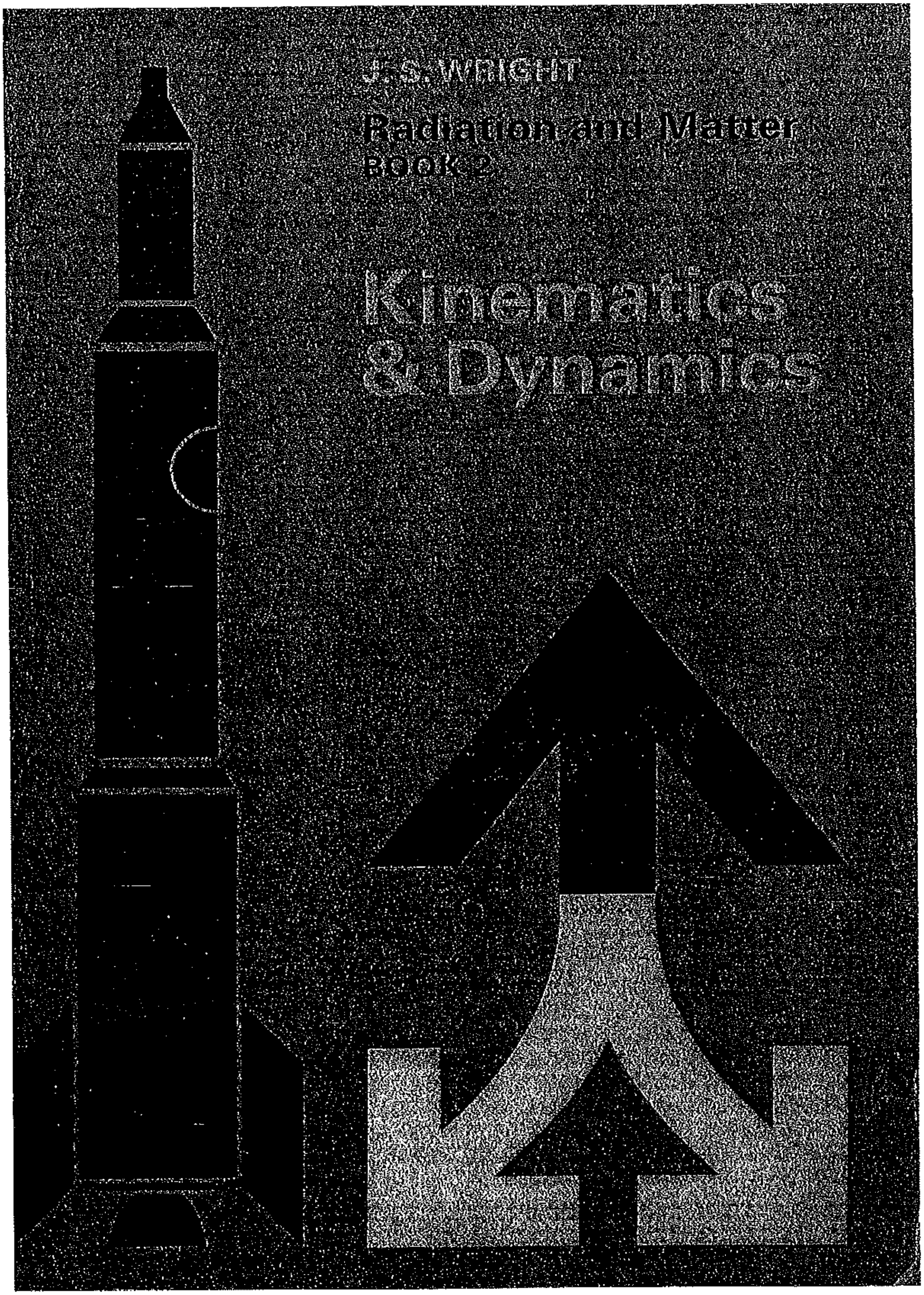
Kinematics & Dynamics

By J. S. Wright



J. S. WRIGHT

Radiation and Matter
BOOK 2

Kinematics
& Dynamics

Chapter 1

Some Basic Ideas

1-1 THE SCOPE OF NEWTONIAN MECHANICS

Early in the second half of the twentieth century, the space age began. Since 1950, successes have been achieved in the field of space travel that were only wild dreams in 1900. We have now arrived at the stage at which, by taking proper precautions, men can travel in space for at least a limited time. And it is quite possible that in the next 50 years men will completely conquer space.

Many of the advances which have been responsible for these achievements have been technological advances. New metals have been discovered which will stand the extreme temperatures encountered, particularly as a satellite re-enters the earth's atmosphere. New communications devices, particularly those involving miniature components, have been devised. New fuels for satellite propulsion have been found. Above all, large amounts of money have been made available for research and development.

Yet the basic laws governing the motions of satellites have been known for at least 250 years; they were first enunciated by Sir Isaac Newton in the 17th century. It is true that these laws have to be modified slightly when we deal with small particles travelling at high speeds, but it is equally true that any description of Physics as we know it today cannot overlook the contributions of Newtonian mechanics.

So this book deals with the laws developed by Newton and others, and develops the ideas necessary to an understanding of the elements of space travel. But the usefulness of Newtonian mechanics does not stop there; the laws of mechanics enable us to understand the motions of objects which we encounter from day to day. Moreover, they enable us to analyse the motions of molecules and atoms and sub-atomic particles.

1–2 THE WORK OF THE PHYSICIST

The physicist is concerned with the discovery of fundamental facts and is not necessarily concerned with applying these facts directly for the service of mankind or for financial gain. Engineers and technologists apply the fundamental knowledge gained by the physicist in building bridges, skyscrapers, automobiles, aircraft, radios, television sets, earth satellites, atomic bombs, etc. Engineers and technologists frequently discover facts on their own, too, and feed these facts back to the physicist. In turn, the physicist may suggest engineering or technological changes. But the main concern of the physicist is with the discovery of fundamental facts, and our concern in this book will be with the discussion of such facts, rather than with an extensive description of their technological applications.

The physicist designs experimental apparatus, performs experiments, assesses experimental data, formulates laws and proposes theories. Each of these activities is important and its role in the over-all process should be understood.

1–3 THE ROLE OF THE LABORATORY

When a physicist sets up an experiment, he usually has a definite goal in mind, and he designs apparatus whose function it is to perform the operations he wishes performed. In making experimental observations, he uses many instruments, some of which are very complex. However, regardless of its complexity, the purpose of any instrument is to extend the experimenter's senses of sight, sound, and touch, and to remove the unreliability which these senses often display. For in the course of the experiment, the physicist, even though he has a goal in mind, must not be influenced by what he hopes will happen.

As a student of Physics you will use the laboratory, and carry out Laboratory Exercises similar to experiments that physicists have done. You will not likely have much part in the designing of the apparatus, but you should have some part in deciding how the apparatus is to be used. You should have some goal or purpose in mind. However, as you perform the experiment, you should not be influenced by this purpose, but should record the results honestly and objectively. Remember, too, that the experiment is not finished when you have recorded the last observation. The data which you have collected must be analysed and interpreted.

1–4 THE ROLE OF MATHEMATICS

In relating, interpreting, and summarizing experimental data, the chief tool of the physicist is mathematics. Physics is a quantitative science involving measurement and calculation, rather than a purely qualitative and descriptive subject. The need for quantitative treatment is very well summarized in the following statement. It is attributed to Lord Kelvin (1827-1907).

"I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be."

The purpose of mathematics is not simply to perform calculations with the numbers resulting from measurement, but also to discover relationships among the quantities involved. In order to interpret the results of the Laboratory Exercises which you will perform, you must be able to recognize such relationships as direct and inverse proportion, either from a table of experimental data, or from the corresponding graph. The physicist is continually searching for relationships such as these, and often for much more complicated relationships. When he finds a relationship which is valid for many sets of experimental data, he formulates a law.

1–5 PHYSICAL LAWS

A physical law is not an instruction that may be obeyed or ignored, as if it were a federal statute. In fact, a physical law is not in any sense responsible for the behaviour of physical objects; all it does is summarize and describe that behaviour. Perhaps we can make the distinction clear by quoting an example.

In Chapter 4 we discuss Newton's second law. This law states, among other things, that the acceleration of an object is proportional to the net force acting on the object. Newton's second law applies to automobiles, airplanes, toboggans, baseballs, tennis balls, lawn mowers—to all objects. But the objects do not behave this way because of the law; rather, experiments have shown that these objects behave in this manner. So the law is simply a summary of experimental facts, a generalization that was possible only after a great deal of experimentation.

We use laws in solving problems, confidently assuming that the laws have been derived from sufficient experimental evidence to ensure their validity in the problem. But there is one danger. Most laws, and the formulas which are the mathematical expressions of these laws, have certain restrictions placed upon them. For example, the formula which expresses Newton's second law is $F = ma$. This formula is easy to learn, but it can be used incorrectly. In order to use it correctly, you must not only know what the symbols F, m, and a stand for, but you must remember that the use of the formula is restricted to cases where F is the net force acting on an object. Moreover, it is valid only for certain units of force, mass, and acceleration.

1–6 UNITS OF MEASUREMENT

Newtonian mechanics has traditionally required a multiplicity of fundamental and derived units. In order to reduce the number of units discussed in this book, we shall use the M.K.S. system of measurement almost exclusively. The M.K.S. system uses the metre, kilogram, and second as units for the fundamental concepts of length, mass, and time. The student should be familiar with these units already; for convenience they are tabulated in the appendix.

The names of units in which derived concepts are measured are combinations of these fundamental units. If, for example, in determining a speed, a distance in metres is divided by a time in seconds, the speed is measured in $\frac{\text{metres}}{\text{seconds}}$, commonly written as metres per second or m/sec. On the other hand, if a quantity of work is calculated by multiplying a force in newtons by a displacement in metres, the work is measured in newtons × metres, commonly written as newton-metres. Moreover, units may be "cancelled" just as numbers are. If metres/sec are multiplied by sec, the result is metres. If newton-metres are divided by newtons, the result is metres.

Some units which could very well be named in terms of fundamental units have abbreviated names. For example, 1 newton-metre is called 1 joule; 1 joule per second is called 1 watt. These examples and others will be discussed in their proper contexts in later chapters.

1–7 HYPOTHESES AND THEORIES

Up to this point we have described the most frequently used elements of scientific procedure. However, there can be useful variations, and even reversals, of the methods outlined.

In Section 1–5 we described how a general law is derived from a large number of experimental observations. This process is called inductive reasoning; it proceeds from the particular to the general. On the other hand, the general law—usually called a hypothesis until it is tested—may be arrived at by what amounts to an intelligent guess. The hypothesis is then tested in particular cases, and if the hypothesis proves correct in a large number of cases, it may become a law. This process of proceeding from the general to the particular is called deductive reasoning. Newton's development of the law of universal gravitation, which we shall discuss in Chapter 6, is an excellent example of the use of deductive reasoning.

At some stage in a series of experiments, perhaps after the law has been enunciated, an attempt is made to explain the observed facts and the general law which describes these facts. That is, a theory is proposed. There are few theories in Mechanics, for the facts and laws seem to be so fundamental as to defy explanation. There is a law of gravity, for example, but no theory as yet to explain it. However, the lack of explanations should not cause us to under-rate the importance of Mechanics. Two all-embracing laws of mechanics—the law of conservation of momentum and the law of conservation of energy—are of fundamental importance to the whole field of Physics.

Chapter 2

Straight Line Kinematics

2–1 INTRODUCTION

Mobility seems to be a prime requirement of twentieth century living. Automobiles travel our highways, airplanes fly through the skies, ships sail the seas, satellites travel through space, and the wheels of industry turn continually. Those who lived in former centuries were concerned with motion too, with the motions of stars and planets in the heavens, with the motions of air masses over the surface of the earth, and, more recently, with the motions of molecules in gases and of electrons in atoms.

Because motion is such a common phenomenon, it is one of the basic concepts of Physics. However, the concept of motion was poorly understood for many centuries, and this lack of understanding hampered the development of many branches of science. Since then, mainly as the result of the work of Galileo Galilei (1564-1642) and Sir Isaac Newton (1642-1727), a system of studying motion has been developed. This system divides the subject into two parts—kinematics and dynamics. Kinematics deals with motion without considering its cause, and dynamics considers both the motion and the forces which affect the motion.

In this chapter we will begin to consider kinematics, that is, a description of motion. We shall confine the discussion to motion along a straight line path.

2–2 AVERAGE SPEED

The average speed for a trip is defined as the total distance travelled divided by the time taken. Suppose that in travelling from Toronto to Windsor the distance of 240 miles is covered in 6 hours. Then the average speed for the entire trip is 40 miles per hour.

Suppose, in another case, that an automobile travelled at a speed of 40 mi/hr for $1\frac{1}{2}$ hours and then reduced speed to 30 mi/hr for the next hour. The distance travelled during the first $1\frac{1}{2}$ hours is 60 miles; the distance travelled during the next hour is 30 miles. The total distance travelled is 90 miles; the total time is $2\frac{1}{2}$ hours. The average speed is thus $90 \div 2\frac{1}{2}$ mi/hr, or 36 mi/hr. Note that the average speed is not the arithmetic average of the two speeds; it is the uniform or constant speed at which the given total distance could be covered in the given time interval.

2–3 MOTION AT CONSTANT SPEED

Automobiles travelling on a street are continually starting, stopping, speeding up, slowing down, ascending or descending hills, and changing direction. Motion at constant (uniform) speed—the type of motion which would occur if the average speed were maintained throughout the trip—occurs rarely but is basic to the understanding of more complicated types of motion.

If the speed of an object is uniform, the object travels equal distances in equal intervals of time. Suppose, for example, that a ground radar station takes a series of readings of the horizontal distance from the station to an aircraft which had previously passed over the station and travelled in a straight line thereafter. The readings might be tabulated as follows:

TIME *(t)*	DISTANCE *(s)*
10.30	20 miles
10.32	25 miles
10.34	30 miles
10.36	35 miles
10.38	40 miles
10.40	45 miles
10.42	50 miles

Examination of these readings indicates that the speed of the aircraft relative to the station is constant at 150 mi/hr. The distance-time graph is shown in Figure 2.1. A study of this graph yields the following information:

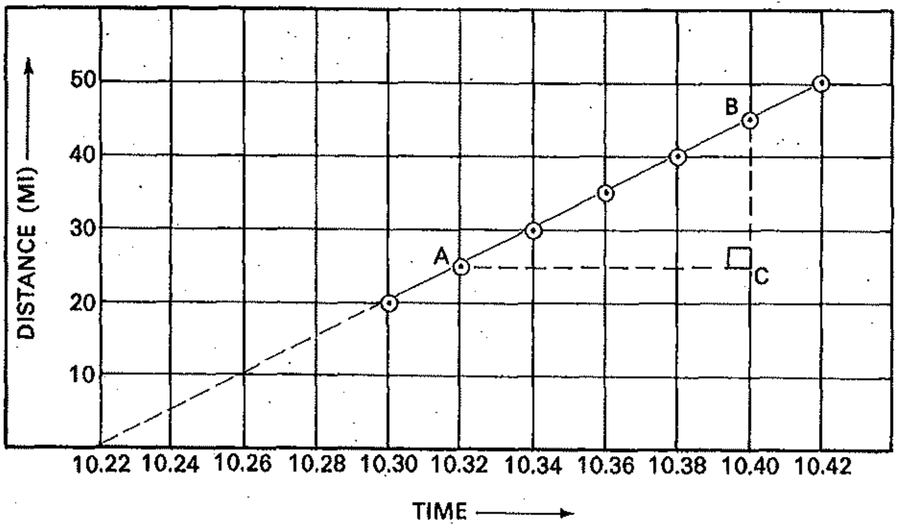


Fig. 2.1. Distance-time graph for constant speed.

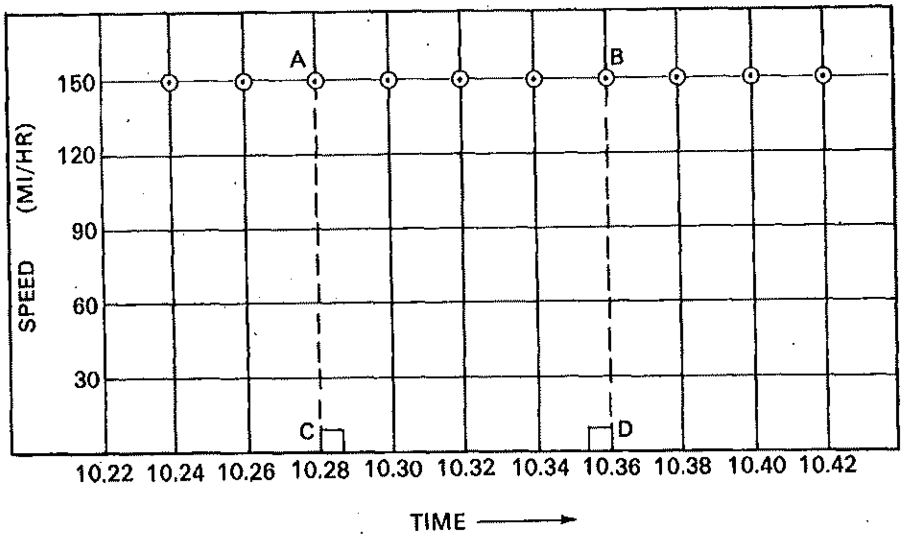


Fig. 2.2. Speed-time graph for constant speed.

(*a*) For uniform speed, the distance-time graph is a straight line.

(*b*) The ratio $BC : AC$, where A and B are any two points on the line and C is the point of intersection of lines drawn through A and B parallel to the axes, is the value of this uniform speed. Note that $BC = \Delta s$, $AC = \Delta t$, and the ratio, $\frac{BC}{AC} = \frac{\Delta s}{\Delta t}$, the slope of the graph. In the case shown, $\Delta s = 20$ mi, and $\Delta t = 8$ min. Therefore the speed is

$$\frac{\Delta s}{\Delta t} = \frac{20}{8} \text{ mi/min} = 150 \text{ mi/hr}$$

(*c*) If the graph is produced to the left, we find by extrapolation that the aircraft passed over the radar station at 10.22. This conclusion is valid if the speed was constant at 150 mi/hr between 10.22 and 10.30.

The speed-time graph is plotted in Figure 2.2. Since the speed is constant, this graph is a straight line parallel to the time axis. If from any two points A and B on this line, perpendiculars are drawn to the time axis, a rectangle $ACDB$ is formed. The area of this rectangle is $CD \times BD$, i.e., the time interval multiplied by the constant speed during that interval. The value of this product is, of course, the distance travelled during the time interval.

We will show later in this chapter that, in general, the area under a speed-time graph is the distance travelled during the time interval. This fact provides a graphical method which is useful for computing distance, particularly in cases in which the speed is not uniform and the graph is not a straight line.

2-4 MEASUREMENT OF UNIFORM SPEED

Motion at uniform speed may be demonstrated in the laboratory with the

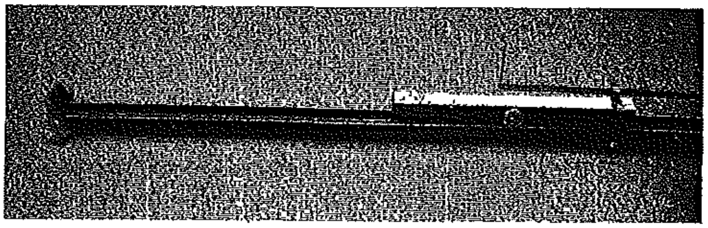


Fig. 2.3. Fletcher's trolley.

Fletcher's trolley apparatus (Fig. 2.3). It consists of a trolley car, about 75 cm long and 8 cm wide, mounted on almost frictionless wheels which run along metal tracks on a rigid metal frame. A strip of spring metal is mounted over the car, and a fine brush is attached to the end of this strip. A strip of paper is fastened on the top of the car, and the brush is adjusted just to touch the surface of the paper. If the brush is inked and the metal strip remains at rest, and if the car is pushed under the brush, the tracing on the paper is a straight line. When the strip is vibrated and the car is put in motion, the inked brush traces a wavy line on the paper. The length of the metal strip can be adjusted to provide different periods of vibration for the brush.

To study uniform speed, one end of the track is raised slightly so that the car will move at uniform speed if once started, but it will not start of its own accord. This adjustment is carried out to make allowance for friction which is unavoidably present. The strip is vibrated, and the car is given a quick push. The tracing on the paper is a uniform wavy line as shown in Figure 2.4. The tracing shows that, when the car moved through a distance *AB* or *BC*, the brush made one complete vibration, and that the distances, *AB*, *BC*, etc., are approximately equal.

The average distance covered during one complete vibration of the brush was 7 cm. The brush vibrated 50 times in 10 seconds. Thus the car travelled 7 cm in $\frac{1}{5}$ sec, and its speed was approximately constant at 35 cm/sec.

