Name: ________________________

PHYS 3010 Test 1D

Evaluate the following and write your answers in simplified form. Each question is worth two marks.

$\cos^2 y + \sin^2 y$

$\cot^2 y + 1 = \csc^2 y$

1. $\frac{d}{dx} Arccot 4x$

$y = \text{Arccot}\, 4x$

$\cot y = 4x$

$-\csc^2 y \frac{dy}{dx} = 4$

$\frac{dy}{dx} = \frac{-4}{\csc^2 y}$

$= \frac{-4}{1+\cot^2 y}$

$\frac{d}{dx} \text{Arccot}\, 4x = \frac{-4}{1+16x^2}$

2. $\int \cos x \sin 2x dx$

$= 2\int \sin x \cos^2 x \, dx \qquad u = \cos x$

$= -2\int u^2 du$

$= -\frac{2}{3}\cos^3 x + C.$

3. $\int_0^\infty y^4 e^{-2y} dy$

$= \left[-\frac{y^4}{2} e^{-2y} - \frac{4y^3}{4} e^{-2y} - \frac{12y^2}{8} e^{-2y} - \frac{24y}{16} e^{-2y} - \frac{24}{32} e^{-2y} \right]_0^\infty$

$= \frac{3}{4}$

y^4	+	e^{-2y}
$4y^3$	−	$\frac{e^{-2y}}{-2}$
$12y^2$	+	$\frac{e^{-2y}}{4}$
$24y$	−	$\frac{e^{-2y}}{-8}$
24	+	$\frac{e^{-2y}}{16}$
0	−	$\frac{e^{-2y}}{-32}$

4. $\int u^2 \ln u du$

$$\begin{array}{ccc} \ln u & & u^2 \\ & + & \\ \frac{1}{u} & - & \frac{u^3}{3} \end{array}$$

$$= \frac{u^3}{3} \ln u - \int \frac{1}{u} \frac{u^3}{3} du$$

$$= \frac{u^3}{3} \ln u - \frac{u^3}{9} + C$$

5. $\int \phi^2 \sin \phi d\phi$

$$\begin{array}{ccc} \phi^2 & + & \sin\phi \\ 2\phi & - & -\cos\phi \\ 2 & + & -\sin\phi \\ 0 & - & \cos\phi \end{array}$$

$$= -\phi^2 \cos\phi + 2\phi \sin\phi + 2\cos\phi + C$$

6. $\int_0^\infty e^{-x} \cos 2x dx$

$$\begin{array}{ccc} \cos 2x & + & e^{-x} \\ -2\sin 2x & - & \frac{e^{-x}}{-1} \\ -4\cos 2x & + & e^{-x} \end{array}$$

$$= \left[-e^{-x} \cos 2x + e^{-x} \sin 2x\right]_0^\infty - 4\int_0^\infty e^{-x} \cos 2x \, dx$$

$$\int_0^\infty e^{-x} \cos 2x \, dx = \frac{1}{5}\left[1\right] = \frac{1}{5}$$

7. $\int_0^\infty \frac{r}{(r^2+16)^{3/2}} dr$

$u = r^2 + 16.$

$du = 2r dr$

$$= \frac{1}{2}\int_{16.}^{\infty} \frac{du}{u^{3/2}}$$

$$= \frac{1}{2} \left. \frac{u^{-1/2}}{-1/2} \right|_{16}^{\infty}$$

$$= \frac{1}{\sqrt{16}} = \frac{1}{4}.$$

8. $\int \tan x dx$ $= \int \frac{\sin x dx}{\cos x}$ $u = \cos x$

$du = -\sin x dx.$

$= - \int \frac{du}{u}$

$= -\ln|\cos x| + c$

$= \ln|\sec x| + c.$

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

Let $x = 4\sec\theta$

$dx = 4\sec\theta\tan\theta$

9. $\int \frac{1}{x\sqrt{x^2-16}}dx$

$= \int \frac{4\sec\theta \tan\theta \, d\theta}{4\sec\theta\sqrt{16\sec^2\theta - 16}}$

$= \frac{1}{4}\int \frac{\tan\theta}{\tan\theta} d\theta$

$= \frac{1}{4} \operatorname{Arcsec}\left(\frac{x}{4}\right) + C.$

10. $\int_0^{4\pi}(\cos^2 3x + \sin^2 3x)dx$ $= \int_0^{4\pi} 1 \, dx$

$= 4\pi$

Total = 20 marks