Assignment 7
Waves

1. Consider the wave $y = 4 \cos(3z - 6t)$. Units are in meters and seconds. Find the following.

a) Amplitude

Amplitude = 4 units

b) Frequency

Angular Frequency $\omega = 6$ rad/sec

Frequency $\nu = \frac{\omega}{2\pi} = 0.95$ Hz

c) Period

Period $T = \frac{1}{\nu} = 1.05$ sec.

d) Wavelength

Wave Vector $k = 3$ unit^{-1}

Wavelength $\lambda = \frac{2\pi}{k} = 2.09$ units

e) Direction

Observer on wave crest sees $3z - 6t = \text{constant}$

$\therefore$ wave goes in $\hat{z}$ direction.

$3\frac{dz}{dt} - 6 = 0$

$\frac{dz}{dt} = 2$

f) Phase Velocity

Phase Velocity $v_p = \frac{\omega}{k}$

$v_p = 2$ units/sec.

2. Consider an organ pipe of length L. The pipe is closed at one end and open at the other allowing a standing wave where a node exists at the closed end and a maximum exists at the open end.

a) What are the resonant wavelengths?

Pipe length = odd # of ~~half~~ quarter wavelengths

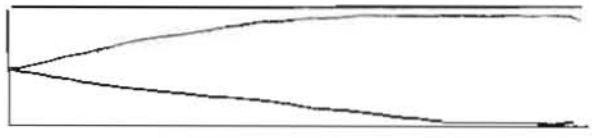
$$L = (2n+1)\frac{\lambda}{4}$$

$$\lambda = \frac{4L}{2n+1} \qquad n = 0, 1, 2, 3, \ldots$$

b) Sketch the resonant nodes corresponding to the 3 longest wavelengths.

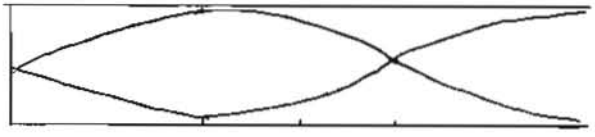
$n = 0$

$\lambda = 4L$



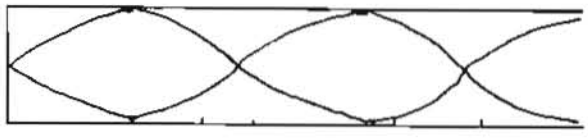
$n = 1$

$\lambda = \frac{4L}{3}$



$n = 2$

$\lambda = \frac{4L}{5}$



c) If L = 2 meters, what is the lowest frequency?

λ_{max} occurs for $n = 0 \Rightarrow \lambda_{max} = 4 \times 2 = 8$ m.

$$\nu_{min} = \frac{v_{air}}{\lambda_{max}} = \frac{330 \text{ m/sec}}{8 \text{ m}} = 41 \text{ Hz}.$$

ν_{min} corresponds to E nearly 2 octaves below middle C.

3. An orchestra wishes to have a listener in front of the conductor hear notes at the same time, played by the violinist located next to the conductor and from a drummer located 50 meters further back.

a) Assuming that the drummer plays as soon as he sees the conductor give the command, how long should the violinist wait before playing her note?

$$\Delta t = \frac{50 \text{ meters}}{330 \text{ m/sec}}$$

$$\approx 0.15 \text{ sec}.$$

b) Why may one assume that the two players see the conductor command at the same time?

Conductor command travels at speed of light

$$\Rightarrow \Delta t_{light} = \frac{50 \text{ m}}{3 \times 10^{8} \text{ m/sec}}$$

$$= 1.67 \times 10^{-7} \text{ sec}$$

$$\Delta t_{light} << \Delta t_{sound}$$