2–5 WORKED EXAMPLES

EXAMPLE 1

Figure 2.5 is a speed-time graph for a car, showing its motion during 5 different time intervals *A*, *B*, *C*, *D* and *E*. (*a*) Describe the motion in words. (*b*) Calculate the distance travelled during each time interval, and the total distance. (*c*) Is such a graph likely in practice?

SOLUTION

(*a*) The car travels for 0.10 hr at 15 mi/hr, then for 0.30 hr at 25 mi/hr, for 0.10 hr at 12.5 mi/hr, for 0.50 hr at 30 mi/hr and finally for 0.10 hr at 12.5 mi/hr.

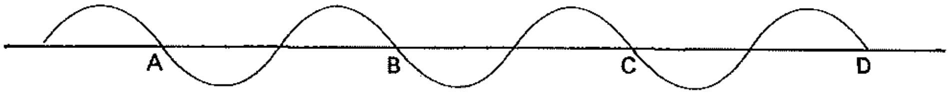


Fig. 2.4. A tracing from a Fletcher's trolley, illustrating uniform speed

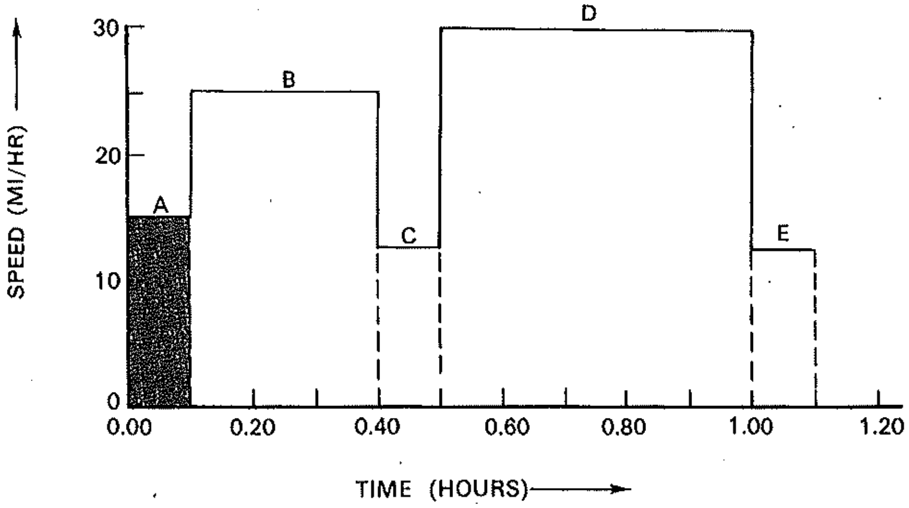


Fig. 2.5. Speed-time record (idealized) of a trip by car.

(*b*) The distance travelled during the time interval *A* may be obtained by multiplying the speed (15 mi/hr) by the time (0.1 hrs) or by finding the area of the shaded rectangle on the graph. (Note that in finding the area from the graph, the length and width of the rectangle must be measured in the units marked on the corresponding axes of the graph.) The distances travelled during intervals *A*, *B*, *C*, *D*, *E* are 1.5 mi, 7.5 mi, 1.25 mi, 15 mi, and 1.25 mi respectively. The total distance is 26.5 mi.

(*c*) Such a graph is unlikely for two reasons. (*i*) The speed is unlikely to remain absolutely uniform for any of the time intervals. (*ii*) The speed cannot possibly change abruptly, for example, from 15 mi/hr to 25 mi/hr. The graph, then, is an idealization of a real situation. Such idealizations are often necessary and frequently useful in physics; they allow us to make a very useful approximation of a complicated real situation.

EXAMPLE 2

Figure 2.6 shows, on the one set of axes, the distance-time graphs for two cars. (*a*) Interpret the graphs in words. (*b*) At what time will car *B* be overtaken by car *A*?

SOLUTION

(*a*) Since the graph for car *B* cuts the distance axis 10 mi above the point where the graph for car *A* cuts this axis, car *B* is 10 mi ahead of car *A* when the timing begins. Since both graphs are straight lines, both cars travel at constant speed. However, since the slope of the graph for car *A* is greater than that for car *B*, car *A* travels faster than car *B* and eventually overtakes car *B*. (The actual speeds of the cars can be found from the slopes of the graph, if desired.)

(*b*) The graphs intersect at time 0.8 hr. This is the time at which car *A* catches up to car *B*. At this time, car *A* has travelled for 20 mi from the start and car *B* for 10 mi.

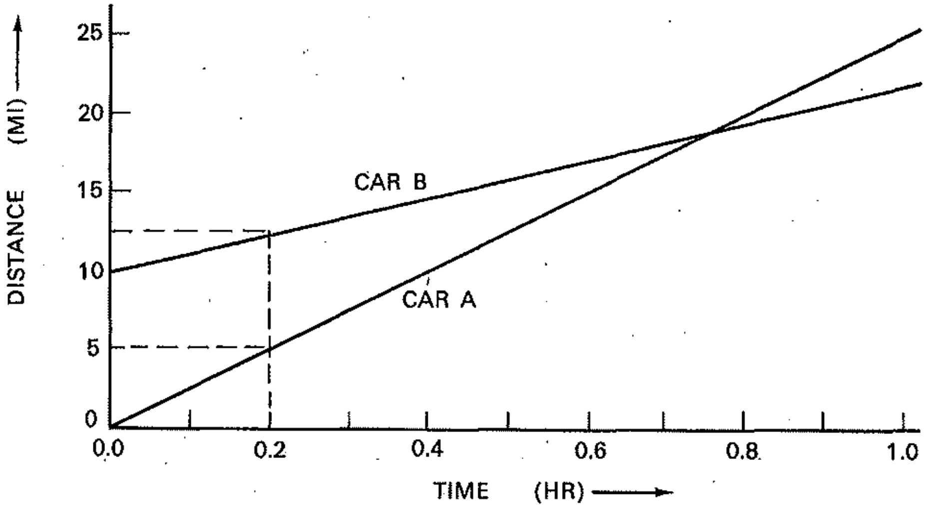


Fig. 2.6. Distance-time graphs for two cars.

2-6 ACCELERATION

It is almost impossible to drive an automobile for a considerable length of time at uniform speed. It is more likely, particularly in city driving, that there will be quick changes in speed, or sudden stops, or quick get-aways. Take, for example, a car moving at a speed of 20 miles per hour; the driver steps on the accelerator and the speed is quickly increased to 30 miles per hour. The speed of the car has been increased by 10 miles per hour; the car has been accelerated.

Suppose that the speed of a car increases from 10 miles per hour to 30 miles per hour in 5 seconds. Assuming that this change takes place uniformly, there has been an increase in speed of 4 miles per hour each second, i.e., the acceleration is 4 miles per hour per second.

Suppose that, in another case, an object moves 5 feet during the first second of its motion from rest, 10 feet during the second second, and 15 feet during the third second of its motion. Its average speeds during these successive seconds are 5, 10, and 15 feet per second respectively. In each second its speed increases 5 feet per second; its acceleration is 5 ft per sec per sec. The first "per sec" is associated with the 5 ft in expressing the increase in speed; the other "per sec" indicates the time required for this increase to take place. The expression ft per sec per sec is frequently written ft/sec^2.

In both of these examples, the acceleration is constant or uniform, and the motion is uniformly accelerated. On the other hand, if a body moves 5 ft in the first second of its motion from rest, 15 ft in the second second, and 30 ft in the third second, the acceleration is variable.

For unidirectional motion, that is, for motion along a straight line path, acceleration may be defined as the rate of change of speed. Acceleration is calculated by dividing the change in speed by the time taken, that is, $a = \frac{\Delta v}{\Delta t}$. If the acceleration is uniform, the speed changes by equal amounts in equal intervals of time; otherwise the acceleration is variable.

2–7 MEASUREMENT OF ACCELERATION

The Fletcher's trolley apparatus may be used to study and measure acceleration. If one end of the track is raised a few inches, the track becomes an inclined plane. The trolley moving down the plane passes under the inked brush, and if the vibrator is put in motion at the same time that the car is released, a tracing such as is shown in Figure 2.7 results. Examination of the tracing shows that BC is greater than AB, CD is greater than BC, etc. The car is accelerating.

The period of vibration of the brush is $\frac{1}{5}$ second. The car moves through each of the following distances AB, BC, CD, DE, etc., during equal, successive intervals of time, that is during $\frac{1}{5}$ second. The distances AB, BC, DC, DE, etc., are measured and found to be 0.97 cm, 1.78 cm, 2.54 cm, 3.28 cm, etc., respectively. When the car is moving from A to B, its speed is increasing. Since it travels 0.97 cm in $\frac{1}{5}$ sec, its average speed in this interval is $0.97 \times 5 = 4.85$ cm/sec. If the speed is increasing uniformly, this average speed will be the speed of the trolley at point (1) between A and B. Similarly, when the car is moving from B to C, its speed is increasing. Since it travels 1.78 cm in $\frac{1}{5}$ sec, its average speed in this interval is $1.78 \times 5 = 8.90$ cm/sec. Again, if the speed of the trolley is increasing uniformly, this average speed will be its speed at point (2) between B and C. Similarly, the speeds at points (3), (4), (5), (6), (7), (8), (9), (10), and (11) of the successive intervals are determined. These speeds are listed in the second column of Figure 2.7.

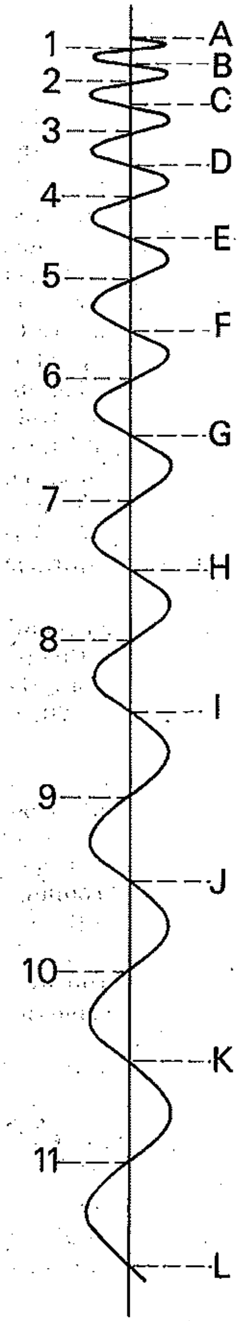
A study of the tracing and of the second column shows that while the car has moved from point (1) of the first interval to point (2) of the second interval, its speed has increased from 4.85 cm/sec to 8.90 cm/sec. The increase in speed is 4.05 cm/sec. Similarly, the further increases in speed are found to be 3.80, 3.70, 3.65, 3.55, 3.85, 3.80, 3.80, 3.85, and 3.80 cm/sec. These increases in speed are the same (within the limits of experimental error), and therefore the car is moving with approximately uniform acceleration.

The increase in speed between points (1) and (2) is 4.05 cm/sec, and this increase occurs in $\frac{1}{5}$ second. Therefore, the acceleration is 4.05×5 cm/sec^2 or 20.25 cm/sec^2.

Similarly, the acceleration for successive intervals from point (2) to (3), from (3) to (4), etc., is determined and found to be 19.00 cm/sec^2, 18.50 cm/sec^2, 18.25 cm/sec^2, etc. (Fig. 2.7, last column). The average of these values for the ten intervals shown on the tracing is 18.9 cm/sec^2. Hence, from the experiment it is concluded that the trolley was moving with approximately uniform acceleration and that the acceleration was 18.9 cm/sec^2.

2–8 DISTANCE-TIME GRAPH FOR UNIFORM ACCELERATION

For the trolley tracing shown in Figure 2.7, the graph of distances from A plotted against the corresponding time intervals is shown in Figure 2.8. Information obtained from a study of this graph is summarized below.



Δs cm	$\frac{\Delta s}{\Delta t}$ cm/sec	Δv cm/sec	$\frac{\Delta v}{\Delta t}$ cm/sec/sec
0.97	4.85		
		4.05	20.25
1.78	8.90		
		3.80	19.00
2.54	12.70		
		3.70	18.50
3.28	16.40		
		3.65	18.25
4.01	20.05		
		3.55	17.75
4.72	23.60		
		3.85	19.25
5.49	27.45		
		3.80	19.00
6.25	31.25		
		3.80	19.00
7.01	35.05		
		3.85	19.25
7.78	38.90		
		3.80	19.00
8.54	42.70		

Average acceleration = 18.9 cm/sec^2
Interval time = 1/5 sec

Fig. 2.7. A trolley tracing, illustrating uniform acceleration.

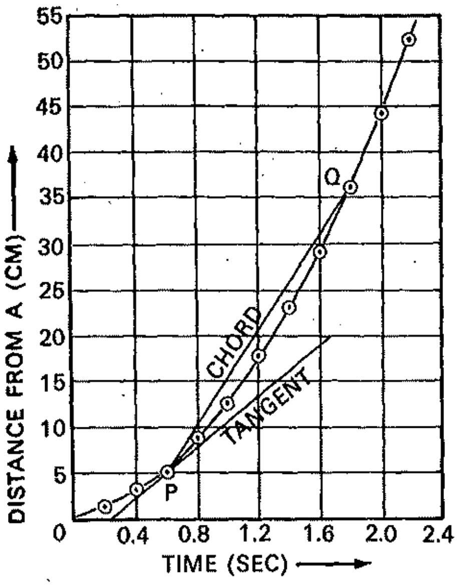


Fig. 2.8. Distance-time graph for uniformly accelerated motion.

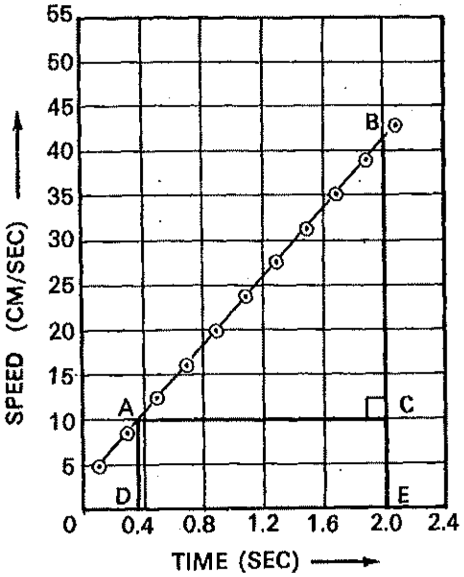


Fig. 2.9. Speed-time graph for uniformly accelerated motion.

(*a*) The distance-time graph for uniform acceleration is curved. The graph is a portion of a curve called a parabola.

(*b*) The slope of the chord joining any two points *P* and *Q* on the curve is the average speed for the time interval involved.

(*c*) If the point *Q* is not close to *P*, the average speed between *P* and *Q* differs considerably from the speed at *P*. However, if the point *Q* is close to *P*, the average speed between *P* and *Q* is very nearly equal to the speed at *P*.

2–9 INSTANTANEOUS SPEED

Instantaneous speed, or speed at a point, may be defined as the average speed over a very short distance which includes the point. In other words, the speed at a point is the value of $\frac{\Delta s}{\Delta t}$ when Δt is very small, i.e., the limit of $\frac{\Delta s}{\Delta t}$ as Δt approaches zero. In symbols

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

As *Q* approaches *P* (Fig. 2.8), Δt approaches zero and the slope of the chord approaches the slope of the tangent at *P*. (You may verify this fact by drawing a small section of the curve near *P* on a large-scale graph.) Thus the speed at *P* may be found by drawing the tangent at *P*, and calculating its slope. In general, an instantaneous speed may be determined from a distance-time graph by drawing the tangent at the appropriate point on the graph. The slope of the tangent is the speed at the point.

Note that uniform speed may now be defined more satisfactorily than was done formerly; speed is uniform if it is the same at all points.

2–10 SPEED-TIME GRAPH FOR UNIFORM ACCELERATION

By drawing a series of tangents at points on the distance-time graph (Fig. 2.8) or by arithmetical calculation similar to that shown in Figure 2.7, a number of instantaneous speeds of the trolley may be determined. The resulting speed-time graph is shown in Figure 2.9. This graph indicates that:

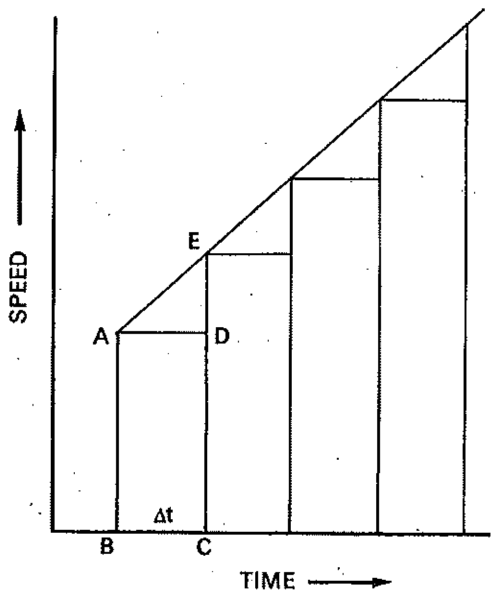


Fig. 2.10. The area of the rectangles in this diagram is slightly less than the area under the graph.

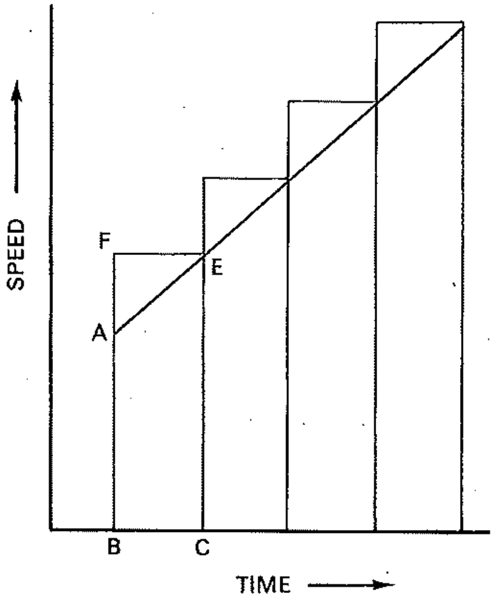


Fig. 2.11. The area of the rectangles in this diagram is slightly more than the area under the graph.

(*a*) The speed-time graph for uniformly accelerated motion is a straight line.

(*b*) The acceleration is obtained by calculating the slope of the graph. In the case shown, the slope of the segment $AB = \frac{BC}{AC} = \frac{30.7 \text{ cm/sec}}{1.64 \text{ sec}} = 18.7 \text{ cm/sec}^2$. (Note also that the slope of the speed-time graph shown in Figure 2.2 is zero, because the acceleration is zero.)

(*c*) The area under the speed-time graph is the distance travelled during the time interval involved. For example, the area of the figure *ADEB* is the distance travelled in time *DE*. If the initial and final speeds *AD* and *BE* are represented by the symbols u and v respectively, if the time *DE* is represented by t, and if the distance travelled is represented by s, then

$$\text{area of } ADEB = s = \left(\frac{u+v}{2}\right)t$$

This fact may not be as obvious for Figure 2.9 as it was for the constant speed graph in Figure 2.2. We may clarify the situation by dividing the area into a series of narrow rectangles and triangles (Fig. 2.10). Suppose that these rectangles are of uniform width Δt. The smallest of these rectangles is labelled *ABCD*; the corresponding triangle is labelled *ADE*. We agree that the area of rectangle *ABCD* is the distance that the object would have travelled if its speed had been equal to its instantaneous speed at *A*. However, the speed increased and the area of rectangle *ABCD* is less than the actual distance travelled during the time Δt. Suppose, then, that we draw our rectangles and triangles as shown in Figure 2.11.

The area of rectangle *FBCE* is the distance the object would have travelled if its speed had been equal to its instantaneous speed at E. Thus the area of rectangle *FBCE* is greater than the actual distance travelled during the time Δt.

As Δt approaches zero, rectangle *ABCD* and rectangle *FBCE* become more nearly equal in area, and triangles *ADE* and *AFE* become less and less significant. The sum of the areas of the rectangles in either case approaches the area under the graph.

Since the distance travelled during time t is the product of the average speed and the time, then the equation $s = \left(\frac{u+v}{2}\right)t$ indicates that the average speed during the time interval is $\frac{u+v}{2}$, i.e., the arithmetical average of the initial and final speeds. This is true only for uniformly accelerated motion.

Further consideration will show that this average speed occurs at the mid-point of the time interval, but not at the mid-point of the distance travelled.

2-11 LABORATORY EXERCISES: CONSTANT SPEED AND CONSTANT ACCELERATION

The Fletcher's trolley, though convenient and accurate, is expensive for student use, and therefore is frequently replaced by less expensive apparatus. A "dynamics cart" with roller skate wheels (Fig. 2.12) replaces the car. A paper tape is attached to the cart, and, as the cart moves, it pulls the tape through a recording timer (Fig. 2.13). The clapper of the timer vibrates, striking a piece of carbon paper above the tape. The resulting series of dots on the tape constitutes a record of the motion of the cart.

