Assignment 3

1. Consider the wave $\psi = 5 \cos(10y + 6t)$. Distances are measured in cm. and time in seconds. Find the following.
   1) amplitude
   2) wavevector $|\vec{k}|$
   3) wavelength λ
   4) angular frequency ω
   5) frequency ν
   6) period T
   7) speed of wave
   8) propagation direction $\hat{k}$

2. Plane Waves in Nonconducting Media

   a) Derive $\nabla^2 \vec{B} - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$ from Maxwell eqns.

   b) Show $\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ is a solution.

   c) What is the relation between $|\vec{E}|$ + $|\vec{B}|$?
   What is relation between direction of $\vec{E}$, $\vec{B}$ + $\vec{k}$?

3. For complex fields show that the energy density averaged over many periods can be written as:

$$\langle u \rangle = \frac{1}{16\pi}\left(\vec{E}\cdot\vec{D}^* + \vec{B}\cdot\vec{H}^*\right)$$

4. Plane Waves in Conducting Medium

Consider the case when the conducting current is much smaller than the displacement current.
i.e. $\frac{4\pi\sigma}{\omega\epsilon} << 1$

Show $\alpha \simeq \frac{\omega}{c}\sqrt{\mu\epsilon}\left[1 + \frac{1}{2}\left(\frac{2\pi\sigma}{\omega\epsilon}\right)^2\right]$

$$\beta \simeq \frac{2\pi\sigma}{c}\sqrt{\frac{\mu}{\epsilon}}$$

The attenuation factor β is then independent of frequency.

5. For copper $\sigma \simeq 5 \times 10^{17}$ ~~cm~~ sec^{-1}
Assuming $\mu = 1$, complete the following table.

Radiation	Frequency	δ - skin depth
UV		
visible		
infrared		
microwave		
TV, FM		
radiofrequency		

6) When going from a more dense to less dense medium (i.e. $n_1 > n_2$) a light beam bends away from the normal. At a so called critical angle θ_c of incidence, the refracted angle is 90°. If $\theta_I > \theta_c$

the wave is totally reflected. Evaluate θ_c for a beam reflecting at a glass ($n=1.5$) air interface.

7. Show that for a s polarized wave, the reflection and transmission coefficients are given by:

$$R = \left(\frac{1-\beta\alpha}{1+\beta\alpha}\right)^2 \qquad \text{where } \beta = \frac{n_2}{n_1}$$

$$T = \frac{4\alpha\beta}{(1+\beta\alpha)^2} \qquad \alpha = \frac{\cos\theta_2}{\cos\theta_0}$$

8. Explain how polaroid sunglasses cut down on glare. (Hint: Think of Brewster's angle)

9. Circular Polarization
Instead of taking $\hat{x}$ + $\hat{y}$ as the 2 orthogonal polarization vectors for a wave propagating in the $\hat{z}$ direction, one can use the following:

$$\vec{\epsilon}_{\pm} = \hat{x}\cos\omega t \pm \hat{y}\sin\omega t$$

a) Show $\langle \vec{\epsilon}_a \cdot \vec{\epsilon}_b \rangle = \delta_{ab}$ $\quad a, b = \pm$

b) Show $\vec{\epsilon}_+$ is a vector rotating counterclockwise in xy plane. Which direction does $\vec{\epsilon}_-$ rotate?

c) Suppose $\vec{E} = \hat{x}E_0\cos\omega t$. What fraction of $\vec{E}$ is polarized "along $\vec{\epsilon}_+$"?