Assignment 4
Lagrangians and Hamiltonians

1. A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k. Find Lagrange's equations of motion.

2. A particle of mass m moves in one dimension under the influence of a force $F(x, t) = k / x^2 e^{-t/\tau}$ where k and τ are constants. Compute the Lagrangian and Hamiltonian functions. Compare the Hamiltonian and the total energy and discuss the conservation of energy for the system.

3. A particle of mass m moves under the influence of gravity along the spiral $z = k\,\theta$, r = constant, where k is a constant and z is the vertical distance. Obtain the Hamiltonian equations of motion.

4. Consider any two continuous functions of the generalized coordinates and momenta $g(q_k, p_k)$ and $h(q_k, p_k)$. The Poisson brackets are defined by

$$[g, h] \equiv \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right)$$

Verify the following properties of the Poisson brackets where H is the Hamiltonian.

a) $\frac{dg}{dt} = [g, H] + \frac{\partial g}{\partial t}$

b) $\dot{q}_j = [q_j, H] \qquad \dot{p}_j = [p_j, H]$

c) $[p_k, p_j] = 0 \qquad [q_k, q_j] = 0$

d) $[q_k, p_j] = \delta_{kj}$