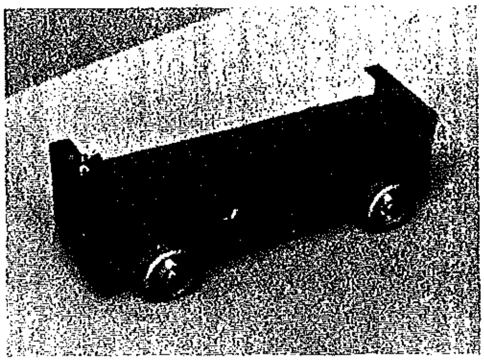


Fig. 2.12. A dynamics cart.

A sheet of $\frac{3}{4}$ inch plywood, 6 to 8 feet in length and $1\frac{1}{2}$ to 2 feet wide, forms a suitable track on which to run the cart. The complete arrangement is shown in Figure 2.14. The track shown in this photograph has plywood sides, the purpose of which is to make the track less flexible and less likely to warp.

(*a*) Elevate the end of the track to which the timer is attached, so that the cart, once started, will run at what you judge to be constant speed. Thread the tape through the timer and attach the end of the tape to the cart. Start the timer, and give the cart a push. Stop the

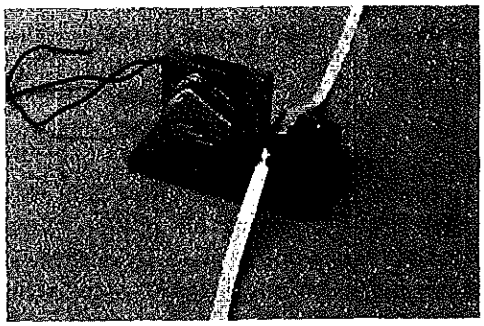


Fig. 2.13. A recording timer.

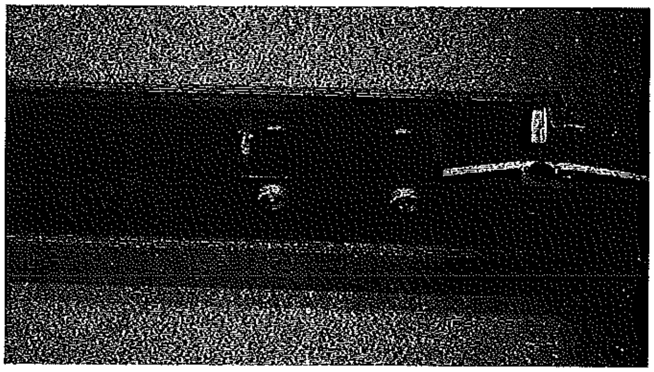


Fig. 2.14. This arrangement of apparatus may be used to record the motion of the cart.

timer when the cart reaches the end of the track. Examine the tape. Does the positioning of the dots on the tape indicate that the speed was constant? Check by measuring the distances between successive dots over the full length of the tape. You may find these distances unequal, because the frequency of the timer may not have been constant. The error due to variation of timer frequency may be reduced as follows. Measure the distances in five-interval groups, i.e., from the first dot to the sixth dot, from the sixth dot to the eleventh, etc. Are these larger distances equal? Was the speed constant?

In order to calculate the speed, you need to settle on a time unit to use. This time unit need not be one second; it can be the period of the timer (1 tick) or the time associated with each of the larger distances mentioned above. We will call this larger time unit 1 tock. Obviously, 1 tock = 5 ticks.

Plot the distance-time graph and the speed-time graph for this motion. You may get the required data by measurement and calculation from the tape, or you may cut the tape up into "one-tock intervals". These smaller pieces of tape are then glued on a graph as shown in Figure 2.15(*a*) and (*b*). You should satisfy yourself that the methods shown are correct. Note that the area under the speed-time graph in Figure 2.15(*b*) is the complete length of the tape, i.e., the distance travelled by the cart.

(*b*) Elevate the end of the track still further, and repeat the procedure outlined in (*a*) above. Let the cart accelerate from rest. Calculate the acceleration from a table similar to that in Figure 2.7. Is the acceleration uniform? What is the average acceleration? Plot the distance-time and speed-time graphs. From the speed-time graph, what values do you obtain for the acceleration, and for the distance travelled?

2–12 EQUATIONS INVOLVING SPEED, ACCELERATION, TIME AND DISTANCE

Consider an object which accelerates from an initial speed u to a final speed v in time t. Since the acceleration a is computed by dividing the change in speed by the time, then

$$a = \frac{v - u}{t}$$

$$\text{or } v = u + at \ldots\ldots (1)$$

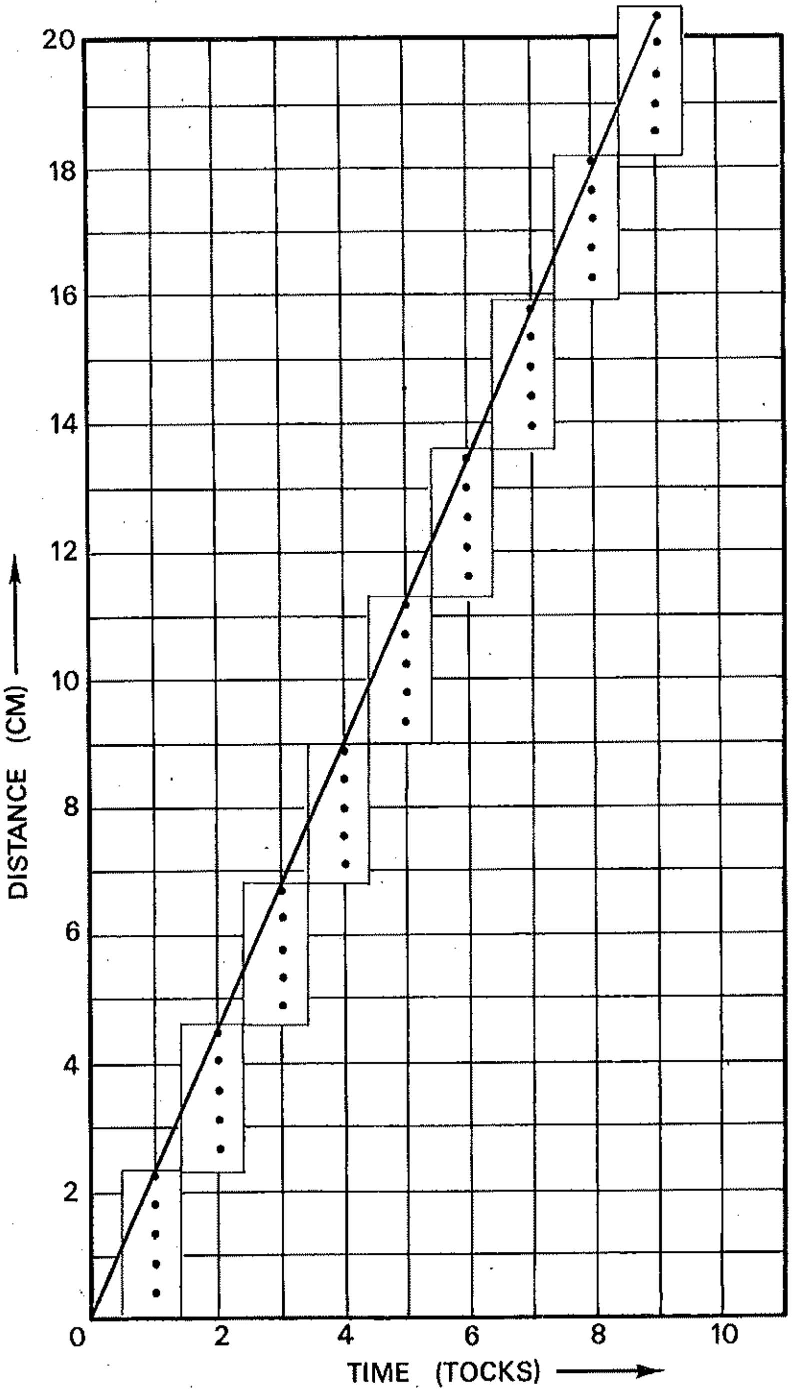


Fig. 2.15(a). Distance-time graph constructed from recording timer tape.

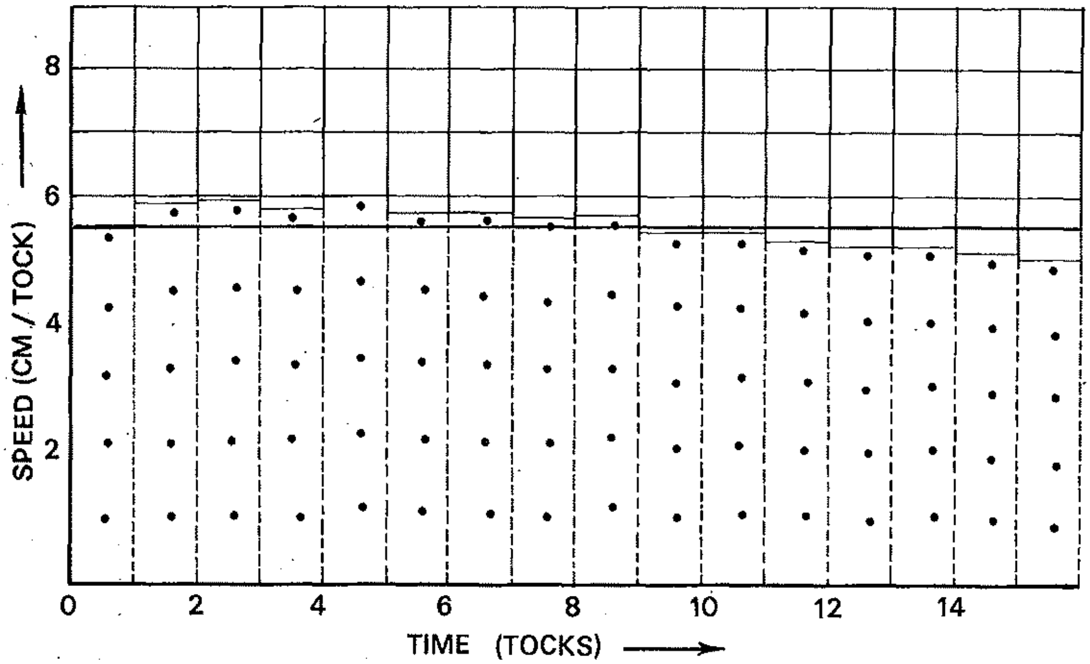


Fig. 2.15(b). Speed-time graph constructed from recording timer tape. The graph is shown as a straight line, on the assumption that the timer frequency was not constant.

If the object is initially at rest ($u = 0$), then $v = at$.

The formula

$$s = \left(\frac{u+v}{2}\right)t\ldots\ldots(2)$$

was developed from the graph in Figure 2.9.

If the value for v in equation (1) be substituted in equation (2),

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$s = ut + \tfrac{1}{2}at^2\ldots\ldots(3)$$

From equation (1)

$$t = \frac{v-u}{a}$$

and substituting this value in equation (2):

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$v^2 = u^2 + 2as\ldots\ldots(4)$$

Elimination of u from equations (1) and (2) yields the formula

$$s = vt - \tfrac{1}{2}at^2\ldots\ldots(5)$$

The five equations enumerated above are very useful in solving problems involving speed, acceleration, time, and distance, and they should be memorized. The following restrictions on their use should be kept in mind. The value of a obtained by substituting values of v, u, and t in (1) is the uniform acceleration if the motion is uniformly accelerated, and the average acceleration if the acceleration is not uniform. Thus equation (1) may be used whether the acceleration is uniform or not. However, equation (2) is valid only if the acceleration is uniform, and therefore equation (2) and equations (3), (4), and (5), which are derived from it, may be used only in cases of uniformly accelerated motion.

Some examples of problems that can be solved using these equations follow.

2–13 WORKED EXAMPLES

EXAMPLE 1

An object moving with uniform acceleration changes its speed from 5 cm per sec to 50 cm per sec in 5 seconds. Find the acceleration.

SOLUTION

$$\begin{aligned} u &= 5 \text{ cm/sec} \\ v &= 50 \text{ cm/sec} \\ t &= 5 \text{ sec} \\ a &= ? \end{aligned}$$

The only equation involving u, v, t, and a is

$$v = u + at$$

Substituting: $50 = 5 + a \times 5$

$$\therefore a = 9$$

The acceleration is 9 cm per sec per sec.

EXAMPLE 2

An object travelling with a speed of 50 cm per sec is moving with a negative acceleration of 10 cm per sec per sec. (*a*) When will it come to rest? (*b*) Where will it come to rest?

SOLUTION

At the beginning of the interval the speed is 50 cm per sec, and at the end of this interval the object is at rest, therefore:

(*a*)
$$\begin{aligned} u &= 50 \text{ cm/sec} \\ v &= 0 \text{ cm/sec} \\ a &= -10 \text{ cm/sec}^2 \\ t &= ? \end{aligned}$$

Equation (1) is selected.

$$\begin{aligned} v &= u + at \\ 0 &= 50 + (-10 \times t) \\ \therefore t &= 5 \end{aligned}$$

It will come to rest in 5 seconds.

(*b*)
$$\begin{aligned} u &= 50 \text{ cm/sec} \\ v &= 0 \text{ cm/sec} \\ a &= -10 \text{ cm/sec}^2 \\ s &= ? \end{aligned}$$

Equation (4) is selected.

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= 2500 - 20s \\ \therefore s &= 125 \end{aligned}$$

Therefore the object will travel 125 cm before coming to rest.

EXAMPLE 3

A car is moving with a uniform acceleration of 6 ft per sec per sec. How long, after attaining a speed of 42 ft per sec, will it take to travel 1440 feet?

SOLUTION

$$\begin{aligned} u &= 42 \text{ ft/sec} \\ a &= 6 \text{ ft/sec}^2 \\ s &= 1440 \text{ ft} \\ t &= ? \end{aligned}$$

Equation (3) is selected.

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ 1440 &= 42t + 3t^2 \\ 3t^2 + 42t - 1440 &= 0 \\ t^2 + 14t - 480 &= 0 \\ (t + 30)(t - 16) &= 0 \\ \therefore t &= 16 \text{ or } t = -30 \end{aligned}$$

It will take 16 seconds to travel 1440 feet. The value $t = -30$ is inadmissible.

2–14 PROBLEMS

1. A car is driven at a speed of 72 km/hr for 0.50 hr, 80 km/hr for 0.25 hr, and 58 km/hr for 0.50 hr. (*a*) Calculate its average speed for the trip. (*b*) Draw the distance-time graph and the speed-time graph for the trip. (*c*) Draw the distance-time and speed-time graphs for a trip of the same duration, at the average speed calculated in (*a*).
2. Figure 2.16 is an idealized speed-time graph for a hitchhiker's trip along a country road. He travelled first on foot, then by car, then by tractor, and then in another car. (*a*) Calculate (*i*) the total distance travelled, (*ii*) the average speed for the trip. (*b*) Draw the distance-time graph for the trip.
3. The period of vibration of the brush on a Fletcher's trolley is 0.22 sec. Successive wave lengths on a tracing measured 4.6, 4.6, 4.5, 4.4, 4.5, and 4.6 cm. (*a*) Is the speed of the trolley uniform? (*b*) Calculate the average speed in each 0.22 second interval and the average speed for the 6 intervals. (*c*) Plot the distance-time graph and from the graph determine the average speed. (*d*) Plot the speed-time graph.

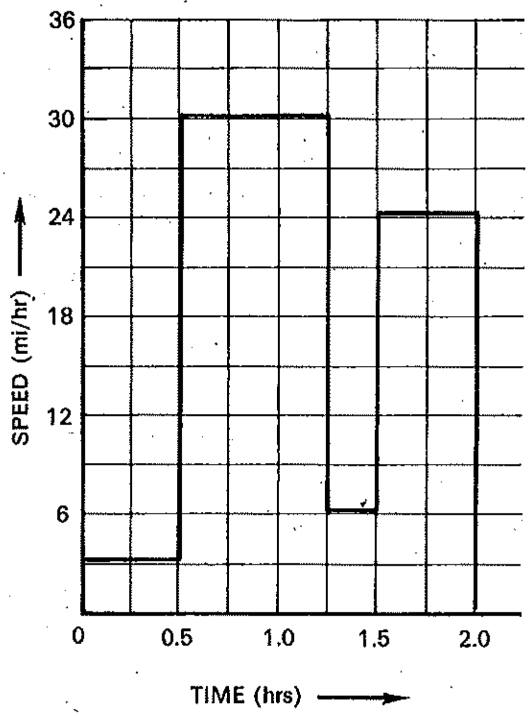


Fig. 2.16. For problem 2.

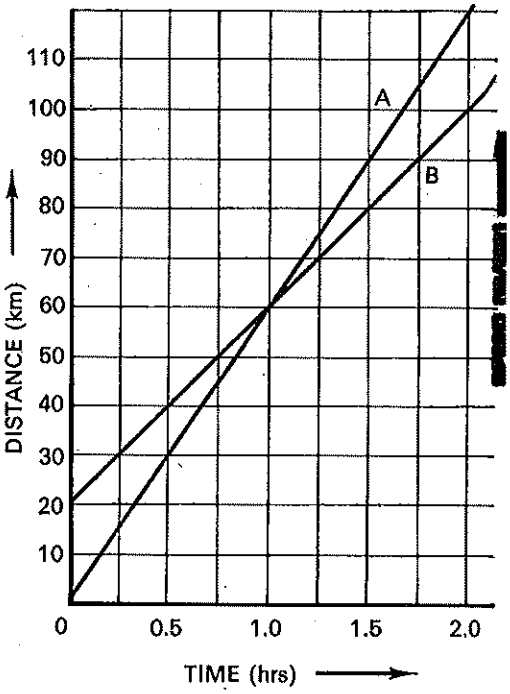
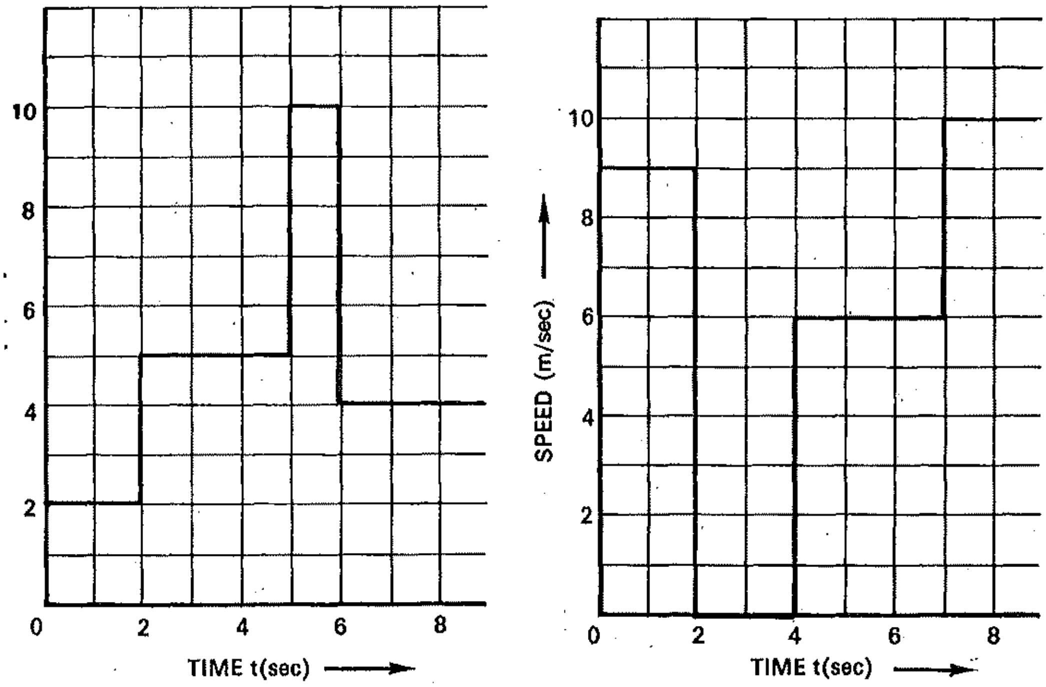


Fig. 2.17. For problem 4.

4. Figure 2.17 shows the distance-time graph for two cars. (*a*) What is the speed of (*i*) car *A*, (*ii*) car *B*? (*b*) When is car *A* (*i*) 10 ~~miles~~ km behind *B*, (*ii*) 10 ~~miles~~ km ahead of *B*? (*c*) When does *A* overtake *B*? (*d*) What distance does (*i*) *A*, (*ii*) *B*, travel in 2.0 hr? (*e*) Draw the corresponding speed-time graphs.
5. For each of the 2 graphs in Figure 2.18, (*i*) calculate the distance travelled between $t = 3$ sec and $t = 7$ sec, (*ii*) calculate the average speed between $t = 3$ sec and $t = 7$ sec, (*iii*) draw the corresponding distance-time graphs.
6. Assume that the speed of light in air is 3.0×10^8 m/sec, and that the index of refraction for light passing from air to glass is 1.5. Draw (*a*) the distance-time graph, (*b*) the speed-time graph, for light traversing a path consisting of 60 cm of air followed by 30 cm of glass.
7. A ball rolling down an incline travels 6 cm in the first 0.25 sec and 24 cm in the first 0.50 sec. Find its average speed in each quarter-second interval, and its acceleration.



