Physics 3010 Test 3

Name: _____

1. (6 marks)

a) (2 marks) How are the vector and scalar potentials related to electric and magnetic fields?

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla \Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

b) (2 marks) Write down the Lagrangian describing a particle having charge q and mass m interacting with an electromagnetic field.

$$L = \frac{m}{2}\dot{\vec{x}}^2 - q\Phi + \frac{q}{c}\vec{A}\cdot\vec{v}$$

c) (2 marks) What is the equation of motion resulting from Lagrange's equation?

Lorentz Force Law $m\ddot{\vec{x}} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$

2. (4 marks)

a) (1 mark) What is the definition of canonical momentum?

$$p_k \equiv \frac{\partial L}{\partial \dot{q}_k}$$

b) (1 mark) What is the definition of a Hamiltonian?

$$H = -L + \sum_k \dot{q}_k p_k .$$

c) (2 marks) Write down the 3 Hamilton equations of motion.

$$\frac{\partial H}{\partial p_k} = \dot{q}_k$$

$$\frac{\partial H}{\partial q_k} = -\dot{p}_k$$

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Total = 10 marks