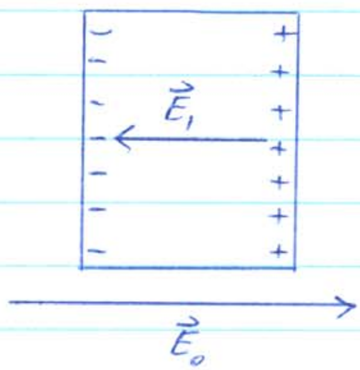
Assignment 1

1. Electrical Conductors

An ideal conductor is a material having an unlimited supply of free charges. Consider a conductor placed in an external electric field $\vec{E}_0$.



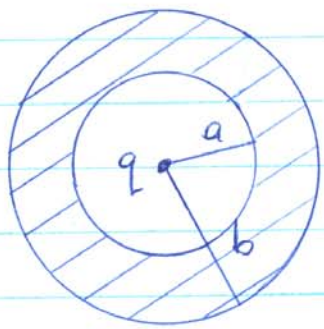
Inside the conductor, $\vec{E}_0$ drives positive charge to the right surface and negative charge to the left surface. These so called induced charges produce a field $\vec{E}_1$ opposing $\vec{E}_0$. Charges continue to move until $\vec{E}_1$ exactly cancels $\vec{E}_0$. The time for this movement of charge to occur is extremely short. Hence we conclude that inside a conductor the electric field is 0.

a) Show the charge density $\rho = 0$ inside conductor.

b) Show the potential Φ is constant inside a conductor.

c) Show that just outside a conductor $\vec{E}$ is perpendicular to the surface and equals $4\pi\sigma$ where σ = surface charge density.

2. A charge q sits in a spherical hollow inside a spherical conductor.



a) Find $\vec{E}$ everywhere.

b) What are charge densities on conductor surfaces?

c) Find potential everywhere taking $\Phi = 0$ at ∞. ~~position of charge q~~.

3. Consider a sphere of radius a of uniform charge density ρ_0.

a) Find $\vec{E}$ everywhere.

b) Find potential everywhere taking $\Phi = 0$ at origin.

c) $a = 2$ cm, $\rho_0 = \frac{3}{2\pi}$ esu/cm^3

i) What is total charge on sphere?

ii) What is electric field 10 cm from sphere center?
" " potential " " in statvolts? 1 statvolt = 1 esu/cm.

iii) A charge of 5 esu is moved from infinity to within 10 cm. from sphere center.

What is work done in ergs in moving charge?

What is force in dynes between charge and sphere at the final position?

4. For a time independent or static situation we showed

$$-\int_a^b \vec{E}\cdot d\vec{l} = \Phi(b) - \Phi(a)$$

depends only on the endpoints a & b.

a) Show that for any closed path $\oint \vec{E}\cdot d\vec{l} = 0$.

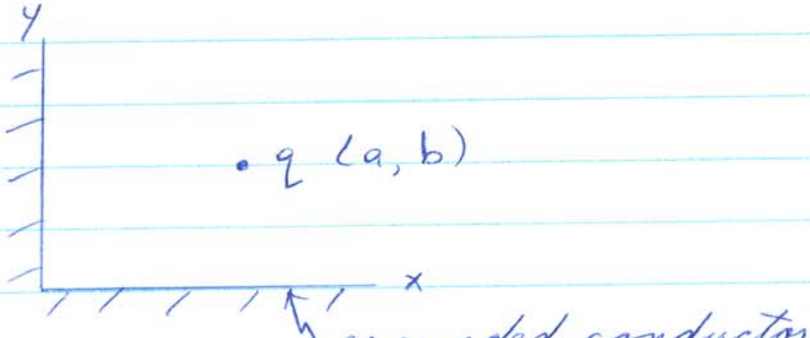
b) Using Stoke's Theorem $\oint \vec{E}\cdot d\vec{l} = 0 \Rightarrow \nabla\times\vec{E} = 0$.

c) $-\int_a^b \vec{E}\cdot d\vec{l} = \Phi(b) - \Phi(a)$ was derived by integrating $\vec{E} = -\nabla\Phi$. Using Cartesian coordinates show this implies $\nabla\times\vec{E} = 0$.

d) Sketch a vector field $\vec{A}$ for which $\oint \vec{A}\cdot d\vec{l} \neq 0$.