(a) (b)

Fig. 2.18. For problem 5.

8. An aircraft, on take-off, starts from rest. Its speed at ten-second intervals thereafter is 10 km/hr, 25 km/hr, 45 km/hr, 70 km/hr, 100 km/hr, and 135 km/hr. (*a*) Calculate, in km/hr/sec, its average acceleration in each ten-second interval. (*b*) Draw the speed-time graph. From the graph, estimate its acceleration at $t = 25$ sec.

9. A ball rolling down a ramp travels 1 metre in the first second, 3 metres in the second second, 5 metres in the third second, and 7 metres in the fourth second. (*a*) Calculate its acceleration. (*b*) Plot the distance-time and speed-time graphs for its motion. Check the accuracy of your graphs by determining from them (*i*) the total distance travelled, and (*ii*) the acceleration.

10. A tracing from a Fletcher trolley experiment revealed the following information. From a position A the trolley moved to B, a distance of 2.1 cm; this distance was traversed during one vibration of the brush. During successive vibrations it moved to C, D, E, and F where $AC = 7.1$ cm, $AD = 15.2$ cm, $AE = 26.4$ cm, and $AF = 40.5$ cm. The brush completed 20 vibrations in 4 seconds. With the aid of a table in which the columns bear proper headings, determine, correct to one place of decimals, the average acceleration of the trolley in cm/sec^2.

11. Using the data given in Question 10, draw both the distance-time graph and the speed-time graph. From the latter graph calculate the average acceleration.

12. A tracing from a Fletcher trolley revealed the following information. From a position A the trolley moved to B a distance of 2.0 cm, during one vibration of the brush. During successive vibrations it moved to C, D, E, F, and G where $AC = 7.1$ cm, $AD = 15.3$ cm, $AE = 26.7$ cm, $AF = 41.3$ cm, and $AG = 58.9$ cm. The period of vibration of the brush was $\frac{1}{8}$ of a second. (*a*) Draw a graph illustrating the motion, plotting distance against time. State the kind of motion represented by the given data. (*b*) From the graph determine the approximate speed of the trolley at the time when the trolley is 35.0 cm beyond A. (*c*) With the aid of a table in which the columns bear proper headings, determine, correct to one place of decimals, the average acceleration of the trolley in cm/sec^2.

13. An object initially moving at 10 m/sec accelerates uniformly. In the next three one-second intervals it travels 12, 16, and 20 m, respectively. Draw the speed-time graph and determine the acceleration of the object.

14. For the speed-time graph shown in Figure 2.19, calculate the distance travelled in 2.0 sec.

15. (*a*) From the distance-time graph in Figure 2.20, determine (*i*) the average speed in each of the four seconds, (*ii*) the acceleration, (*iii*) the speed at $t = 2.5$ sec. (*b*) Draw the speed-time graph and determine from it the distance travelled in 4 sec. Check your answer by referring to Figure 2.20.

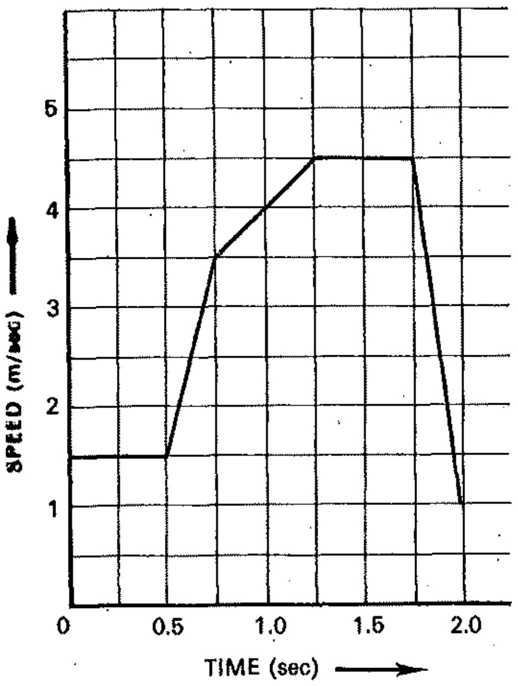


Fig. 2.19. For problem 14.

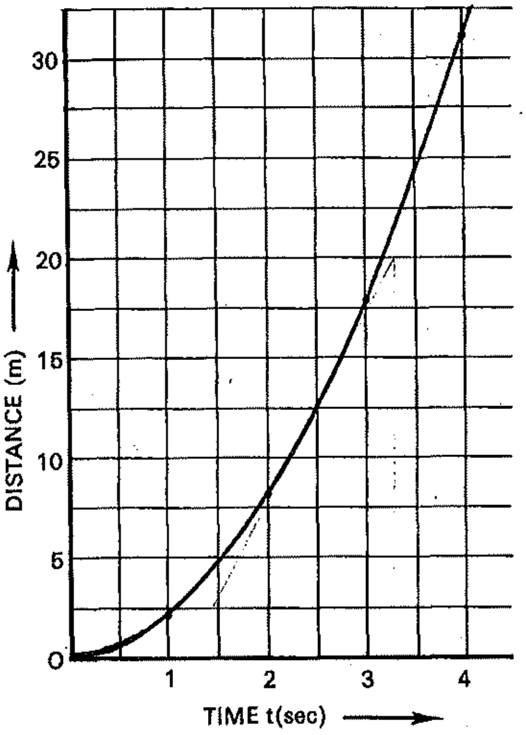


Fig. 2.20. For problem 15.

16. Consider the relationship $\Delta v = a\,\Delta t$. What is the effect on Δv of (*a*) doubling Δt, (*b*) tripling a?

17. For the relationship $s = \frac{1}{2}at^2$, what is the effect on s of (*a*) changing t by a factor of 3, (*b*) changing a by a factor of 0.7?

18. Consider the relationship $v^2 = 2as$. What is the effect on v of (*a*) changing s by a factor of 4, (*b*) changing a by a factor of 3?

19. What is the average acceleration of a baseball which, starting from rest, rolls 50 m down a hill in 10 sec? Find its speed at the end of the 10th sec.

20. A yard engine shunts a freight car along a level siding. If the car stops in 50 seconds, 250 m from the point where it was released, calculate the speed of the engine at the instant the car was released.

21. An object moves for 3 sec with constant acceleration, during which time it travels 81 m. The acceleration then ceases and during the next 3 seconds it travels 72 m. Find its initial speed and its acceleration.

22. An object has an initial speed of 4 m/sec and a uniform acceleration of 2 m/sec^2. How far does it travel in 10 sec?
23. A skier starts down a slope 0.5 km long at a speed of 4 m/sec. If he accelerates at a constant rate of 2 m/sec^2, find his speed at the bottom of the slope.
24. A cyclist moving with a uniform speed of 6 m/sec passes a motor car that is just starting. If the motor car has a uniform acceleration of 2 m/sec^2, when and where will the car overtake the cyclist? Check your algebraic solution by means of a graphical solution.
25. A car moving with uniform acceleration travels 65 m in the tenth second of observation and 95 m in the fifteenth second. Calculate the acceleration and the initial speed.

2–15 SUMMARY

1. Average speed

$$= \frac{\text{total distance travelled}}{\text{elapsed time}}$$

2. If speed is uniform,
   (*a*) equal distances are travelled in equal intervals of time,
   (*b*) the distance-time graph is a straight line,
   (*c*) the slope of the distance-time graph is equal to the constant speed,
   (*d*) the speed-time graph is a straight line parallel to the time-axis,
   (*e*) the area under the speed-time graph is equal to the distance travelled.
3. For unidirectional motion, acceleration

$$= \frac{\text{change in speed}}{\text{elapsed time}} = \frac{\Delta v}{\Delta t}.$$

4. For uniformly accelerated motion,
   (*a*) equal changes in speed occur in equal time-intervals,
   (*b*) the distance-time graph is parabolic,
   (*c*) the slope of a chord of the distance-time graph is equal to the average speed for the time-interval,
   (*d*) the slope of the tangent at a point on the distance-time graph is equal to the instantaneous speed at that time,
   (*e*) the speed-time graph is a straight line,
   (*f*) the slope of the speed-time graph is equal to the acceleration,
   (*g*) the area under the speed-time graph is equal to the distance travelled,
   (*h*) the following formulae may be used to solve problems, if and only if the motion is uniformly accelerated:

$$\begin{aligned} v &= u + at \\ s &= \left(\frac{u+v}{2}\right)t \\ v^2 &= u^2 + 2as \\ s &= ut + \tfrac{1}{2}at^2 \\ s &= vt - \tfrac{1}{2}at^2 \end{aligned}$$

Chapter 3

Vectors and Vector Kinematics

3–1 INTRODUCTION

In Chapter 2 we considered the motions of several different objects each of which moved along a straight line path. We did not at any time mention the position of that path relative to other objects, or the direction of that path. Frequently, however, the position and direction of a path are important. For example, suppose that a plane is to make a trip of 200 miles. The position of the path is certainly important; it is hardly likely that the pilot will choose a path 6 inches above the ground. And the direction is important too, if the pilot hopes to arrive at the proper destination. When we take these factors into consideration, we are led to a discussion of relative motion and vectors.

3–2 RELATIVE MOTION

Suppose that a traveller, before leaving home, puts his suitcase in the trunk of his car. He travels 100 miles, stops, opens the trunk, and finds the suitcase still there. Has the suitcase moved? With respect to the floor of the trunk it has moved very little, if at all; with respect to the owner's home, it has moved 100 miles.

This example illustrates the principle that the position of an object and the motion of that object are, consciously or unconsciously, considered with reference to the position of some other object. In general, one point is said to be in motion with respect to another point when the line joining the two points changes in length or direction. Thus, a passenger seated in a moving train is not moving with respect to his seat. However, he is moving with respect to the ground, for the line joining him to a point on the ground is changing in length. Two children on a moving merry-go-round are moving relative to each other, because the line joining them is changing in direction. Similarly, points on opposite wing tips of an aircraft are moving with respect to one another when the aircraft turns or is tilted, because the line joining them is changing in direction.

3–3 DISPLACEMENT AND DISTANCE TRAVELLED

Although a trip in an automobile by road from Meaford to Midland covers a distance of about 70 miles, the actual distance in a straight line across country is only about 36 miles. Seventy miles is the distance travelled by the automobile. Thirty-six miles is the magnitude of the displacement of the automobile. The direction of displacement is from Meaford to Midland.

Displacement, rather than distance travelled, is the important factor in most cases of motion. Distance travelled, or path length, is an example of a scalar quantity—a quantity having magnitude only. Displacement, on the other hand, is an example of a vector quantity—a quantity having direction as well as magnitude. Further examples of scalar and vector quantities will be discussed in this and later chapters.

A displacement may be represented by a directed line segment. The length of the line indicates the magnitude of the displacement; the direction in which the line is drawn is the direction from the initial to the final position of the object, and is indicated by an arrowhead on the line segment.

3–4 RESULTANT DISPLACEMENT

Suppose that at a given instant an object is at position B (Fig. 3.1) and that later it moves to position C. The object has been displaced, and the amount of the displacement and the direction of the displacement are represented by the directed line segment $\overrightarrow{BC}$, which is called a displacement vector. The arrow above the letters BC indicates that we are dealing with a vector, rather than with a scalar quantity. Later the object moves from C to D, so that the directed line segment $\overrightarrow{CD}$ represents a further displacement. In each case, the length of the line from the initial point to the arrowhead represents the magnitude of the displacement, and the direction of the line on the paper represents the direction of the displacement.

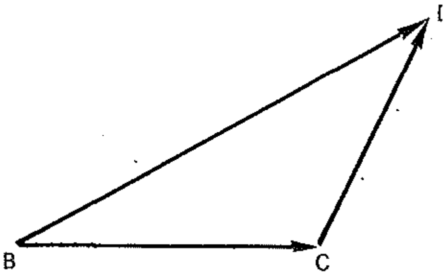


Fig. 3.1. $\overrightarrow{BC}$ and $\overrightarrow{CD}$ represent successive displacements; $\overrightarrow{BD}$ is their resultant.

Now join BD to complete the triangle BCD in Figure 3.1. The net effect or resultant of the two displacements of the object is represented in magnitude by the line BD, and in direction by the arrowhead on BD pointing toward D. That is, $\overrightarrow{BD}$ is the resultant of $\overrightarrow{BC}$ and $\overrightarrow{CD}$. This construction for finding the resultant of two vectors is called the vector triangle.

The resultant of two displacements can be found in another way. The two vectors, for example $\overrightarrow{BC}$ and $\overrightarrow{BE}$ in Figure 3.2, are drawn from a common point B. A parallelogram is then drawn with these vectors as sides. The diagonal $\overrightarrow{BD}$ is the resultant of $\overrightarrow{BC}$ and $\overrightarrow{BE}$. This method for finding the resultant of two vectors is called the vector parallelogram.

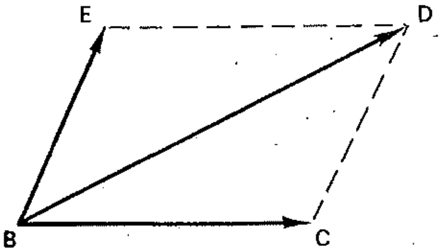


Fig. 3.2. The parallelogram of displacements. $\overrightarrow{BD}$ is the resultant of $\overrightarrow{BC}$ and $\overrightarrow{BE}$.

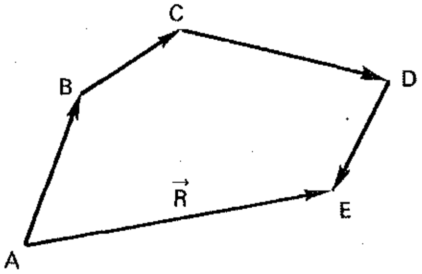


Fig. 3.3. The polygon of displacements. $\vec{R}$ is the resultant of four successive displacements.

The resultant of more than two displacements is found by the method shown in Figure 3.3. $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, and $\overrightarrow{DE}$ are vectors representing successive displacements of an object. The vector $\overrightarrow{AE}$, formed by joining the foot of the first vector to the head of the last vector, represents the magnitude and direction of the resultant. This construction for finding the resultant of more than two vectors is called the vector polygon. $\overrightarrow{AE}$ is the resultant of $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, and $\overrightarrow{DE}$.

3–5 ADDITION OF VECTORS

Finding the resultant of several vectors is called vector addition. In spite of its name, vector addition may differ radically from ordinary addition. Let us consider several cases:

(*a*) If an object undergoes successive displacements of 3 ft, 7 ft, 6 ft, and 4 ft, all in the same direction, the resultant is obviously a displacement of 20 ft in that direction. Thus the resultant of displacements in the same direction is obtained by simple addition; in this case vector addition is the same as arithmetic addition.

(*b*) If an object undergoes successive displacements of 3 ft east, 7 ft west, 6 ft east and 4 ft west, the resultant is obviously a displacement of 2 ft west. If we assign a plus sign to vectors directed east, and a minus sign to vectors directed west, the resultant is the sum of +3 ft, −7 ft, +6 ft, and −4 ft, that is, −2 ft, or 2 ft west. Apparently, then, the resultant of several vectors, some of which have one direction and others of which have exactly the opposite direction, can be found by algebraic addition as for positive and negative numbers, after assigning a positive sign to one of the directions, and a negative sign to the other.

(*c*) In all other cases, vector addition differs completely from addition of numbers, since plus and minus signs can be applied only to directions which are exactly opposite. The triangle, parallelogram or polygon method for finding the resultant may be used. In certain cases the magnitude and direction of the resultant can be calculated mathematically; these calculations will be discussed after we consider subtraction of vectors.

3–6 SUBTRACTION OF VECTORS

In order to subtract 3 from 7, we ask ourselves the question: What number must be added to 3 to give 7? That is,

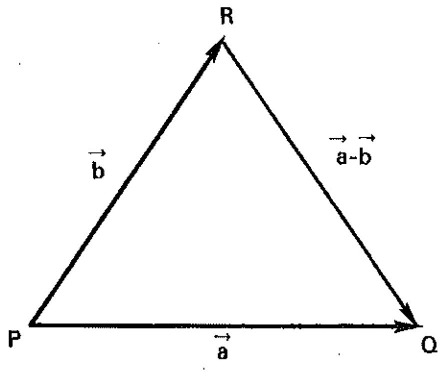


Fig. 3.4. Vector subtraction. $\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR}$.

to evaluate the difference 7 − 3, we determine what number must be added to the second number (3) to give the first number (7). We may follow the same procedure in finding the value of $\vec{a} - \vec{b}$, the difference between two vectors $\vec{a}$ and $\vec{b}$. Place the feet of the two vectors together (Fig. 3.4), thus forming two sides PQ and PR of the triangle PQR. The vector which must be added to $\overrightarrow{PR}$ in order to produce a resultant $\overrightarrow{PQ}$ is obviously $\overrightarrow{RQ}$. That is, $\overrightarrow{RQ} = \vec{a} - \vec{b}$. Similar reasoning shows that $\overrightarrow{QR} = \vec{b} - \vec{a}$.

3–7 CALCULATION OF RESULTANTS

The method of calculating the resultant of displacement vectors in two special cases is outlined below.

(*a*) Suppose that we wish to calculate the resultant of displacements of 3 ft east and 4 ft north (Fig. 3.5). Since B is a right angle,

$$AC^2 = AB^2 + BC^2 = 9 + 16 = 25$$
$$AC = 5 \text{ ft.}$$

Also, $\angle A$ is such that $tan\ A = \frac{4}{3} = 1.33$

$$\therefore\ A = 53.1^\circ \text{ approximately.}$$

Thus the resultant $\overrightarrow{AC}$ is 5 ft in the direction 53.1° north of east.

(*b*) Suppose that we wish to calculate the resultant of displacements of 6 ft east and 5 ft northwest. We begin by sketching a diagram like the accurate diagram drawn in Figure 3.6. The magnitude r of the resultant may be calculated from the trigonometric relationship

$$r^2 = p^2 + q^2 - 2pq\ cos\ R$$

Here, $p = 6$, $q = 5$, $R = 45^\circ$ and $cos\ R = 0.71$. Hence $r = 4.2$ approximately. If R is obtuse, its cosine is negative and equal in magnitude to $cos\ (180^\circ - R)$.

Angle Q may be calculated from the trigonometric relationship

$$\frac{sin\ Q}{q} = \frac{sin\ R}{r}$$
$$\therefore\ sin\ Q = \frac{q\ sin\ R}{r}$$
$$= \frac{5\ sin\ 45^\circ}{4.2}$$
$$= \frac{5 \times 0.71}{4.2}$$
$$= 0.845$$
$$\therefore\ \angle Q = 57.7^\circ \text{ approximately.}$$

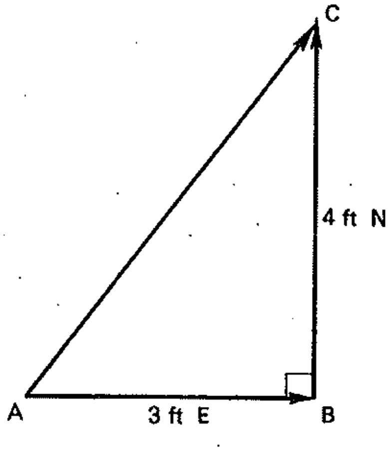


Fig. 3.5. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$.

Thus the resultant $\overrightarrow{QP} = 4.2$ ft in the direction 57.7° north of east.

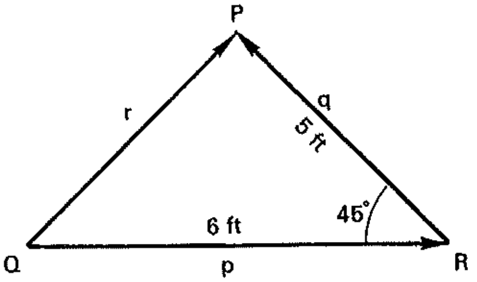


Fig. 3.6. $\vec{r} = \vec{p} + \vec{q}$.

3-8 COMPONENTS OF A VECTOR

Two specific vectors can have only one resultant, but any vector may be the resultant of any pair of an infinite number of pairs of vectors. Each member of each pair is called a component of the original vector. If both the magnitude and direction of one component are given, the magnitude and direction of the other component may be found; if the directions of both components are given, the magnitudes of both components may be found. (You should verify these facts by experimenting with a vector parallelogram or triangle.)

The most useful and most often used components of a vector are those which are perpendicular to each other. Suppose, for example, that we wish to find the horizontal and vertical components of a vector $\vec{R}$ directed at an angle of θ to the horizontal. We may resolve this vector into horizontal and vertical components $\vec{H}$ and $\vec{V}$ (Fig. 3.7) by drawing on R a rectangle $ABCD$ having horizontal and vertical sides. Noting that $CB = AD$, three facts are at once apparent:

$$(1) \quad R^2 = H^2 + V^2$$

$$(2) \quad \sin \theta = \frac{CB}{AC}$$

$$\therefore V = R \sin \theta$$

$$(3) \quad \cos \theta = \frac{AB}{AC}$$

$$\therefore H = R \cos \theta$$

Note also that if $\theta = 90°$, $\sin \theta = 1$ and $\cos \theta = 0$, and as a result $H = 0$ and $V = R$. In general, a vector has its full effect in its own direction, and no effect or component in a direction at right angles to itself.

