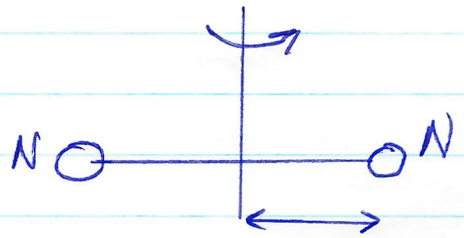
Assignment 9

1a)



$$r = \frac{1.5 \times 10^{-10}}{2} \text{ m}$$

$$I = 2 m_N r^2$$

$$= 2 \times 14 \times 1.67 \times 10^{-27} \text{ kg} \left(0.75 \times 10^{-10} \text{ m}\right)^2$$

$$= 2.6 \times 10^{-46} \text{ kg m}^2$$

b) Energy of first excited rotational state is:

$$\Delta E = \frac{L^2}{2I}$$

$$= \frac{(\sqrt{2}\hbar)^2}{2I}$$

$$= \frac{(1.06 \times 10^{-34})^2}{2.6 \times 10^{-46}}$$

$$= 4.3 \times 10^{-23} \text{ J}$$

Excitation Probability $P(\Delta E) = e^{-\Delta E / k_B T}$

$$e^{-1} = e^{-\Delta E / k_B T}$$

$$\therefore T = \frac{\Delta E}{k_B} = \frac{4.3 \times 10^{-23}}{1.38 \times 10^{-23}} = 3 \text{ K}$$

2a) $\vec{v}_{ave} = 0$ i.e. there is no preferred direction of motion

b) $\bar{v} = \left(\frac{8kT}{\pi m}\right)^{1/2}$

$= \left(\frac{8 \times 1.38 \times 10^{-23} \times 300}{\pi\ 28 \times 1.67 \times 10^{-27}}\right)^{1/2}$

$= 475.1$ m/sec.

c) $v_{RMS} = \sqrt{\frac{3kT}{m}}$

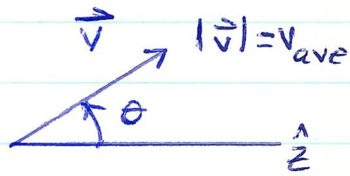
$= 515$ m/sec

d) $v_{Most\ Probable} = \sqrt{\frac{2kT}{m}}$

$= 421$ m/sec.

3a) Consider gas of density n where particles each have speed v_{ave} but velocity is in random direction. Half of particles corresponding to density $\frac{n}{2}$ move in z direction. These particles have average velocity z component given by:

$$\bar{v}_z = \frac{1}{2\pi}\int_0^{2\pi} d\phi \int_0^{\pi/2} v_z \sin\theta \, d\theta$$



$$= \frac{1}{2\pi}\, 2\pi \int_0^{\pi/2} v_{ave} \cos\theta \sin\theta \, d\theta$$

$$\therefore \bar{v}_z = \frac{v_{ave}}{2}$$

Particle Flux in z direction $= \frac{n}{2} \cdot \bar{v}_z$

$$= \frac{n\, v_{ave}}{4}$$

$$= \frac{2.7 \times 10^{25} \times 524}{4}$$

$$= 3.5 \times 10^{27} \text{ part/m}^2\text{/sec}$$

b) Leak Rate $= 3.5 \times 10^{27}\, \pi \left(\frac{0.1 \times 10^{-3}}{2}\right)^2$

$$= 1.1 \times 10^{20} \text{ part/sec.}$$

4a) Mean free path $\lambda = \dfrac{k_B T}{\sqrt{2}\,\pi d^2 P}$

$$P = \frac{N k_B T}{V} \Rightarrow \lambda = \frac{V}{\sqrt{2}\,\pi d^2 N}$$

$\therefore$ λ is independent of temperature.

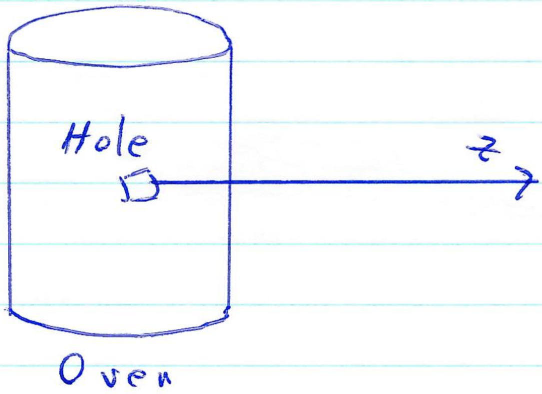
b) Mean time between collisions $\tau = \dfrac{\lambda}{\bar{v}_{Rel}}$

$$\bar{v}_{Rel} \propto T^{1/2} \Rightarrow \tau \propto T^{-1/2}$$

$\therefore$ collision frequency $\tau^{-1} \propto T^{1/2}$

If T doubles, τ^{-1} increases by factor of $\sqrt{2}$.

5 a)



For small hole, one may assume leak rate is small & doesn't significantly perturb velocity distribution in oven.

Particle flux $J_z \propto v_z \underbrace{P(v_z)}$

Probability atom in oven has velocity v_z

$$\therefore J_z \propto v_z e^{-m v_z^2 / 2 k_B T}$$

b) Velocity Probability Distribution in beam is

$$P_B(v_z) = N J_z$$

where N is normalization constant defined by

$$\int_0^\infty P_B(v_z)\, dv_z = 1$$

$$N \int_0^\infty v_z e^{-mv_z^2/2k_BT} dv_z = 1$$

$$N \frac{1}{2} \frac{2k_BT}{m} = 1$$

$$N = \frac{m}{k_BT}$$

$$\therefore \quad P_{Beam}(v) = m\beta v e^{-m\beta v^2/2} \quad \text{where } \beta \equiv \frac{1}{k_BT}$$