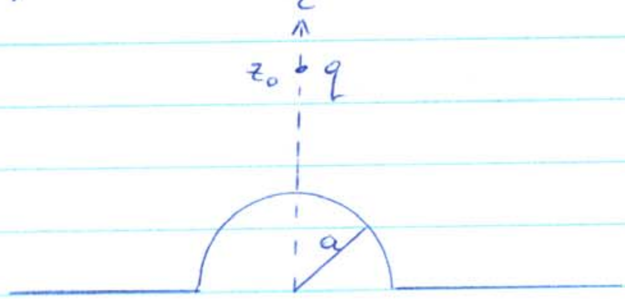
5. Consider an empty 3 dimensional rectangular cube having all sides at 0 potential. What is potential inside cube and how do you know this is the only possible answer?

6. An infinite conducting sheet is bent into a 90° corner. A point charge q is placed near the corner as shown.



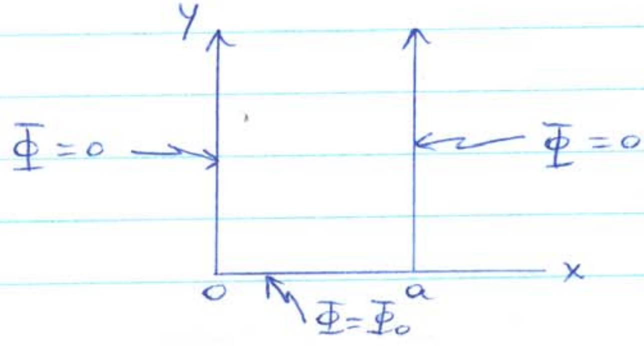
Find the potential everywhere.

7. An infinite conducting sheet has a hemispherical bubble of radius a.



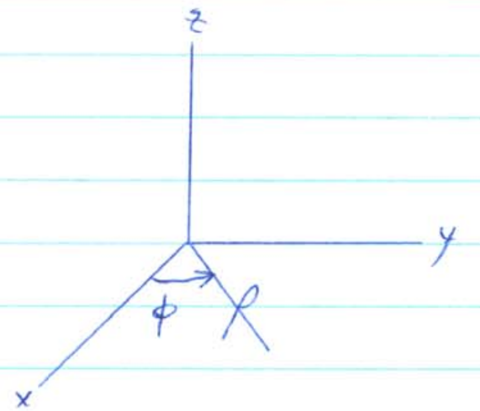
Find potential everywhere.

8. An infinitely deep trough has its two sides at 0 potential and its bottom at Φ_0. Find potential everywhere in trough.



9. Laplace equation in cylindrical coordinates is

$$0 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$



$z = z$

$y = \rho \sin\phi$

$x = \rho\cos\phi$

Consider the case where Φ is independent of z.

a) Let $\Phi = R(\rho)\,Q(\phi)$ and find differential eqns. for R + Q.

b) Constant $= 0$ Show $R = A\ln\rho + B$
$Q = C\phi + D$

Constant $k^2 > 0$ Show $R = A\rho^k + B\rho^{-k}$
$Q = C\cos k\phi + D\sin k\phi$

Constant $-k^2 < 0$ Show $Q = Ce^{k\phi} + De^{-k\phi}$

c) Suppose $\Phi(\rho,\phi) = \Phi(\rho, \phi+2\pi) \Rightarrow Q(\phi) = Q(\phi+2\pi)$.
Show that:

i) $k = n$ an integer and

ii) $\Phi(\rho,\phi) = A\ln\rho + B + \sum_{n=1}^{\infty}(A_n\rho^n + B_n\rho^{-n})(C_n\cos n\phi + D_n\sin n\phi)$