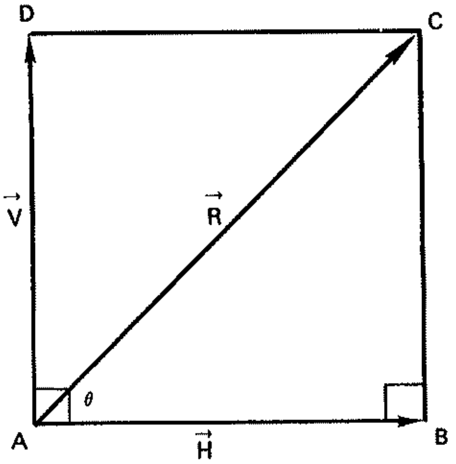


Fig. 3.7. H and V are the horizontal and vertical components, respectively, of R.

3-9 VELOCITY

Often, when there is occasion to consider the vector displacement of an object, there is also occasion to consider the length of time during which this displacement takes place. The quotient obtained by dividing the displacement by the time taken is called the velocity of the object. Like displacement, velocity is a vector quantity.

The average velocity for a trip is defined as the resultant (net) displacement divided by the time taken. Suppose an automobile sets out from point A and travels by a circuitous route to a point B, 30 miles north of A. If the trip takes 5 hours, the average velocity for the trip is 6 mi/hr north. The average velocity is the uniform or constant velocity at which the given displacement would occur in the given time interval.

The facts that have been discussed so far in this chapter concerning displacement vectors apply equally well to velocity vectors. This fact is obvious when we realize that, to obtain a velocity vector, we simply divide a displacement vector by a time. Perhaps the best known application of vector methods to velocity vectors is in connection with aerial navigation.

3–10 THE NAVIGATOR'S PROBLEM

Before takeoff, the navigator of a plane has available to him the following information: (*a*) the speed, relative to the air, at which the pilot intends to fly the plane; (*b*) an estimated wind speed and direction, supplied by a meteorologist; (*c*) the direction on the ground from the airport from which he takes off to the one at which he intends to land. However, if he lets the pilot point the plane in this latter direction, the wind will blow the plane "off course" and the plane will not arrive at its intended destination. Therefore the navigator must calculate (*a*) in what direction to have the pilot point the plane, and (*b*) the speed of the plane relative to the ground. He can accomplish both of these calculations by means of a vector triangle such as that shown in Figure 3.8. From any point O he draws a line v_g of indefinite length, in the direction in which the plane must travel relative to the ground. Also from O he draws a vector $\overrightarrow{OP}$, representing the velocity $\vec{v_w}$ of the wind relative to the ground. With centre P and radius equal to the intended speed of the plane relative to the air, he draws an arc cutting v_g at Q. The length of OQ is the speed of the plane relative to the ground; the direction of PQ is the direction in which the pilot must point the plane. That is, if all goes according to plan.

But flights seldom go according to plan, because (among other things) v_w rarely turns out to be as the meteorologist predicted. After a few minutes in flight, the navigator finds that his position relative to the ground is not what he expected. From his observed position he can calculate both the magnitude and direction of $\vec{v_g}$. The pilot can tell him (presumably) what the magnitude and direction of $\vec{v_a}$ have been, and the navigator draws another vector diagram to find what $\vec{v_w}$ actually is. Then he draws a diagram such as Figure 3.8 again. The procedure is repeated at regular intervals throughout the trip. Nowadays electronic devices do most of these operations automatically.

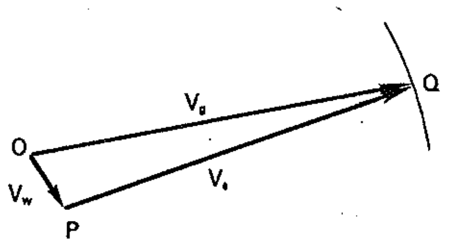


Fig. 3.8. The navigator's vector triangle.

3-11 MULTIPLYING VECTORS BY NUMBERS AND BY SCALARS

The usual meaning of 5 × 3 is that three 5's are to be added together. Following the same reasoning, we conclude that, when a displacement of 5 ft north is multiplied by 3, the product is a displacement of 15 ft north. That is, when a vector is multiplied by a number, the magnitude of the vector is multiplied by that number, and the direction of the vector and its units remain unchanged.

When a vector such as 40 mi/hr east is multiplied by a scalar quantity such as 5 hr, the above rules apply with the one exception that the units change. The magnitude of the product is obviously 200; the direction is east; but the units of the result, obtained by multiplying mi/hr by hr, are mi. The product is 200 mi east.

3-12 VECTOR ACCELERATION

In Chapter 2, we defined the acceleration of an object travelling along a straight line path as the rate of change of its speed with time. This definition serves very well when the direction of the path does not change. However, consider such cases as these: a ball is thrown straight up and then returns to earth; a car coasts part way up a hill, comes to rest, and then coasts back down again; a stone rotates in a circle on the end of a string. In order to deal with these motions we must consider acceleration as a vector quantity. Acceleration is then defined as rate of change of velocity, and is calculated by dividing the change in velocity by the time.

In cases where part of the motion of an object is in one direction and part is in exactly the opposite direction, the motion formulae derived in Chapter 2 may be used, provided s, u, v and a are treated as vector quantities. It is particularly important to remember that s must be treated as a displacement rather than as the total path length or distance travelled.

3-13 WORKED EXAMPLE

A boy, gliding on skates in a given direction at a speed of 6 m/sec, suddenly encounters a headwind which causes him to slow down at a constant rate of 1.5 m/sec². (*a*) When and where will he come to rest? (*b*) What will be his velocity and position 6 seconds after he encounters the wind?

Solution

Consider the direction of the boy's original motion as the positive vector direction.

(*a*)
$\vec{s} = ?$
$\vec{u} = 6$ m/sec
$\vec{v} = 0$
$\vec{a} = -1.5$ m/sec^2
$t = ?$

Using the formula $\vec{v} = \vec{u} + \vec{a}t$

$$0 = 6 - 1.5t$$
$$t = 4$$

Using the formula $\vec{s} = \dfrac{\vec{u} + \vec{v}}{2}t$

$$\vec{s} = \frac{6+0}{2} \times 4 = 12$$

The boy comes to rest in 4 sec, 12 m from the position where he first encountered the wind.

(*b*) The solution which follows is valid only if the boy's acceleration is the same after he comes to rest as before. The vector values given below apply from the time when he first encountered the wind.

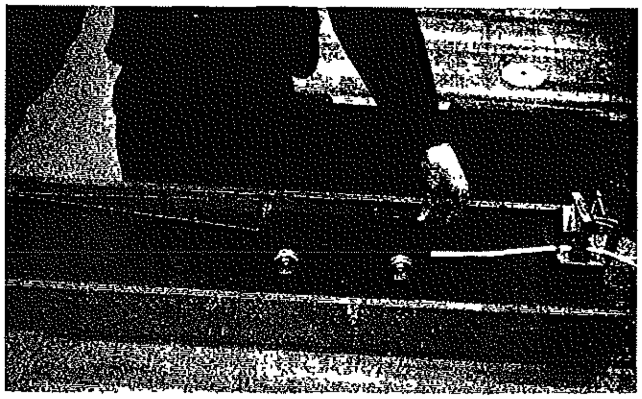


Fig. 3.9. When the cart is released, it will move with varying speed.

$\vec{s} = ?$
$\vec{u} = 6$ m/sec
$\vec{v} = ?$
$\vec{a} = -1.5$ m/sec
$t = 6$ sec

Using the formula $\vec{v} = \vec{u} + \vec{a}t$
$\vec{v} = 6 - 1.5 \times 6$
$\vec{v} = -3$

Using the formula $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\vec{s} = 6 \times 6 - \frac{1}{2} \times 1.5 \times 36$
$= 36 - 27$
$= 9$

Thus at the end of 6 seconds he will be 9 m from his starting point, in the direction of the original motion, but he will be moving backward with a speed of 3 m/sec.

3-14 LABORATORY EXERCISES: ACCELERATED MOTION

1. Attach a tape from a recording timer to one end of a dynamics cart, and a rubber band to the other end (Fig. 3.9). Hold the cart stationary with one hand, and stretch the rubber band with the other hand, as shown in the photograph. Have your partner start the timer, then release the cart. Try not to move the hand holding the rubber band; simply let the band go slack as the cart moves along the track past your hand.

Use the tape to plot both the distance-time graph and the speed-time graph for the motion of the cart. Try to relate each part of each graph to what you saw happening to the cart.

2. Attach the tape from a recording timer to a block of wood (Fig. 3.10). Start the timer, then allow the block to fall freely. Make the necessary measurements on the tape, and calculate the acceleration of the block as it fell.

3-15 FREE FALL

The second laboratory exercise in Section 3-14 suggests a method for finding the acceleration of a falling object. Since this acceleration is caused by the force of gravity, it is called the acceleration due to gravity and is given the symbol $\vec{g}$. In a vacuum, the magnitude of $\vec{g}$ is the same for all objects, and is approximately 32 ft/sec^2, 9.8 m/sec^2, or 980 cm/sec^2. The direction of g is down, whether the object is moving up or down.

Where an object falls through the air, the resistance of the air reduces the magnitude of the acceleration. The effect of air resistance depends on the shape of the object, on its volume, density and surface area, and on its speed. In many cases, the effect of air resistance is negligible, and we will consider this to be the case through the remainder of this book, unless we explicitly state otherwise.

The analysis of the vertical motion of an object under the influence of gravity provides a good example of the use of vectors in motion problems.

3–16 WORKED EXAMPLES

EXAMPLE 1

From a point 70 m above the ground an object is projected vertically upward with a velocity of 25 m/sec. Assuming that $g = 10 \text{ m/sec}^2$, calculate how long it will take to reach the ground.

SOLUTION

Step 1. Consider the upward portion of the trip, and consider vectors directed upward as positive.

$$\begin{aligned} \vec{v} &= \vec{u} + \vec{a}t \\ 0 &= 25 - 10t \\ t &= 2.5 \end{aligned}$$

That is, the object ascends for 2.5 seconds.

$$\begin{aligned} \text{Also, } 2as &= v^2 - u^2 \\ -20s &= 0 - 25^2 \\ s &= 31.25 \end{aligned}$$

That is, the object rises to a height of 31.25 m + 70 m = 101.25 m.

Step 2. Consider the fall from this 101.25 m level, and consider vectors directed downward as positive.

$$\begin{aligned} \vec{s} &= \vec{u}t + \tfrac{1}{2}\vec{a}t^2 \\ 101.25 &= 0 + 5t^2 \\ t &= 4.5 \end{aligned}$$

That is, the object falls 101.25 m in 4.5 sec. Thus the total time of flight is 2.5 sec + 4.5 sec, or 7.0 sec.

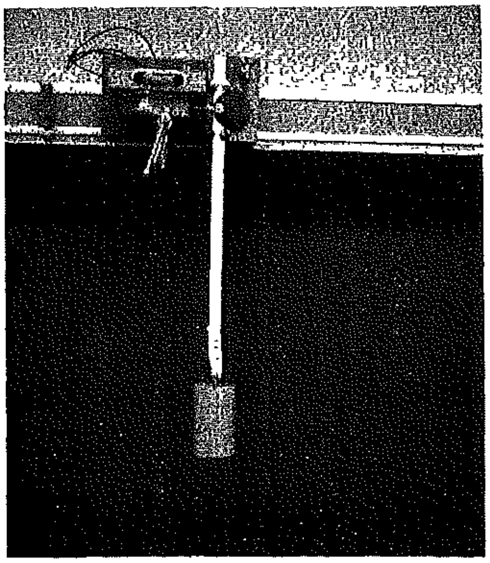


Fig. 3.10. A recording timer may be used to determine the acceleration of a falling object.

The problem may be solved in one step. Consider the whole flight, and consider vectors directed downward as positive.

$$\begin{aligned} \vec{s} &= \vec{u}t + \tfrac{1}{2}\vec{a}t^2 \\ 70 &= -25t + 5t^2 \\ 5t^2 - 25t - 70 &= 0 \\ t^2 - 5t - 14 &= 0 \\ (t - 7)(t + 2) &= 0 \\ t &= 7 \text{ or } t = -2 \end{aligned}$$

Thus, the time of flight is 7 sec. (The negative root is inadmissible in this case).

EXAMPLE 2

An object is projected vertically upward with an initial speed of 128 ft/sec. When will it reach a height of 240 feet above the ground?

SOLUTION

Note that the object may reach the 240 ft level on the way up and again on the way down. However, in either case its displacement from its initial position is 240 ft up.

$$\vec{s} = -240 \text{ ft}$$
$$\vec{u} = -128 \text{ ft/sec}$$
$$\vec{a} = \vec{g} = 32 \text{ ft/sec}^2$$
$$t = ?$$

Using the formula $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$-240 = -128t + 16t^2$$
$$t^2 - 8t - 15 = 0$$
$$(t-3)(t-5) = 0$$
$$t = 3 \text{ or } t = 5$$

The object is at the 240 ft level on the way up 3 sec after projection, and on the way down 5 sec after projection.

3–17 THE PATH OF A PROJECTILE

Vector methods are particularly useful in analysing the motion of an object, say a thrown ball, which moves horizontally at the same time as it falls (or rises) vertically. The photograph in Figure 3.11 compares the motions of two balls. The ball on the left was dropped at the same time as the ball on the right was projected horizontally. The vertical component of the initial velocity of each ball was zero. Examination of the photograph yields the following information.

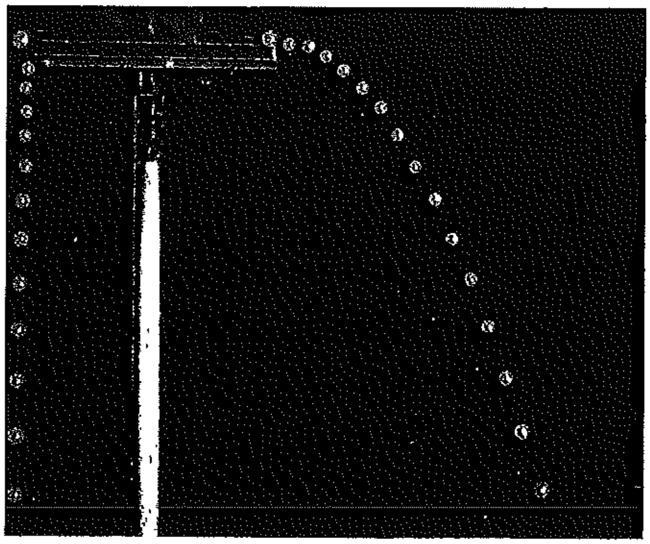
(*a*) For any given time interval, the vertical components of the displacements of the two balls are equal.

(*b*) In equal time intervals, the right hand ball undergoes equal horizontal displacements.

Though these facts may seem startling at first glance, they are nevertheless true. In Chapter 5 we will discuss the reasons for them; for the present we will simply take them for granted as a result of Figure 3.11. What they mean is this:

(*a*) For a projectile whose motion has both horizontal and vertical components, the two components may be considered separately, each as if the other did not exist.

(*b*) The horizontal component of the projectile's velocity remains constant.



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University of Western Ontario

Fig. 3.11. The ball on the left was dropped at the same time as the ball on the right was projected horizontally At each flash the two balls are at the same level.

3–18 WORKED EXAMPLES

EXAMPLE 1

A bomb is dropped from an aircraft flying horizontally at a speed of 600 km/hr at a height of 490 m. When and where does the bomb strike the ground? (Neglect air resistance).

SOLUTION

Consider first the vertical components of the vectors and consider vectors directed downward as positive.

$$\vec{s} = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$
$$490 = 0 + 4.9t^2$$
$$t = 10$$

That is, the time of fall of the bomb is 10 sec.

Next, consider the horizontal motion. The horizontal speed remains constant at 600 km/hr during the 10 sec ($\frac{1}{360}$ hr) while the bomb falls. Therefore the horizontal distance travelled = $\frac{600}{360}$ km = 1.7 km. The bomb strikes the ground 1.7 km from the point on the ground directly below the point of release.

EXAMPLE 2

A helicopter is rising vertically at a uniform speed of 48 ft per sec. When it is 640 ft from the ground, a ball is projected horizontally with a speed of 30 ft per sec. Calculate (*a*) when the ball will reach the ground, (*b*) where it will reach the ground, (*c*) the magnitude of its resultant velocity when it strikes the ground.

SOLUTION

Consider first the vertical component of the motion of the ball, and consider vectors directed downward as positive.

(*a*)
$$\vec{u} = -48 \text{ ft/sec}$$
$$\vec{a} = 32 \text{ ft/sec}$$
$$\vec{s} = 640 \text{ ft}$$
$$\vec{s} = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$
$$640 = -48t + 16t^2$$
$$16t^2 - 48t - 640 = 0$$
$$t^2 - 3t - 40 = 0$$
$$(t - 8)(t + 5) = 0$$
$$t = 8 \text{ or } t = -5$$

The negative root is inadmissible.

∴ the time taken to reach the ground is 8 sec.

(*b*) The horizontal component of the velocity is constant; the horizontal distance covered = 8 × 30 = 240 ft.

(*c*) The vertical component of the velocity at ground is given by $\vec{v} = \vec{u} + \vec{a}t$. ∴ $\vec{v} = -48 + (32 \times 8) = 208$ ft/sec. The horizontal component is 30 ft/sec. The resultant velocity $\vec{r}$ is obtained by applying the parallelogram of velocities (Fig. 3.12). The magnitude of the resultant velocity = $\sqrt{30^2 + 208^2} = 210$ ft/sec.

Further calculations show that the ball projected from the helicopter reaches a

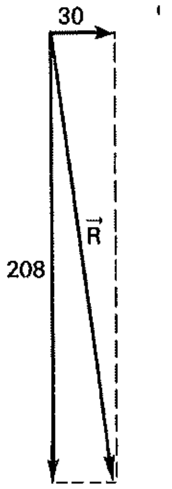


Fig. 3.12. The resultant velocity is found by means of the parallelogram of velocities.

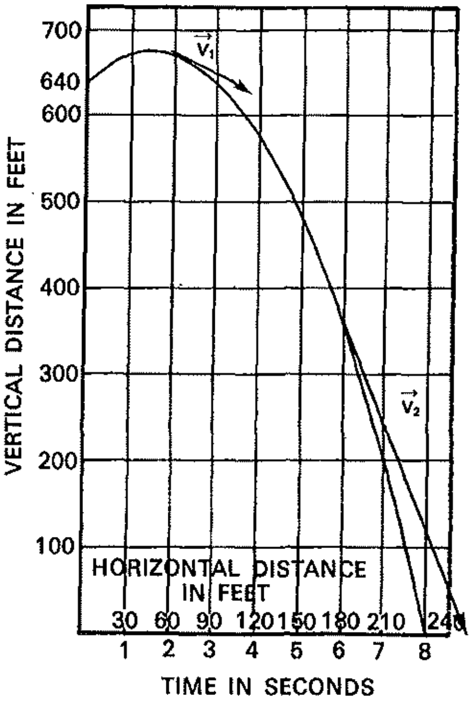


Fig. 3.13. A graph showing the path of a projectile, projected horizontally with a speed of 30 ft per sec, from a helicopter which is rising vertically at 48 ft per sec.

height of 676 ft and then loses altitude until it reaches the ground 8 seconds after projection. Other altitudes and times are shown in the following table.

TIME (sec)	ALTITUDE (ft)
0	640
1	672
$1\frac{1}{2}$	676
2	672
3	640
4	576
5	480
6	352
7	212
8	0

This information is summarized in Figure 3.13. In addition, the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ at times 2.0 sec and 6.0 sec are shown. They were calculated as was the resultant velocity at the ground in the worked example above. The first velocity is 34 ft/sec in a direction making an angle of approximately 28° with the horizontal, and the second is approximately 147 ft/sec in a direction making an angle of approximately 78° with the horizontal. These two vectors are shown as AB and AC in Figure 3.14. $\vec{BC}$ then is $\vec{v}_2 - \vec{v}_1$, that is $\vec{\Delta v}$. Measured on the scale to which velocities were drawn, $\vec{\Delta v}$ seems to be approximately 128 ft/sec, and is directed down. The corresponding time interval Δt is 4 sec. Therefore the

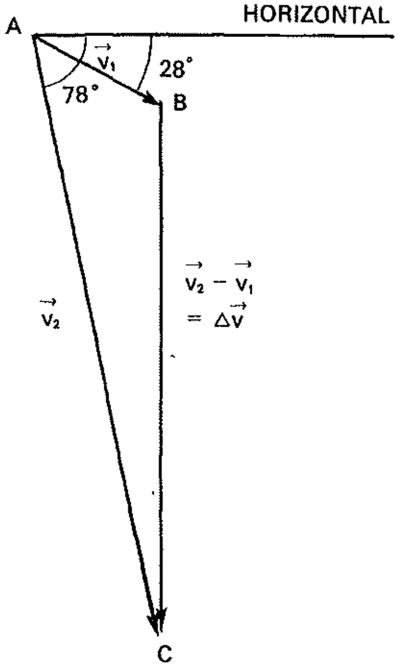


Fig. 3.14. Vector triangle for the velocity vectors from Figure 3.13.

acceleration, $\frac{\Delta\vec{v}}{\Delta t}$, is 32 ft/sec² down. We knew this in the beginning of course, but our calculations have made two facts plain. (*a*) The acceleration vector $\vec{a}$ has the same direction as $\Delta\vec{v}$, and this direction is not necessarily the same as the direction of either $\vec{v}_1$ or $\vec{v}_2$. (*b*) The above is a valid method for calculating the acceleration of an object which follows a curved path. We will find it useful in Chapter 5 when we analyse circular motion.

