarizabilities are predicted to scale as n^{*7} and the theoretical data can therefore be fitted to the function²⁴

$$\alpha_0/n^{*7} = A + B/n^*. \quad (5)$$

A least-squares fit to the data yielded $A = -0.120$ and $B = 8.30$. The results obtained by Eq. (5) agree with the computed values given in Table 1 to within a few percent.

Unfortunately, no measured polarizabilities were found in the literature. Khan et al¹⁴ estimated the scalar polarizability of the ground state using a variational technique to be either 349 or 402 a_0^3 depending on the value taken for the effective principal quantum number of the ground state. Both of these estimates are considerably larger than that reported here. Khan et al¹⁴ also reported values for the potassium, rubidium and cesium ground state polarizabilities that are considerably higher than the measured results as shown in Table 2. In contrast, results obtained by the Coulomb approximation agree closely with the measured values of all alkali ground state polarizabilities.

The only francium state lifetime that has been reported is that for the $7P_{3/2}$ state which was found to be 21.02 ± 0.16 ns.¹³ This is in excellent agreement with the result given in Table I. This lifetime has also been estimated by two separate many body perturbation calculations to be 20.8⁴ and 21.8 ns.²⁷ The inconsistency is surprising given that these calculations are much more elaborate and time consuming than the present work.

In conclusion, this paper reports the first estimates of lifetimes as well as the scalar and tensor polarizabilities of over 60 low lying francium states. These results were obtained using a Coulomb approximation which has been shown to work quite well for the other alkali atoms. Hence, the data will facilitate future investigations of francium spectroscopy.

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