Physics 3010 Test 2

Name: _____

1. (2 marks) What is meant by a conservative potential?

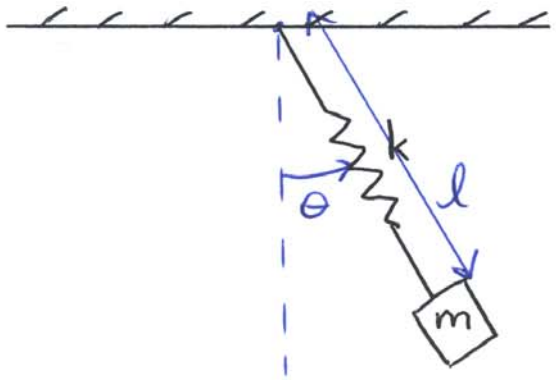
Force $\vec{F} = -\nabla U$

This also implies energy conservation.
Change in pot. energy is independent of path taken.

2. (2 marks) Explain why or why not the generalized velocity in Lagrangian mechanics is the same as the velocity one encountered in first year Classical Mechanics.

A generalized coordinate may be an angle θ.
" " velocity " " $\dot{\theta}$
$\dot{\theta}$ is an angular velocity with units of rad/sec not m/sec.

3. (6 marks) Consider a mass connected to a spring hanging from the ceiling as shown below. The mass oscillates in a fixed plane.



a) Define suitable generalized coordinate(s).

Let l be distance of mass from pivot point.
Let θ be angle shown above.

b) Write down the Lagrangian

$$T = \frac{m}{2}\left(\dot{l}^2 + l^2\dot{\theta}^2\right)$$

$$U = -mgl\cos\theta + \frac{k}{2}\left(l - l_{eq}\right)^2$$

↑ spring equilibrium length

$$\therefore L = T - U$$

$$= \frac{m}{2}\left(\dot{l}^2 + l^2\dot{\theta}^2\right) + mgl\cos\theta - \frac{k}{2}\left(l - l_{eq}\right)^2$$

c) Write down the equations of motion. (Do not solve them.)

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0$$

$$-mgl\sin\theta - \frac{d}{dt}\left(ml^2\dot{\theta}\right) = 0$$

$$-gl\sin\theta - l^2\ddot{\theta} - 2l\dot{l}\dot{\theta} = 0 \qquad (1)$$

$$\frac{\partial L}{\partial l} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{l}}\right) = 0$$

$$ml\dot{\theta}^2 + mg\cos\theta - k(l - l_{eq}) - m\ddot{l} = 0$$

$$\ddot{l} = l\dot{\theta}^2 + g\cos\theta - \frac{k}{m}(l - l_{eq}) \qquad (2)$$

Total = 10 marks