Physics 3010 Test 5

Name: _____

1. (2 marks) What is a Central force?

Central force is derivable from potential that only depends on distance between two particles, eg. gravitational or Coulomb forces.

2. (2 marks) What is the difference between the electron mass and the reduced mass in the hydrogen atom?

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p}$$

$$\mu = \frac{m_e m_p}{m_e + m_p}$$

$$= m_e \frac{1}{1 + \frac{m_e}{m_p}}$$

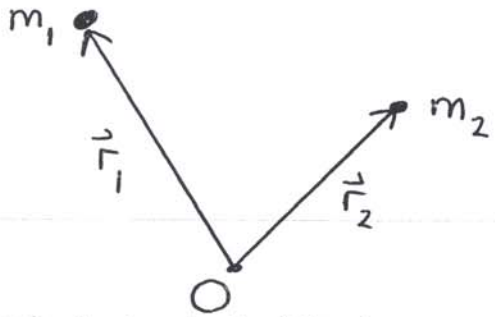
$$= m_e \left(1 + \frac{m_e}{m_p}\right)^{-1}$$

$$\simeq m_e \left(1 - \frac{m_e}{m_p}\right)$$

$$\simeq m_e \left(1 - \frac{1}{1836}\right)$$

$$\therefore \mu = 0.9995\, m_e$$

3. (6 marks) Consider two particles with masses and positions as shown below interacting via a central potential.



The Lagrangian is given by the following expression.

$$L = \frac{m_1}{2}\dot{\vec{r}}_1 \cdot \dot{\vec{r}}_1 + \frac{m_2}{2}\dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2 - U(|\vec{r}_1 - \vec{r}_2|)$$

Derive an expression for the Lagrangian in terms of the coordinates $\vec{r} = \vec{r}_1 - \vec{r}_2$ and the center of mass position.

Center of Mass Position $\vec{R} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

One can show $\vec{r}_1 = \vec{R} + \dfrac{m_2}{M}\vec{r}$ where $M \equiv m_1 + m_2$

$$\vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$$

$$\therefore L = \frac{m_1}{2}\left(\dot{\vec{R}} + \frac{m_2}{M}\dot{\vec{r}}\right)^2 + \frac{m_2}{2}\left(\dot{\vec{R}} - \frac{m_1}{M}\dot{\vec{r}}\right)^2 - U(r)$$

$$= \frac{m_1}{2}\left(\dot{\vec{R}}^2 + \frac{2m_2}{M}\dot{\vec{R}}\cdot\dot{\vec{r}} + \frac{m_2^2}{M^2}\dot{\vec{r}}^2\right)$$

$$+ \frac{m_2}{2}\left(\dot{\vec{R}}^2 - \frac{2m_1}{M}\dot{\vec{R}}\cdot\dot{\vec{r}} + \frac{m_1^2}{M^2}\dot{\vec{r}}^2\right) - U(r)$$

Total = 10 marks

$$\therefore L = \frac{M}{2}\dot{\vec{R}}^2 + \frac{\mu}{2}\dot{\vec{r}}^2 - U(r)$$