3–19 PROBLEMS

Use $g = 9.8$ m/sec² unless otherwise instructed.

1. At a particular instant, car *A*, with a uniform speed of 45 mi/hr, is 0.5 miles behind car *B*, which has a uniform speed of 30 mi/hr. What is the speed of *A* relative to *B*, and how long will *A* require to overtake *B*?
2. A stone is dropped from a point on the ceiling to the floor of a railway car which is travelling with constant velocity on a level track. At what point will the stone strike the floor of the car? Give a reason for your answer.
3. Why do aircraft take off and land into the wind? ?
4. In order to take off successfully from an aircraft carrier, a certain type of aircraft must attain an air speed of 90 mi/hr, but can attain a speed of only 60 mi/hr relative to the deck. What steps can be taken to attain the necessary air speed? Under what circumstances would a take-off be inadvisable?
5. The hour hand of a kitchen clock is 6.0 cm long. (*a*) Calculate the distance its tip travels (*i*) between 12:00 noon and 3:00 P.M., (*ii*) between 12:00 noon and 6:00 P.M., (*iii*) between 12:00 noon and 12:00 midnight. (*b*) Calculate its displacement in each of the time intervals mentioned in (*a*).
6. A man walks 2 miles east, stops, turns through 120° to his left, and walks 4 miles in this new direction. What is the resultant of the two displacements?
7. Compare the resultant of displacements of 5 km north and 6 km east with the resultant of displacements of 6 km east and 5 km north.
8. Compare the resultant of displacement of 10 metres east, 6 metres north-west and 5 metres west, with the resultant of displacements of 100 metres east, 60 metres north-west, and 50 metres west.
9. Use diagrams to show that (*a*) $\vec{a}+\vec{b} = \vec{b}+\vec{a}$, (*b*) $n\vec{a}+n\vec{b}+n\vec{c} = n(\vec{a}+\vec{b}+\vec{c})$
10. $\vec{p}$ represents a displacement of 10 m east and $\vec{q}$ a displacement of 15 m north-east. Use trigonometric tables to calculate (*a*) $\vec{p}+\vec{q}$, (*b*) $\vec{p}-\vec{q}$, (*c*) $\vec{q}-\vec{p}$.
11. A snail travels 2.0 metres north, turns 40° left, and proceeds 3.0 metres further before stopping to rest. Calculate the resultant displacement.
12. Evaluate $\vec{a}-\vec{b}$ for each of the following pairs of displacement vectors:
    (*i*) $\vec{a} = 5$ ft east, $\vec{b} = 3$ ft east,
    (*ii*) $\vec{a} = 5$ km east, $\vec{b} = 7$ km east,
    (*iii*) $\vec{a} = 4$ m north, $\vec{b} = 3$ m west.

13. A firecracker explodes, breaking into two unequal pieces. The larger part undergoes a displacement of 30 m north-west. The smaller part lands 80 m south-east of the larger part. What was the displacement of the smaller part?

14. If you travel 200 metres south-east, what are the southerly and easterly components of your motion?

15. An aircraft, with a ground speed of 500 mi/hr, is climbing steadily at 700 ft/min. What are the horizontal and vertical components (*a*) of its velocity, (*b*) of its displacement during 0.2 hr?

16. A car is driven at a velocity of 72 km/hr east for 0.50 hr, 48 km/hr north for 0.25 hr, and 62 km/hr west for 0.50 hr. Calculate (*a*) the magnitude of its displacement, (*b*) the magnitude of its average velocity.

17. The second hand on a classroom clock is 15 cm long. (*a*) Calculate the speed of its tip as it rotates. (*b*) State the velocity of the tip at 15 sec and at 30 sec. (*c*) Calculate the change in its velocity between 15 sec and 30 sec.

18. Using vector diagrams, find the magnitude of the resultant of two simultaneous velocities of 30 cm/sec and 50 cm/sec (*a*) at an angle of 90°, (*b*) at an angle of 45° to each other.

19. What is the air speed of a plane which takes $1\frac{3}{4}$ hrs to travel the 630 mi between two cities when it has a 70 mi/hr tail wind?

20. A ship is moving east at 5.5 m/sec. A passenger strolls on the deck at a rate of 1.5 m/sec. Find the magnitude of the velocity of the passenger relative to the earth (*a*) when he walks toward the bow, (*b*) when he walks toward the stern, (*c*) when he walks across the deck.

21. A passenger in a boat finds that the speed of the boat relative to the water is 5 mi/hr, and that the boat is pointing north-east. The water is flowing north at 10 mi/hr. Find the velocity of the boat relative to the ground.

22. The pilot of an airplane wishes to travel west with a ground speed of 800 km/hr. He knows that the wind is blowing from the north at 60 km/hr. In what direction should he point the airplane, and what airspeed should he maintain?

23. For the displacement-time graph shown in Figure 3.15, (*a*) calculate the average velocity during the first 4 seconds, (*b*) calculate the instantaneous velocity at $t = 4$ sec, (*c*) draw the corresponding velocity-time graph.

24. Base your answers to this question on the graph shown in Figure 3.16. This graph shows the velocity of an object travelling along a straight line. (*a*) Which portion of the graph represents a constant positive acceleration? (*b*) Which portion of the graph represents zero acceleration? (*c*) During which portion of the motion was the displacement decreasing? (*d*) At what point was the displacement a maximum? (*e*) Sketch (*i*) the corresponding displacement-time graph, (*ii*) the corresponding acceleration-time graph.

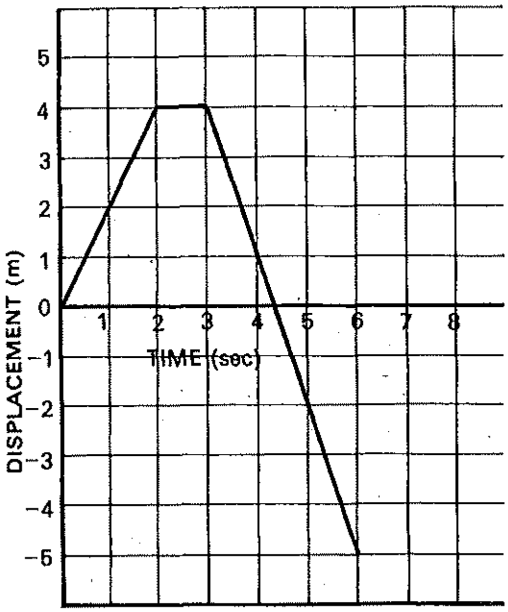


Fig. 3.15. For problem 23.

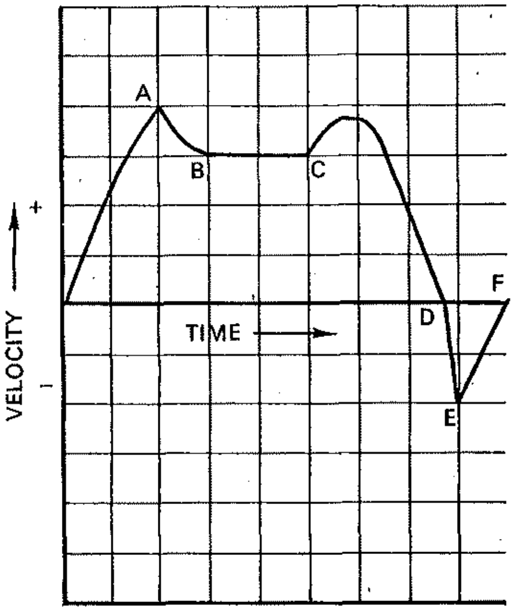


Fig. 3.16. For problem 24.

25. The second hand on a watch is 1.5 cm long. (*a*) Calculate the speed of its tip as it rotates. (*b*) State the velocity of the tip at 30 sec and at 45 sec. (*c*) Calculate the change in its velocity between 30 sec and 45 sec. (*d*) Calculate its average acceleration between 30 sec and 45 sec.

26. The velocity of a car changes from 30 mi/hr north to 40 mi/hr east in 20 sec. Calculate its average vector acceleration.

27. An airplane flying at a constant speed of 1000 km/hr executes a slow turn which changes its direction of travel from east to west. If the turn takes 80 seconds, calculate its average vector acceleration.

28. Describe qualitatively the motion represented by the acceleration-time graph in Figure 3.17. Sketch the corresponding velocity-time graph.

29. For the acceleration-time graph shown in Figure 3.18, determine the rate of change of acceleration at $t = 3$ sec and $t = 5$ sec.

30. The initial speed of an object is 16 m/sec to the right. It has a constant acceleration of 4 m/sec^2 to the left. At what times is it at a position 30 m to the right of its starting point? Interpret the two answers. Check by drawing the velocity-time graph.

31. In question 30, how long would it take the object to reach a position 30 m to the left of its starting point?

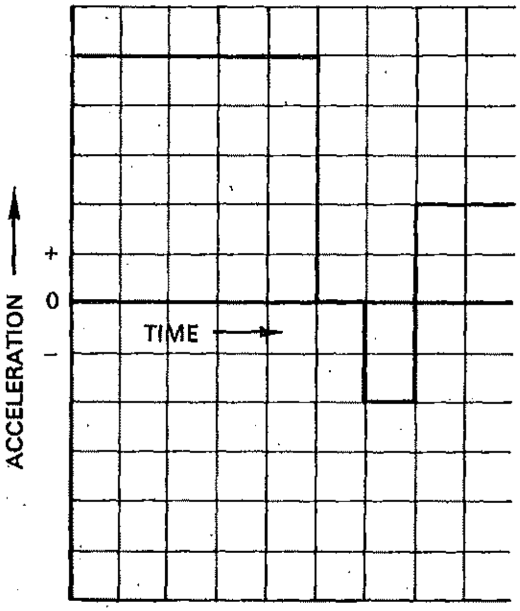


Fig. 3.17. For problem 28.

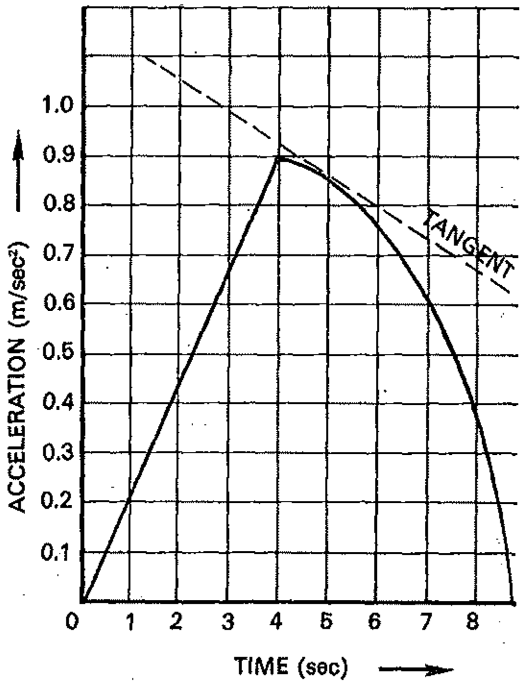


Fig. 3.18. For problem 29.

32. During the first quarter of the journey from a station A to a station B a train is uniformly accelerated and during the last quarter it is uniformly decelerated. During the middle half of the journey the speed is uniform. Show that the average speed of the train is $\frac{2}{3}$ of the maximum speed.

33. A train starts from rest, accelerates uniformly for 18 sec, travels for 0.5 min at constant speed, and decelerates uniformly to rest in 10 sec. The total distance travelled is 880 m. (*a*) Calculate the maximum speed attained. (*b*) Plot the speed-time graph.

34. A car is observed to cross a street in 4.0 sec. The street is 120 ft wide, and the car is accelerating at 4.0 ft/sec^2. Calculate its speed when it is half-way across the street.

35. Calculate the displacement of a ball during the fourth second of its fall from rest.

36. A stone is thrown vertically upward with an initial speed of 24.5 m/sec. (*a*) Find (*i*) its velocity, and (*ii*) its displacement, after 1, 2, 3, 4 and 5 sec. (*b*) Plot the displacement-time graph, the velocity-time graph, and the acceleration-time graph.

37. A rifle is fired horizontally from a point 2.0 metres above the ground. The muzzle velocity of the bullet is 300 m/sec. Calculate its time of flight, and the horizontal component of its displacement.

38. A baseball thrown from shortstop to first base travels 30 m horizontally and rises and falls 5.0 m. Find the horizontal and vertical components of the initial velocity of the ball. (Use $g = 10$ m/sec².)

39. An object projected with a horizontal velocity of 30 m/sec takes 4.0 sec to reach the ground. Assuming that air resistance is negligible, and that $g = 10$ m/sec², calculate (*a*) the height from which the object was projected, (*b*) the magnitude of the object's resultant velocity just before the object strikes the ground, (*c*) the horizontal component of the object's displacement.

40. An airplane, executing a shallow dive, releases a bomb. At the time of release, the bomb has velocity components of 160 m/sec horizontally and 40 m/sec vertically. (*a*) If the height of release is 4.8 km, and if air resistance reduces the vertical acceleration to an effective value of 8.0 m/sec², calculate the time of fall. (*b*) If air resistance reduces the horizontal velocity at the rate of 0.5 m/sec², calculate the horizontal displacement of the bomb during its fall.

3-20 SUMMARY

1. One point is in motion with respect to another if the line joining them is changing in length or direction.
2. A scalar quantity has magnitude only; a vector quantity has magnitude and direction. Distance, speed, and the acceleration associated with unidirectional motion are scalar quantities. Displacement, velocity, and the acceleration defined as rate of change of velocity, are vector quantities.
3. To find the sum (resultant) of 2 vectors, place the foot of the second vector on the head of the first. The resultant is the line segment from the foot of the first vector to the head of the second.
4. To find the difference between two vectors, place their feet together. Their difference is the line segment joining the head of the second vector to the head of the first.
5. The product of a vector and a number is a vector having the same direction and units as the original vector. The product of a vector and a scalar is a vector having the same direction as the original vector, but different units.
6. The components of a vector are two vectors (usually mutually perpendicular), whose resultant is the original vector.
7. The horizontal and vertical components of the motion of a projectile may be considered separately.
8. If the motion of an object is not unidirectional, but if the vector acceleration is constant, the formulae developed in Chapter 2 may be used, provided that s, u, v, and a are treated as vectors.

Chapter 4

Newton's Laws of Motion

4–1 INTRODUCTION

In Chapters 2 and 3 we have discussed only the description, or the kinematics, of motion. We have made no attempt to answer such reasonable and vital questions about motion as the following. Why does an object start to move? Under what circumstances is its velocity constant? What factors affect its acceleration? The first clear answers to these questions were stated by Sir Isaac Newton, and they involve dynamics rather than kinematics. Before we consider Newton's contributions, we shall consider some pre-Newtonian ideas about the causes of motion.

4–2 EARLY IDEAS ABOUT MOTION

The early philosophers' ideas about the causes of motion were much like our own ideas when we first started thinking about the subject. In many cases we would agree quite readily that, in order to cause an object to start moving, stop moving, speed up, slow down, or change direction, something else must push or pull on the object. In other words, an object will not accelerate unless an external force is exerted on it. But in certain situations we might have some reservations about this general statement—probably fewer reservations than the early philosophers would have had, for we have been conditioned to recognize forces which they did not know existed.

Horizontal motion on a rough surface presented considerable difficulty. It was known that an object rolling or sliding along such a surface eventually comes to rest without the application of any obvious external force; indeed a constant applied force is necessary to cause the object to move with constant velocity. Aristotle (384-322 B.C.) therefore concluded that a constant force was necessary to maintain constant velocity, and that, if a force did not act on a moving object, that object would come to rest. Since Aristotle's time we have learned to recognize the existence of a force of friction exerted by the surface on the object rolling or sliding on it. The moving object, then, comes to rest under the retarding action of the force of friction. Moreover, if the object is to move at constant velocity, we must apply a force sufficient to balance the force of friction. The resultant or net force acting on the object is then zero, and the object's velocity remains constant.

The ancients were puzzled also by the fact that an object accelerates as it falls. Apparently they did not recognize the existence of the force of gravity. They were puzzled too by the motions of the sun, the moon, and the stars in paths that were not straight lines, apparently with no force acting. Thanks to Newton, we now explain celestial motion in terms of gravitational force. Before Newton's time it was common practice to explain the acceleration of a falling object by saying that it was part of "the internal urge of bodies to seek the place proper to their scheme of things," and to explain celestial motion by attributing to "celestial matter" properties not possessed by earthly matter. This explanation obviously would not be accepted today when earthly matter is projected regularly into space and behaves predictably there. But this explanation was questioned long before the twentieth century; one of the most noteworthy of the questioners was Galileo Galilei (1564-1642).

4–3 NEWTON'S FIRST LAW

Galileo reasoned that, since a ball rolling uphill slows down and a ball rolling downhill speeds up, then a ball rolling on a horizontal frictionless surface should continue to move with constant velocity indefinitely. Sir Isaac Newton, who was born in the same year that Galileo died, recognized the truth of Galileo's assumption and included it in his famous book, Philosophiae Naturalis Principia Mathematica, published in 1687. It is known now as Newton's First Law of Motion, and is stated as follows:

Every body continues in its state of rest or of motion at uniform speed in a straight line, unless an unbalanced force acts upon it.

This law is a purely negative statement that the body will undergo no acceleration unless an unbalanced force acts upon it. It is impossible of proof and did not readily gain general acceptance by many contemporaries of Galileo and Newton. However, indirect evidence, similar to the following, seems to indicate its validity.

(*a*) As has already been noted, no stationary object begins to move of its own accord. Indeed, in cases where magicians seem to demonstrate otherwise, the sceptical observer immediately begins to search for hidden wires or other devices which exert the necessary forces.

(*b*) A hockey player, particularly a goal-tender, knows that a force is required to stop or even to slow down a fast-moving puck. Although he has never observed a puck which was subject to no forces whatsoever, he does know that if the ice is smooth the puck will slide farther than if the ice is rough. The thoughtful goalie may suspect that if the ice were perfectly smooth, i.e., if there were no friction, the puck would continue at constant speed in a straight line indefinitely.

(*c*) In baseball, a batter realizes that a force is necessary to change the direction of motion of the ball thrown by the pitcher, and that the greater the change in direction (a well hit ball as compared to a foul tip) the greater is the force required. If a ground ball takes a "crazy hop," fielders know that some object or irregularity on the ground exerted a force to produce the change in direction. Moreover, the usual explanation for the fact that a baseball can be made to curve is simply an explanation of the fact that an unbalanced force is acting on the ball. In all cases, the players assume that if no unbalanced force acts on the ball, its direction of motion will not change.

4–4 INERTIA

Newton's first law implies that any object resists a change in its velocity. This resistance to acceleration is called inertia. Many simple experiments may be performed to illustrate the existence of inertia, and hence, to illustrate Newton's first law.

If a tablecloth is spread on a table and a book is placed upon it, the cloth may be removed by a rapid jerk without moving the book. Indeed, an expert at this trick can pull a silk cloth from under a full set of dishes.

When a steady pull is exerted on a cord attached to a heavy weight that is resting on the floor, the weight may be lifted. On the other hand, a quick jerk may break the cord.

A sixteen-pound shot with screw eyes attached on opposite sides is suspended by a loop of stout string; a similar loop hangs below the shot (Fig. 4.1). When a rod is placed within the lower loop and steady pressure is exerted on the rod, the string will break above the ball. If the rod is raised a few inches within the loop and brought down with a quick jerk, the lower string will break.

Many everyday experiences demonstrate the inertia of stationary or moving objects. A person shovelling snow can stop the shovel suddenly but the snow, because of its inertia, continues forward. Passengers standing on a bus brace themselves or grasp a firm support to avoid being "thrown" forwards or backwards as the bus stops or starts suddenly. When the vehicle turns sharply, the passengers tend to continue in a straight line with the result that they seem to be "thrown" to one side.

4–5 FORCE—A VECTOR QUANTITY

The word "force" was used repeatedly in the discussion above, even though it had not been previously defined. Nor will it be defined here. In a sense Newton's first law defines force as that which is necessary to accelerate an object.

