Assignment 2
Calculus of Variations

1. Show explicitly that the function $y(x) = x$ produces a minimum path length by using the varied function $y(\alpha,x) = x + \alpha \sin \pi(1-x)$. Use the first few terms in the expansion of the resulting elliptic integral to show $\partial J / \partial \alpha_{\alpha=0} = 0$.

2. Consider light passing from one medium with index of refraction n_1 into another medium with index of refraction n_2. Use Fermat's principle to minimize time and derive the law of refraction.

3. Using the method of lagrange multipliers, find the dimensions of the parallelepiped of maximum volume circumscribed by a) a sphere of radius R and b) an ellipsoid with semiaxes of length a, b and c.

4. Using the method of lagrange multipliers, find the ratio of the radius R to the height H of a right circular cylinder of fixed volume V that minimizes the surface area A.