Assignment 5

1. Show that the real and imaginary parts of the plane wave $\psi = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

   satisfy the three dimensional wave equation.

2. Write down Maxwell's equations in differential form and state in words what each means.

3. Derive the following wave equation from Maxwell's Laws in vacuum.

$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

4. Sketch the electric and magnetic fields corresponding to a plane wave propagating in the z direction having electric field in the y direction.

5. Circular Polarization: Instead of taking $\hat{x}$ & $\hat{y}$ as the two orthogonal polarization vectors for a wave propagating in the $\hat{z}$ direction, one can use the following.

$$\hat{\epsilon}_{\pm} = \hat{x} \cos \omega t \pm \hat{y} \sin \omega t$$

   a) Show $\langle \hat{\epsilon}_a \cdot \hat{\epsilon}_b \rangle = \delta_{ab}$

   where a, b = +, - and the angular brackets denote a time average over many optical periods.

   b) Show $\hat{\epsilon}_+$ rotates counterclockwise in the xy plane.

   c) Which direction does $\hat{\epsilon}_-$ rotate in?

   d) Consider $\vec{E} = \hat{x} E_0 \cos \omega t$

   What fraction of this wave is polarized along $\hat{\epsilon}_+$?