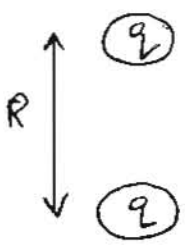
Assignment 9
Electrostatics

1. Two equally charged 1 gram masses repel each other. The lower mass is held fixed.

a) What is the charge on each mass for the Coulomb force to balance the gravitational force ~~of the Earth~~ on the upper mass?



Gravitational Force = Coulomb Force

$$\frac{Gm^2}{R^2} = \frac{kq^2}{R^2}$$

$$q = \sqrt{\frac{G}{k}}\, m$$

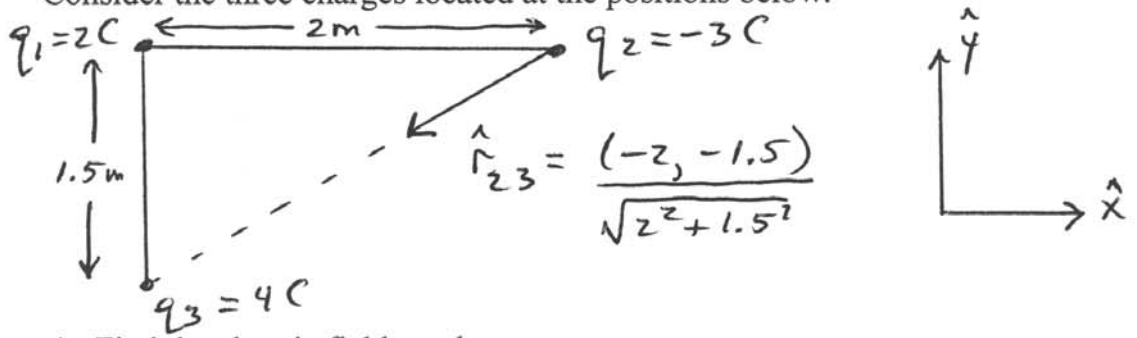
$$\therefore\ q = \left(\frac{6.67 \times 10^{-11}}{9 \times 10^{9}}\right)^{1/2} 10^{-3} = 8.61 \times 10^{-14}\ \text{Coulomb}$$

b) How many electrons does this charge represent?

$$8.61 \times 10^{-14}\ C = \frac{8.61 \times 10^{-14}}{1.6 \times 10^{-19}}\ \text{electrons}$$

$$= 5.4 \times 10^{5}\ \text{electrons}$$

2. Consider the three charges located at the positions below.



a) Find the electric field on charge q_3.

$$\vec{E}_{\text{on } q_3} = \frac{k q_1}{r_{13}^2}\hat{r}_{13} + \frac{k q_2}{r_{23}^2}\hat{r}_{23}$$

$$= 9\times 10^9 \left\{ \frac{2}{1.5^2}(-\hat{y}) + \frac{(-3)}{2^2+1.5^2} \cdot \frac{(-2,-1.5)}{\sqrt{2^2+1.5^2}} \right\}$$

$$= (3.46, -5.40)\times 10^9 \text{ Nt/Coul.}$$

b) Find the force on q_3.

$$\vec{F}_{\text{on } q_3} = q_3 \vec{E}_{\text{on } q_3}$$

$$= 4C \times (3.46, -5.40)\times 10^9 \text{ Nt/Coul}$$

$$= (1.38, -2.16)\times 10^{10} \text{ Nt.}$$

c) What happens if the charges are free to move?

Charges fly apart because forces are nonzero.

3. Two metal plates have a uniform charge density of 10 Coulombs/meter^2. The plates have dimensions of 0.6 x 0.6 meter^2 and are separated by 2 mm.

a) Ignoring edge effects, the electric field (volts/meter) between the two plates is given by $E = 1.13 \times 10^{11} Q / A$ where Q is the charge in Coulombs on one plate and A is the area in meters^2. Evaluate the field.

$E = 1.13 \times 10^{11} \times 10 \text{ Coul/m}^2$

$= 1.13 \times 10^{12} \frac{\text{Coul}}{\text{m}^2}$

b) What is the voltage between the two plates?

Voltage = E x plate separation

$= 1.13 \times 10^{12} \times .002$

$= 2.26 \times 10^{9}$ volts

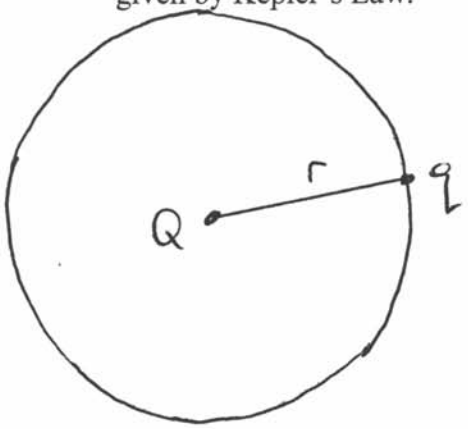
c) What is the charge stored on one plate?

$Q = 10 \frac{\text{Coul}}{\text{m}^2} \times (0.6 \text{ m})^2$

$= 3.6$ Coul.

4. Consider two opposite charges. One is very heavy and may be assumed to be fixed while the other one has a mass m and orbits the second at a radius r and velocity v.

a) Show that the relation between the orbital radius and the orbital period is given by Kepler's Law.



Centripetal Force = Coulomb Force

$$\frac{mv^2}{r} = \frac{kqQ}{r^2}$$

$$rv^2 = \frac{kqQ}{m}$$

$$r\left(\frac{2\pi r}{T}\right)^2 = \frac{kqQ}{m}$$

$\therefore$ Kepler's Law: $\boxed{\frac{r^3}{T^2} = \text{constant}}$

b) If the radius is doubled what happens to the period?

$$T^2 \propto r^3$$

or $T \propto r^{+3/2}$

If $r \rightarrow 2r$ then $T \rightarrow 2^{3/2}T = 2.83\,T$