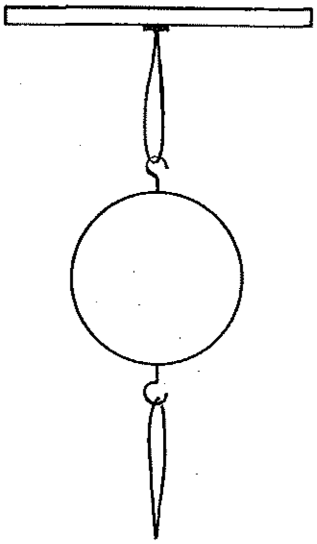


Fig. 4.1. Illustrating the inertia of an object at rest.

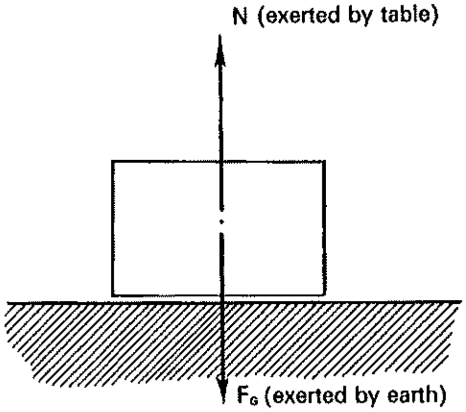


Fig. 4.2. Forces acting on a block at rest on a table.

Forces may vary in magnitude and act in different directions. Hence, force is a vector quantity and the method for finding the resultant of several forces is the same as has been described in Chapter 3 for displacement and velocity vectors. The resultant of several forces acting on an object may very well be zero, in which case the acceleration of the object will be zero. Such is the case for a block at rest on a table (Fig. 4.2) and for a car travelling at constant speed on a straight and level road (Fig. 4.3).

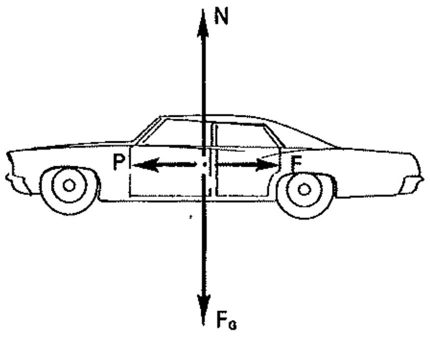


Fig. 4.3. Forces acting on a car moving at constant velocity. $\vec{P}$ is the propelling force, $\vec{F}$ is the force of friction, $\vec{N}$ is the vertical force exerted by the road, and $\vec{F_G}$ is the force of gravity.

In predicting the motion of an object, we frequently draw what is called a force diagram for the object. In drawing such diagrams, we must remember that only those forces which act on the object can have any effect on the motion of the object, and these are the only forces shown on the force diagram. These forces are applied by some agent outside the object, and are therefore called external forces. Moreover, the force which determines the motion of the object is the net force—the resultant of all of the forces shown in the force diagram.

4–6 SOME COMMON FORCES

The forces which act on objects to cause them to accelerate may be of many types, including a physical push or pull. One of the most common forces is the force of friction, a force which always acts so as to retard motion and which is rarely absent from any system of objects in motion.

The cause of friction between two solid surfaces sliding over one another is evident from a study of Figure 4.4. A surface may appear to be perfectly smooth to the unaided eye, but even the smoothest surface when examined under a microscope

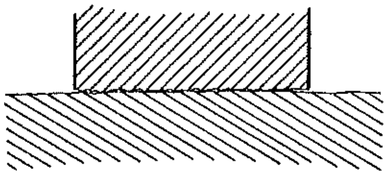


Fig. 4.4. When two surfaces are in contact, their small projections interlock.

shows little projections with hollows between them. When two plane surfaces are in contact some of the projections of each surface fit into the hollows in the other surface. Before sliding can take place, the projections must be broken off or forced clear of the hollows. Thus, when a force is applied to make one surface slide over another, there is a resistance (force of friction) which opposes the applied force. This force of friction is less if the projections are small, i.e., if the surfaces are smooth.

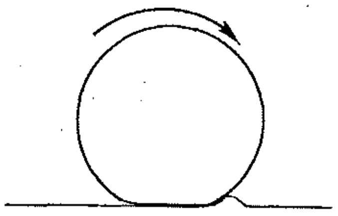


Fig. 4.5. *Illustrating the cause of rolling friction.*

The cause of friction in the case of a solid object rolling on a solid surface is shown in Figure 4.5. If a heavy ball rests on a surface, it makes a depression in the surface. In addition, the portion of the ball which touches the surface is flattened to some extent. Before rolling can take place, the ball must either be forced out of the depression, or the bulge of the surface in front of the ball must be forced out of the way. Thus, there is again a force of friction which opposes the applied force. This force of friction is less if the surfaces are hard.

Other forces which frequently have to be considered are magnetic and electric forces. A magnetized or electrified object can produce an effect on another object even though there is no physical connection between them.

The most common force producing effects at a distance is the force of gravity. This force is discussed fully in Chapter 6. However, some facts concerning the force of gravity must be discussed here.

4-7 GRAVITATIONAL FORCE AND GRAVITATIONAL MASS

All of the objects whose motions we will consider are composed of matter in one of its three forms: solid, liquid or gas. You are no doubt familiar with the word mass, used rather vaguely to measure the quantity of matter which an object contains. You will be familiar too with the use of a pan balance of some sort to measure the mass of an object. Equilibrium is attained when the earth exerts equal gravitational forces on the masses on each of the two pans. When the balance "balances" we say that the mass of the object being "weighed" is equal to the mass of the "standard masses" placed on the other pan of the balance. The mass obtained in this way is called the gravitational mass of the object. Several facts concerning gravitational mass should be noted. (*a*) Gravitational mass is independent of the object's position. If the balance "balances" at one place, it will balance at any and all positions in the universe. (*b*) For a given type of material, gravitational mass varies directly as the volume of the object. The constant ratio of mass to volume is called the density of the material. (*c*) The weight $\vec{F_G}$ of an object is the gravitational force which the earth exerts on it. The magnitude of $\vec{F_G}$ is directly proportional to the mass of the object (see Chapter 5). (*d*) An

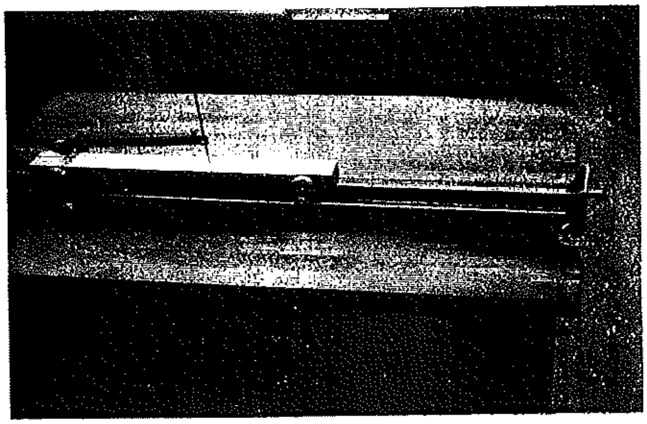


Fig. 4.6. The trolley (mass 1800 gm) and the suspended weights (mass 200 gm) are accelerated by a force equal to the weight of 200 gm.

object's inertia depends on its gravitational mass. You may verify this fact by finding the gravitational mass of a cannon ball and of a balloon, and by kicking each in turn. The greater the mass, the greater is the resistance to acceleration.

4-8 ACCELERATION AND NET FORCE

Newton's first law is a negative statement to the effect that, if the resultant force acting on an object is zero, the acceleration is also zero. This law implies that if the resultant force is other than zero the object will undergo acceleration. Newton's second law outlines the factors upon which this acceleration depends and the quantitative relationships between each of these factors and the acceleration.

Everyday experience indicates that (*a*) the greater the net force applied to an object, the greater is the acceleration of that object, and (*b*) the greater the mass of an object, the smaller is the acceleration produced by the action of a given unbalanced force. Moreover, the acceleration seems to depend only on these two factors, mass and unbalanced force. The quantitative nature of these relationships will now be discussed.

A Fletcher's trolley may be used to study the relationship between force and acceleration. The mass of the trolley car can be altered by adding additional masses to specially-built receptacles in the body of the car. To cancel the effect of friction, one end of the track is raised slightly, so that the car will not start of itself yet will continue moving if once started. A string is now attached to the trolley and is passed over a pulley; on the end of the string a mass M is attached (Fig. 4.6).

If M consists of two 100-gram masses, then the force which sets the car in motion is the attraction of the earth on the 200-gram mass. Both the car and the 200-gram mass are accelerated by this force. A tracing (Fig. 4.7) is made in the manner described in Section 2.4, and the acceleration is found to be 98 cm/sec^2. A 100-gram mass is now removed from M and placed in a slot in the car. Thus the force producing the acceleration has one-half its former value, but the total mass accelerated is the same as in the first case. The acceleration is now found to be 49 cm/sec^2, one-half of its former value.

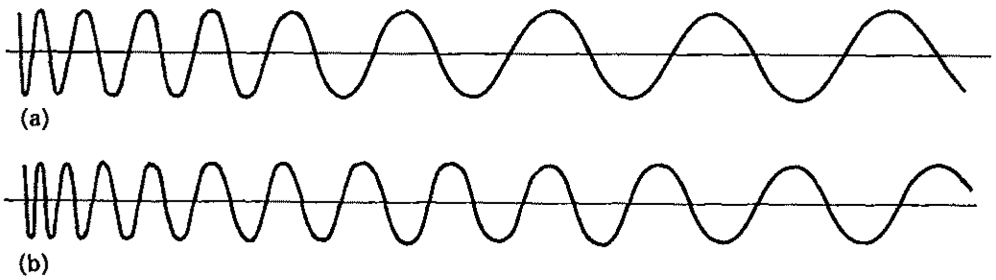


Fig. 4.7. The force which produced the trace *(a)* was double that in *(b)*. The total mass accelerated was the same in both cases.

Further experiments with the trolley confirm that the acceleration a of an object of mass m is directly proportional to the net force F acting on the object, that is

$$a \propto F \text{ if } m \text{ is constant.}$$

4-9 ACCELERATION AND MASS

The way in which acceleration varies with mass for a constant applied force may be demonstrated with Fletcher's trolley. The mass may be varied by placing additional masses in the slots in the trolley. The accelerating force is the weight of the masses attached to the end of the string (Fig. 4.6), and is kept constant. The acceleration is found to be inversely proportional to the mass if the net force is constant. That is

$$a \propto \frac{1}{m} \text{ if } F \text{ is constant.}$$

4-10 LABORATORY EXERCISES: ACCELERATION, FORCE, AND MASS

A dynamics cart, of the type used in the Laboratory Exercises in Sections 2-11 and 3-14, may be used to investigate the relationships among acceleration, force and mass. Elastic bands are used to provide the accelerating forces; a recording timer is used to record the motion; and the accelerated mass may be varied by placing bricks on the cart. The masses of the cart and bricks may be obtained by weighing them; it is convenient to use bricks each of which weighs twice as much as the cart. Sand in plastic bags may be used in place of the bricks. The most convenient unit of mass to use is "one cart."

1. Attach a tape from a recording timer to one end of the cart, and an elastic band to the other end (Fig. 4.8). Have your partner hold the cart in position. Use a metre stick to stretch the elastic band to a total length of about 70 cm., as shown in the photograph. If this extension of the band is maintained, the band will exert a constant force on the cart as the cart moves down the track, after your partner releases the cart. Your job is to move along with the cart and to maintain this constant extension of the band throughout the motion. When you are ready, signal to your partner to start the timer and release the cart. Maintain the extension of the elastic band until the cart nears the end of the track.

When you are satisfied that you have carried out the above instructions reasonably well, repeat the procedure using two elastic bands in parallel, rather than one. Do not change the accelerated mass, and use the same extension of the elastic bands as in the first case. Then repeat the procedure using three elastic bands, and four elastic bands.

From each tape, calculate the acceleration of the cart. Then, assuming that the force exerted on the cart by the stretched bands is proportional to the number of bands, draw a graph of force (in bands) plotted against acceleration (probably in cm/tock^2). What is the relationship between acceleration and force? Is the force exerted by the bands the only force acting on the cart? Is the assumption that the force exerted by the bands is proportional to the number of bands, a valid assumption?

2. Use the procedure outlined in 1. above, but this time keep the force (number of elastic bands) constant, and vary the accelerated mass by placing bricks or bags of sand on the cart. Calculate the acceleration from the tapes for at least four different masses. Plot acceleration against mass. Replot the information in an attempt to obtain a straight-line graph. What is the relationship between acceleration and mass?

4–11 NEWTON'S SECOND LAW

Both the acceleration and the force are vectors, and they have a common direction. That is, the acceleration vector has the same direction as the force vector. Also,

since $a \propto F$ when m is constant,

and $a \propto \frac{1}{m}$ when F is constant,

then $a \propto \frac{F}{m}$ when both F and m vary.

This relationship is Newton's second law: When an unbalanced (net) force acts on an object, the resulting acceleration is directly proportional to the net force and is inversely proportional to the mass of the object.

4–12 INERTIAL MASS

The ratio $\frac{F}{a}$, a constant for any given object, is frequently called the inertial mass of that object. Since the ratio $\frac{F}{a}$ varies from object to object, different objects have different inertial masses. Suppose that a certain force causes object A to accelerate at 0.5 m/sec^2 whereas the same force causes object B to accelerate at 1.0 m/sec^2. Then the inertial mass of A is double that of B. But, since the acceleration of an object varies inversely as the object's gravitational mass, the

Fig. 4.8. If the rubber band is kept extended a constant amount, it applies a constant force to the cart.

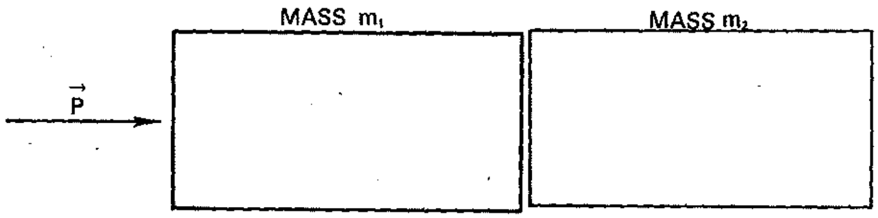


Fig. 4.9. A horizontal force is applied to the first of two blocks, and both blocks move with the same acceleration.

$\vec{a} = 0$. Moreover, the second law may be used to develop the third law, which is not so much a law of motion as a law describing the forces of interaction of objects. Suppose, for example, that you are standing in a very crowded street car. You may justly claim that your neighbour is pushing you, but he may claim, equally correctly, that you are pushing him. Each of you is in fact pushing the other; each is pushing and being pushed. The relationship between these forces of interaction may be discovered as follows.

Suppose a horizontal force $\vec{P}$ pushes an object of mass m_1, which in turn pushes a second object of mass m_2, all on a horizontal frictionless surface (Fig. 4.9). Both masses move with the same acceleration, which we may calculate by using the formula $\vec{F} = m\vec{a}$ for the whole system. (We shall ignore any vertical forces, because they balance each other, and as a result there is no vertical acceleration.) Thus

$$\vec{a} = \frac{\vec{P}}{m_1 + m_2} \qquad (1)$$

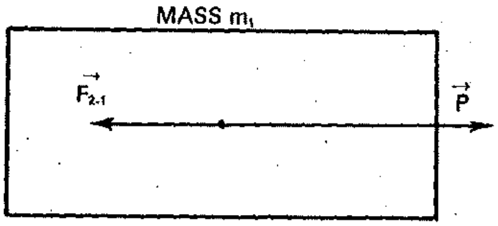


Fig. 4.10. Force diagram for the block whose mass is m_1.

Now consider the force diagram for the block of mass m_1 (Fig. 4.10). The horizontal force $\vec{P}$ exerted on this block is balanced in part by the force $\vec{F}_{2-1}$ exerted by the second block. The net force is $\vec{P} + \vec{F}_{2-1}$. (The plus sign indicates a vector sum.)

$$\vec{P} + \vec{F}_{2-1} = m_1 \vec{a}$$

$$\vec{a} = \frac{\vec{P} + \vec{F}_{2-1}}{m_1} \qquad (2)$$

For the block of mass m_2 (Fig. 4.11), the only force acting is the force $\vec{F}_{1-2}$ exerted by the first block. Using $\vec{F} = m\vec{a}$ in this case we obtain

$$\vec{F}_{1-2} = m_2 \vec{a}$$

and

$$\vec{a} = \frac{\vec{F}_{1-2}}{m_2} \qquad (3)$$

Equating the right sides of (1) and (2)

$$\frac{\vec{P}}{m_1 + m_2} = \frac{\vec{P} + \vec{F}_{2-1}}{m_1}$$

$$\vec{P}m_1 + \vec{P}m_2 + \vec{F}_{2-1}m_1 + \vec{F}_{2-1}m_2 = \vec{P}m_1$$

$$\therefore \vec{F}_{2-1}(m_1 + m_2) = -\vec{P}m_2$$

$$\therefore \vec{F}_{2-1} = -\frac{\vec{P}m_2}{m_1 + m_2} \qquad (4)$$

Equating the right sides of (1) and (3)

$$\frac{\vec{P}}{m_1 + m_2} = \frac{\vec{F}_{1-2}}{m_2}$$

$$\therefore \vec{F}_{1-2} = \frac{\vec{P} m_2}{m_1 + m_2} \qquad (5)$$

Comparing (4) and (5)

$$\vec{F}_{1-2} = -\vec{F}_{2-1}$$

Thus the forces which m_1 and m_2 exert on each other are equal in magnitude but opposite in sign.

Equal and opposite pairs of forces occur whenever two objects interact, even though the objects are not in contact with one another; the forces may be magnetic, electric or gravitational. There can be no force unless two objects are involved; each exerts a force on the other. In general, for every force exerted by one object on a second object, there is an equal and opposite force exerted by the second object on the first. This is Newton's third law.

One of the forces is commonly called an action force and the other a reaction force and Newton's third law is sometimes stated: reaction is always equal and opposite to action. This statement omits one important point: the action and reaction forces are exerted on and by different objects. The reaction to the force exerted by A on B is the force exerted by B on A.

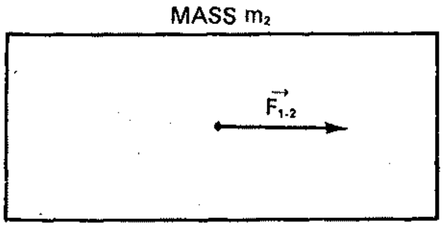


Fig. 4.11. Force diagram for the block whose mass is m_2.

4–18 EXAMPLES OF ACTION-REACTION PAIRS

The list of examples of Newton's third law is endless, for the law applies to any situation. A few everyday examples follow.

When a bat strikes a ball, the bat exerts a force on the ball and the ball exerts an equal and opposite force on the bat.

If a finger is pressed against the surface of a table, the table exerts on the finger an equal force in the opposite direction.

The reaction to the weight of an object (the gravitational force which the earth exerts on the object) is the force with which the object attracts the earth. If the object is free to move, it will be accelerated towards the earth, and at the same time the earth will be accelerated towards the object. However, because of the great mass of the earth, its acceleration is too small to be observed.

When a person steps ashore from a small boat, the boat moves away from shore. The force exerted by the person on the boat causes the boat's acceleration away from shore; the force exerted by the boat on the person causes his acceleration towards the shore. If the boat is large, it will experience little acceleration.

Consider the forces acting on a block placed on a table (Fig. 4.2). The weight $\vec{F}_G$ of the book acts vertically downwards; a force $\vec{N}$ exerted by the table acts vertically upwards. These forces are equal and opposite, not because they constitute an action-reaction pair, but because the acceleration of the book, and hence the resultant force acting on the book, is zero.

The reaction to $\vec{F_G}$ is the gravitational force exerted by the book on the earth, and the reaction to $\vec{N}$ is the force exerted by the book on the table.

The reaction force never acts on the same object as the action force; hence, an action-reaction pair of forces never cancel one another.

