Physics 3010 Test 9

Name: ____________________________

1. (3 marks) What are the units of the following items expressed in meters, kilograms and/or seconds.

a) Moment of Inertia

$[I] = [mR^2] = \text{kg meter}^2$

b) Angular Momentum

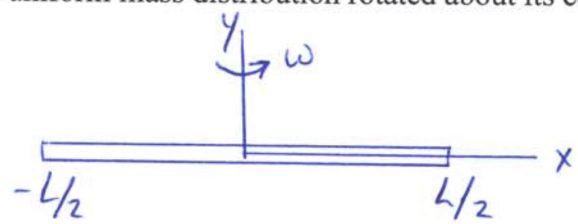
$[L] = [mvr] = kg\,\frac{m^2}{sec}$

c) Angular Velocity

$[\omega] = [\dot{\theta}] = \frac{rad}{sec.}$

2. (3 marks) Derive the moment of inertia of a bar of length L and mass M having a uniform mass distribution rotated about its center of mass.

Mass per unit length $\lambda = \frac{M}{L}$



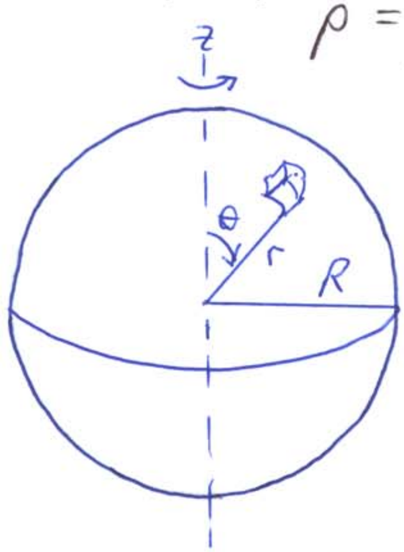
Consider bar mass from x to $x+dx$.

$$dI = x^2\,dm$$
$$= x^2 \lambda\,dx$$
$$I = \int_{-L/2}^{L/2} \lambda x^2\,dx$$
$$= \lambda \left[\frac{x^3}{3}\right]_{-L/2}^{L/2}$$
$$= \frac{\lambda L^3}{12}$$

$$\therefore I = \frac{ML^2}{12}$$

3. (4 marks) Find the moment of inertia of a rotating sphere of radius R whose density is given by

$$\rho = \frac{\rho_0 r}{R}$$



Consider infinitesimal volume element $dV = r^2 \sin\theta \, d\phi \, dr$

Mass of dV is $dM = \rho \, dV$

$$dI = (r \sin\theta)^2 \, dM$$

$$I = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin^2\theta \, \frac{\rho_0 r}{R} \, r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= \frac{\rho_0 2\pi}{R} \int_0^R r^5 \, dr \int_0^\pi \sin^3\theta \, d\theta$$

$$= \frac{\rho_0 2\pi}{R} \frac{R^6}{6} \cdot \frac{4}{3}$$

$$I = \frac{4\pi}{9} \rho_0 R^5$$

One can show sphere mass $M = \pi \rho_0 R^3$. $\Rightarrow I = \frac{4}{9} M R^2$

Total = 10 marks

(Uniform sphere has $I = \frac{2}{5} M R^2$)