**Physics 3010 Test 1**

Name: _____

1. (3 marks) Express the following units in terms of meters, second and/or kilogram.

a) Newton $= kg \cdot \frac{m}{sec^2}$

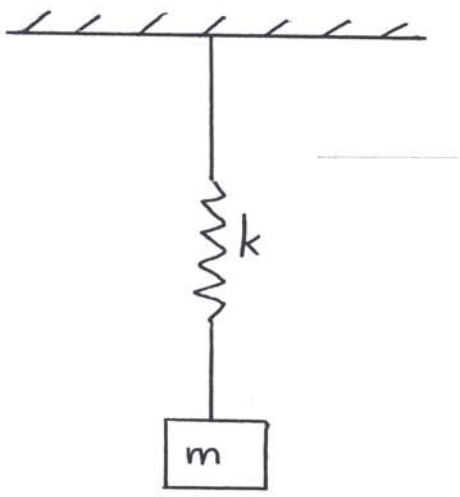
b) Joule $= kg \cdot \frac{m^2}{sec^2}$ (Nt. - meter)

c) Watt $= kg \frac{m^2}{sec^3}$ (Joule/sec)

2. (2 marks) Write down Hamilton's Principle, defining any terms.

A system follows path that minimizes the action; i.e. $\delta \int_{t_1}^{t_2} L \, dt = 0$

3. (6 marks) Consider the following mass that only moves up and down i.e. one dimensional motion.




a) Define the generalized coordinates.

Let $x$ be extension of spring from $= m$.

b) Write down the Lagrangian.

$$T = \frac{m}{2}\dot{x}^2$$

$$U = \frac{k}{2}x^2 - mgx$$

$$L = T - U$$

$$= \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2 + mgx$$

c) Solve Lagrange's equation of motion.

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

$$-kx + mg - \frac{d}{dt}(m\dot{x}) = 0$$

$$m\ddot{x} = -kx + mg.$$

$$\ddot{x} = -\frac{k}{m}x + g \qquad (1)$$

Let $x = x_0 + A\sin\omega t$

$$\dot{x} = +A\omega\cos\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t$$

$$\therefore (1) \Rightarrow \quad -A\omega^2\sin\omega t = -\frac{k}{m}x_0 \; -\frac{kA}{m}\sin\omega t + g$$

Equating constant terms $\Rightarrow \quad 0 = -\frac{k}{m}x_0 + g$

$$x_0 = \frac{mg}{k}$$

Equating coefficients of $\sin\omega t \Rightarrow \quad -A\omega^2 = -\frac{k}{m}A$

$$\omega = \sqrt{\frac{k}{m}}$$

Total = 10 marks

$$\therefore x = \frac{mg}{k} + A\cos\omega t$$