Assignment 2

1. Prove that magnetic fields can't do any work.

2. $\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$

However many vector potentials $\vec{A}$ give rise to the same magnetic field $\vec{B}$, eg. $\vec{A}$ & $\vec{A} + \nabla \chi$. This freedom to choose $\nabla \chi$ permits us to arbitrarily choose the value of $\nabla \cdot \vec{A}$. In the Coulomb gauge $\nabla \cdot \vec{A} = 0$. Show that this is possible by finding χ such that:

$$\nabla \cdot (\vec{A} + \nabla \chi) = 0.$$

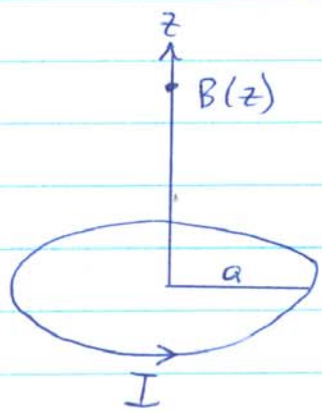
Hint: Remember solution to Poisson's Eqn.

For further reference we state the Lorentz Gauge.

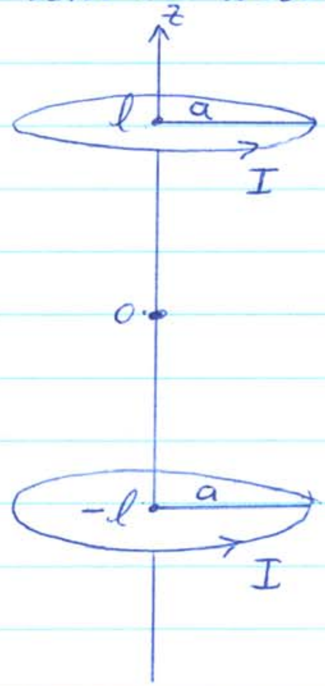
$$\nabla \cdot \vec{A} = -\frac{1}{c} \frac{\partial \Phi}{\partial t} \qquad \Phi = \text{electric potential}$$

3. The magnetic field at height z above a single loop of current I is:

$$\vec{B} = \frac{2\pi I a^2}{c (a^2 + z^2)^{3/2}} \hat{z}$$



It is important in many experiments to have a uniform field. Helmholtz coils are used for this purpose as shown below.



a) What is field on the z axis due to both coils?

b) Why are all odd derivatives of $B(z)$ at the origin equal to 0?

c) Show that the second derivative of $B(z)$ evaluated at the origin is 0 if $2l = a$.
(coil separation distance = coil radius)
This is the so called Helmholtz criterion.

d) $a = 30$ cm.
$I = 20$ amps.
\# turns in each coil $N = 50$
Units of magnetic field in cgs system are gauss.

$$1 \text{ gauss} = \frac{\text{esu/sec}}{\text{cm cm/sec}} = \frac{\text{esu}}{\text{cm}^2}$$

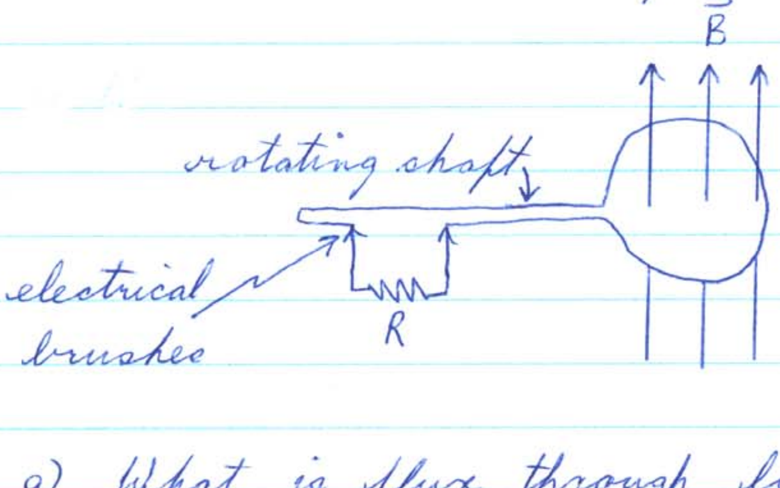
i) Convert current 20 amps to current in esu/sec.

Verify that $\frac{I(esu/sec)}{c} = \frac{I(amps)}{10}$

ii) What is field at center of Helmholtz coils?

iii) Earth's magnetic field is about 1/2 gauss. What current is needed to generate a field that can cancel the Earth's field?

4. Consider a loop of wire of area A at the end of a rotating shaft. The shaft rotates at angular frequency ω. The current loop is immersed in a uniform field $\vec{B}$.



a) What is flux through loop as a function of time?

b) What is voltage created by changing flux?

c) What is current flowing in resistor as a function of time and plot it?

5. Consider the example of the conducting sphere with charge Q surrounded by dielectric.

a) Find bound surface charge densities σ_b at $r = a$ & b, a = rad. of sphere, b = rad. of dielectric mat. surrounding sphere

b) What is electric potential at center of sphere taking potential to be 0 at infinity.

6. Ampere's equation + displacement current in material is:

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{free} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

One may wonder why this last term isn't $\frac{1}{c}\frac{\partial \vec{E}}{\partial t}$.

Show that the above equation gives rise to the continuity equation of charge $\nabla \cdot \vec{J}_{free} = -\frac{\partial \rho}{\partial t}$.

7. The differential forms of Maxwell's equations in a nonpolarizable, nonmagnetizable media are:

$$\nabla \cdot \vec{E} = 4\pi \rho \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

State the integral form of each and give a brief qualitative description of each of these "complicated mathematical equations".