4–19 PROBLEMS

1. If an object is subject to no forces whatsoever, its velocity will remain constant. Is the converse necessarily true?
2. Draw the force diagram for a block of wood floating on water.
3. Why does a solid immersed in liquid appear to lose weight? Draw the force diagram for a stone suspended underwater from a string.
4. An aircraft is flying at a uniform speed of 600 km/hr relative to the air. Draw the force diagram for the aircraft.
5. A brick is pushed along a rough floor. What forces are exerted by the brick on the floor? What forces are exerted by the floor on the brick?
6. Calculate the magnitude of the resultant of forces of 10 newtons north and 20 newtons east. Use a graphical method to find the direction of the resultant.
7. A force of 100 newtons north and a force of 100 newtons west act on an object. What is their resultant?
8. The same net force F imparts an acceleration of 6 m/sec^2 to a 4-kg object and an acceleration of 2.4 m/sec^2 to a second object. What is the mass of the second object? What is the value of F?
9. A net force of 0.6 newtons gives a mass m an acceleration of 0.18 m/sec^2, and another net force F gives the same mass an acceleration of 0.45 m/sec^2. Calculate F and m.
10. A net force of 20 newtons acts on an object whose mass is 4 kg. What is the object's acceleration?
11. What force will give a mass of 10 kg an acceleration of 50 cm/sec^2?
12. Calculate the force required to give a 0.49-kg mass an acceleration of 10 cm/sec^2.
13. What will be the acceleration of a 150-kg motorcycle if the net force acting on it is (*a*) 75 newtons, (*b*) 225 newtons, (*c*) 22.5 newtons? In what direction does the acceleration take place in each case?
14. A net force of 0.6 newtons causes an object to accelerate at a rate of 0.3 m/sec^2. What is the object's mass?
15. A horizontal force F is applied to a 2-kg block at rest on a table. When F is $\frac{1}{4}$ of the weight of the block, the block moves at constant speed. Calculate the value of F required to accelerate the block from rest to a speed of 3 m/sec in 4.0 sec.

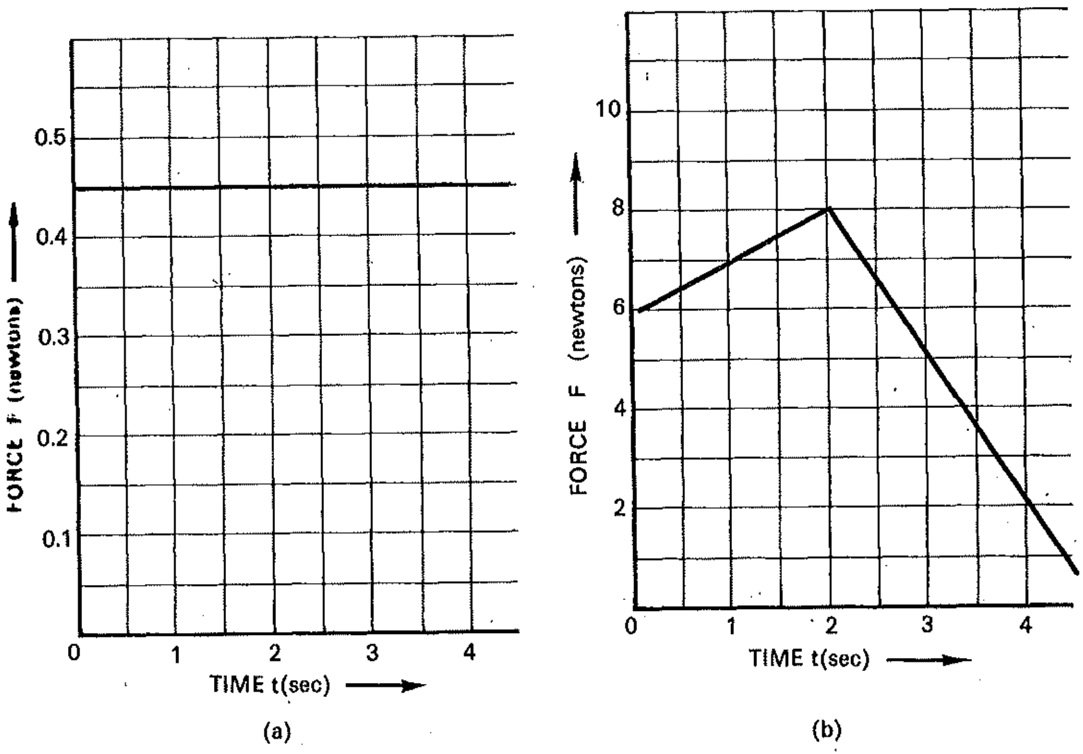


Fig. 4.12. For problem 17.

16. A shell of mass 1 kg is discharged with a speed of 4.5×10^2 m/sec from a gun having a barrel of length 2.0 m. Calculate the average force exerted on the shell while it is in the barrel.

17. For each of the two graphs in Figure 4.12, calculate (*a*) the impulse of the force between $t = 1$ sec and $t = 3$ sec, (*b*) the change in the momentum of the object on which the force acts, between $t = 0$ and $t = 4$ sec.

18. What is the magnitude of the impulse imparted to an object by a force of 7.0 newtons acting for 5.0 sec? By how much will the momentum of the object change during these 5.0 sec?

19. Calculate the magnitude of the impulse which causes the velocity of a 6.0-kg mass to change by 50 cm/sec.

20. Suppose that an impulse of 5.0 newton-sec is applied to an object. By how much does the velocity of the object change if its mass is (*a*) 5.0 kg, (*b*) 2.5 kg, (*c*) 2.0 kg?

21. A constant force is applied to a 3.0-kg object initially at rest. The object moves 25 m during the first 5.0 sec. Calculate the impulse of the force.
22. A 50-gm golf ball is hit by a club and given a speed of 40 m/sec. (*a*) Calculate the impulse imparted to the ball. (*b*) If the club is in contact with the ball for 0.10 sec, calculate the magnitude of the average force exerted by the club on the ball. (*c*) What is the magnitude of the average force exerted by the ball on the club?
23. For each of the following cases, specify the reaction to the force mentioned, making clear what the reaction is exerted by, and what it acts on: (*a*) the force exerted by a bat striking a baseball; (*b*) the force exerted by the earth on a freely falling body; (*c*) the force exerted by the earth on the moon.
24. Refute the following argument:
    No object can ever accelerate, for each of the forces acting on it is balanced by the corresponding reaction force.

4-20 SUMMARY

1. Newton's First Law: An object will not accelerate unless an external, unbalanced force acts upon it.
2. A force is a push or a pull; its effect is to cause any object on which it acts to accelerate. Force is a vector quantity.
3. The inertia of an object is its resistance to acceleration.
4. Newton's Second Law: The acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to the gravitational mass of the object. The acceleration takes place in the direction of the net force.
5. The inertial mass of an object is the $\frac{\text{force}}{\text{acceleration}}$ ratio for that object. The inertial mass of an object is proportional to its gravitational mass.
6. The formula $\vec{F} = m\vec{a}$ expresses Newton's Second Law mathematically. It applies, for example, if F is in newtons, m in kilograms and a in metres/sec^2.
   $$1 \text{ newton} = 1 \text{ kg-m/sec}^2$$
7. The impulse of a force is the product of the force and its time of action. Impulse units are newton-sec.
8. The momentum of an object is the product of its mass and its velocity. Momentum units are kg-m/sec.
   $$1 \text{ kg-m/sec} = 1 \text{ newton-sec}$$
9. Newton's Second Law: The rate of change of an object's momentum is proportional to the net force applied to the object. It may be written in the form
   $$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t}$$
10. Newton's Third Law: For each force exerted by an object A on another object B, there is an equal and opposite force exerted by B on A.

Chapter 5

Motion Near the Surface of the Earth

5–1 INTRODUCTION

The force of gravity is perhaps the commonest force which we know, and as a result the acceleration of a falling object is perhaps the most readily observable acceleration. In Chapter 3 we gave this acceleration the symbol $\vec{g}$ and stated that $\vec{g}$ was the same for all objects. In Chapter 4 we defined the weight $\vec{F}_G$ of an object as the gravitational force which the earth exerts on it and stated that the magnitude of $\vec{F}_G$ is directly proportional to the mass m of the object. For the present we shall continue to assume the truth of this last statement, and use it to investigate the factors affecting the value of $\vec{g}$.

5–2 FACTORS AFFECTING THE ACCELERATION OF A FALLING OBJECT

For an object falling in a vacuum, the only force acting on the object is its weight $\vec{F}_G$ acting down, and the downward acceleration is $\vec{g}$. The formula $\vec{F} = m\vec{a}$, applied in this case, becomes $\vec{F}_G = m\vec{g}$, or $\vec{g} = \frac{\vec{F}_G}{m}$. But the statement that F_G is directly proportional to m means that $\frac{F_G}{m}$ is constant. Therefore $\vec{g}$ is constant; all objects, regardless of mass, fall with the same acceleration in a vacuum.

Historically, the order of the reasoning in the above paragraph was reversed. Galileo is said to have dropped two metal balls, the mass of one being ten times that of the other, from the top of the leaning tower of Pisa. He found that they struck the ground simultaneously. Newton released a guinea and a feather simultaneously at the top of a long vacuum tube, and found that the coin and the feather reached the bottom of the tube at the same time. Thus, in cases where air resistance is negligible or non-existent, g is independent of m. Note that this experimental result, coupled with

Newton's second law, indicates that F_G is directly proportional to m, a conclusion which is by no means intuitively obvious.

Prior to Galileo's time it had been assumed that the acceleration of a falling object was dependent on the mass of the object. This is a natural enough assumption, for when objects fall in air the force of gravity is balanced in part by air resistance. This air resistance is proportionally much greater for an object such as a feather than for a coin or a heavy metal ball. However the acceleration observed in air cannot properly be called an acceleration due to gravity, since it is due to the resultant of gravity and air resistance.

The term "acceleration due to gravity" should be reserved, therefore, for cases in which air resistance is negligible. Though the magnitude of $\vec{g}$ is independent of mass, it is dependent on the object's elevation —its distance from the centre of the earth. The greater the elevation, the less the weight of the object and therefore the less the acceleration due to gravity becomes. Conversely, as an object falls, its elevation continually decreases, its weight continually increases, and therefore its acceleration due to gravity continually increases. However, in the case of objects falling near the earth's surface, the vertical displacement is so small in comparison with the radius of the earth that the variation in g is negligible.

Values of g have been determined in many localities throughout the world. At sea level on the equator, $g = 9.781$ m/sec^2; at the poles 9.831 m/sec^2, and at Toronto 9.806 m/sec^2. In the problems in this chapter, as in Chapter 3, we shall use $g = 9.8$ m/sec^2 or 32 ft/sec^2, and assume in all cases that the effect of air resistance is negligible.

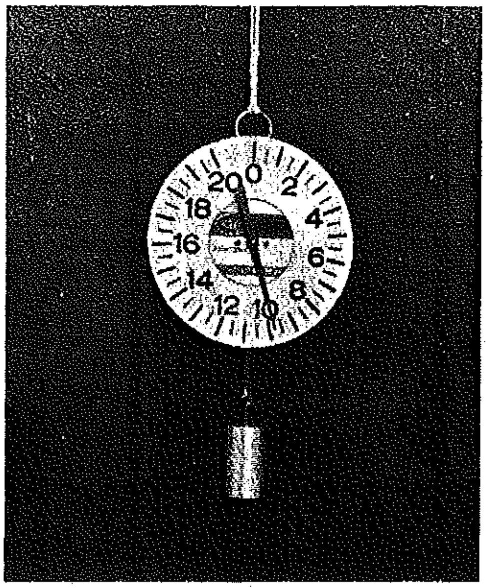


Fig. 5.1. A newton balance records the weight of a one-kilogram mass to be about 9.8 newtons.

5-3 THE EARTH'S GRAVITATIONAL FIELD

For an object falling in a vacuum, we have already noted that the equation $\vec{F} = m\vec{a}$ becomes $\vec{F}_G = m\vec{g}$. Thus at a location where $g = 9.8$ m/sec^2, the weight of a one-kilogram mass is 9.8 newtons. Figure 5.1 shows this fact recorded by a newton balance—a spring balance calibrated in newtons. The weight vector, of course, is directed down toward the centre of the earth, as is the acceleration vector. At places where the magnitude of $\vec{g}$ is 9.7 m/sec^2, the magnitude of $\vec{F}_G$ for a one-kilogram mass is 9.7 newtons. The gravitational force per unit mass is then 9.7 newtons per kilogram. The vectors drawn in Figure 5.2 show, to scale, the gravitational force per unit mass at distances r, $1.5r$ and $2r$ from the centre of the earth, r being the radius of the earth. Figure 5.2 then shows a part of the gravitational field of the earth; the magnitude and direction of each field vector depends on its position in the field.

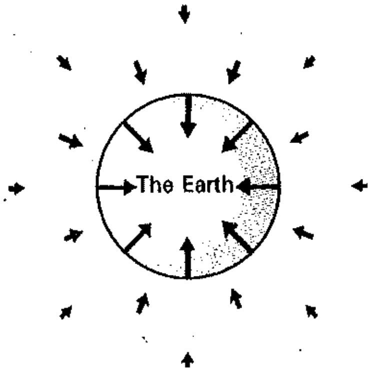


Fig. 5.2. A portion of the gravitational field of the earth.

5-4 THE PATH OF A PROJECTILE

In Chapter 3 we concluded, after examining Figure 3.11, that (*a*) for a projectile whose motion has both horizontal and vertical components, the two components may be considered separately, and (*b*) the horizontal component of the projectile's velocity remains constant.

We may now use Newton's second law to verify these conclusions. In the absence of air resistance, the only force acting is the force of gravity. Since this force acts down, it has no horizontal component and therefore the acceleration vector has no horizontal component. As a result the horizontal velocity remains constant. Moreover, the downward force is the same as if the projectile were falling vertically; therefore the vertical acceleration is the same as for a vertical fall. Thus the vertical acceleration is independent of the horizontal motion; the two components may be considered separately.

The equation of the path of a projectile projected horizontally may be determined from Figure 5.3. Suppose that the constant horizontal speed is v, and that the projectile is at a point $P(x, y)$ at a time t sec after projection.

$$\text{Then } x = vt \qquad (1)$$

$$\text{and } y = -\tfrac{1}{2}gt^2 \qquad (2)$$

$$\text{From (1), } t = \frac{x}{v}$$

Substituting in (2)

$$y = -\tfrac{1}{2}g\frac{x^2}{v^2}$$

$$\text{or } x^2 = -\frac{2v^2y}{g}$$

This equation is of the form $x^2 = -4py$,

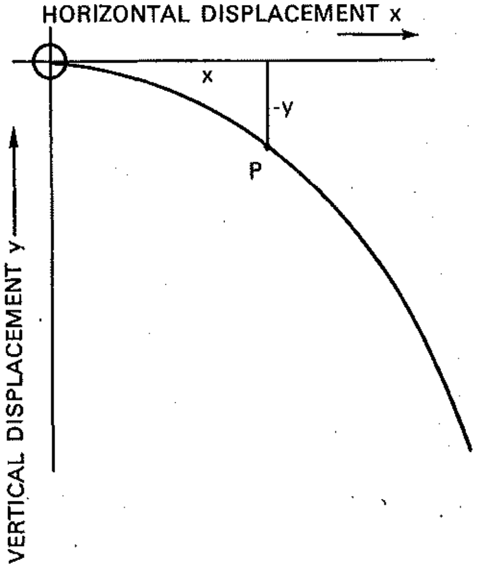
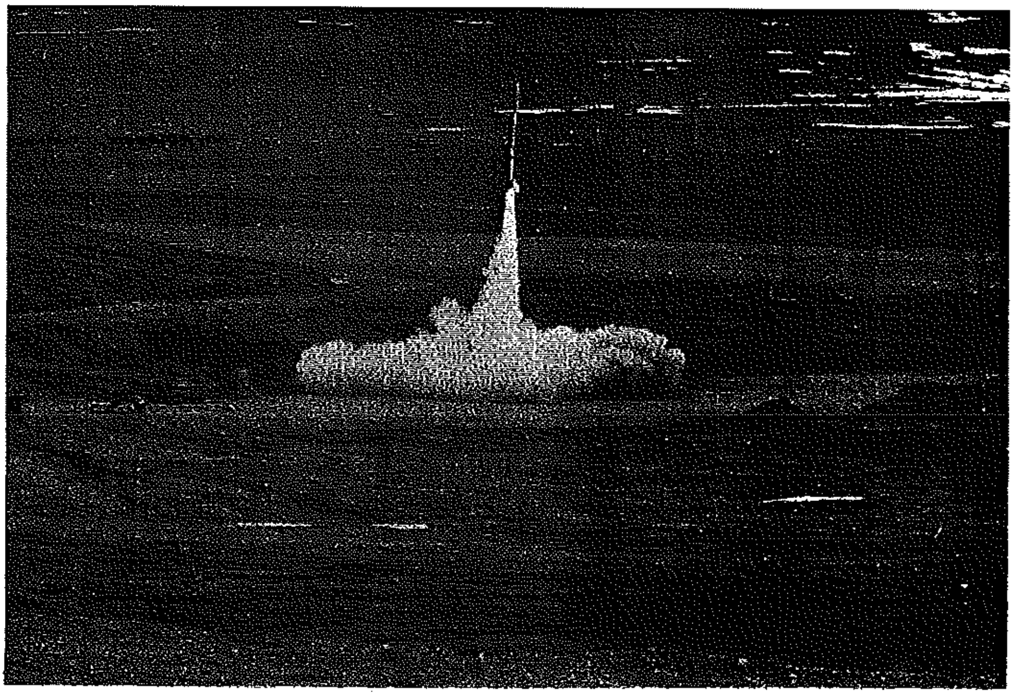


Fig. 5.3. The path of a projectile.



National Research Council, Space Facilities Branch

Fig. 5.4. A Canadian Black Brant III rocket is fired from a special launch site at Resolute on Cornwallis Island in the Canadian Arctic. The nose cone carried a special detector system for measuring cosmic X-rays.

and is a portion of a parabola having its vertex at the point of projection, and which is symmetrical about the vertical line through this point.

The exercise for this chapter contains further problems on projectile motion, problems of a type first encountered in Chapter 3. The basic methods outlined in Chapter 3 still apply of course, and, in addition, Newton's second law has to be used in some cases. These problems are of a type basic to short range artillery work. But nowadays rockets (Fig. 5.4) and earth satellites have much greater range, and the dynamical problems involved are much more complicated. Before we can begin to consider this latter type of problem, we must become familiar with circular motion.

5–5 CIRCULAR MOTION

Acceleration has been defined as rate of change of velocity, velocity being a vector quantity and therefore having both magnitude and direction. Most of the instances of acceleration discussed earlier involved changes in the magnitude of the velocity vector. However, we have seen that acceleration may result from a change in the direction of the velocity vector.

Consider a stone attached to a string and swung about the hand, so that it travels in a circle with constant speed. Although the speed of the stone remains constant, the direction of motion is continually changing and therefore the stone is being accelerated. The force necessary to cause this acceleration is obviously exerted on the stone by the string.

5–6 CENTRIPETAL FORCE

Figure 5.5 represents an object of mass M moving with uniform speed in a circle whose centre is O. In the position shown, the instantaneous direction of motion of M is along the tangent MA. Therefore MA represents the direction of the object's velocity vector at this instant. If, while in this position, the string were cut, M would move off along the line MA. However, if the string remains intact it exerts a force on M which causes M to move out of this straight path and travel the curved path. This force, because it appears to cause M to "seek the centre," is known as the centripetal force (centrum, centre; petere, to seek).

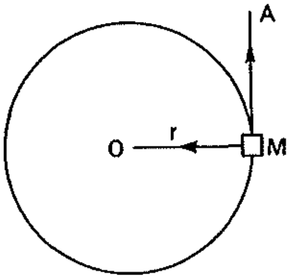


Fig. 5.5. Uniform circular motion. The instantaneous velocity vector is tangent to the circle.

Centripetal force is the force which must be exerted on an object to cause it to follow a circular path. It may be exerted as a tension in a string, as a gravitational, magnetic, or electric force, by means of friction, or in other ways. Centripetal force acts towards the centre of rotation, and hence at right angles to the direction of motion. For, if it did not, it would have a component in the direction of motion and the speed of the object would change. Centripetal force is therefore called a central force. Since the acceleration vector has the same direction as the force vector (Newton's Second Law), the acceleration produced by the centripetal force is directed to the centre of the circle. It is called the centripetal or central acceleration. Its effect is not to cause the radius of rotation to decrease, but to cause the object to move closer to the centre than it would if the force were not acting.

5–7 MAGNITUDE OF CENTRIPETAL FORCE

The magnitude of the centripetal force required depends on three factors: the mass of the object, its speed, and its radius of rotation. The greater the mass, the faster the movement, or the smaller the radius of rotation, the greater will be the centripetal force required. It can be shown mathematically that the centripetal force necessary to cause an object of mass m to rotate at a constant speed v in a circle of radius r is given by the formula

$$F_c = \frac{mv^2}{r}$$

If m is in kilograms, v in m/sec, and r in m, then F_c is in newtons.

The mathematical development of this relationship follows. P_1 and P_2 (Fig. 5.6*a*) are two positions on the circular path of the rotating object; $\vec{v_1}$ and $\vec{v_2}$ are the velocity vectors at P_1 and P_2 respectively. The vectors $\vec{v_1}$ and $\vec{v_2}$ are equal in magnitude but differ in direction; they are perpendicular to the corresponding radii OP_1 and OP_2. Let angle $P_1OP_2 = \theta$, chord $P_1P_2 = x$ and arc $P_1P_2 = s$.

In Figure 5.6*b* the vectors $\vec{v_1}$ and $\vec{v_2}$ are drawn in their proper directions, originating from a common point A.

Then $\vec{BC} = \vec{\Delta v}$

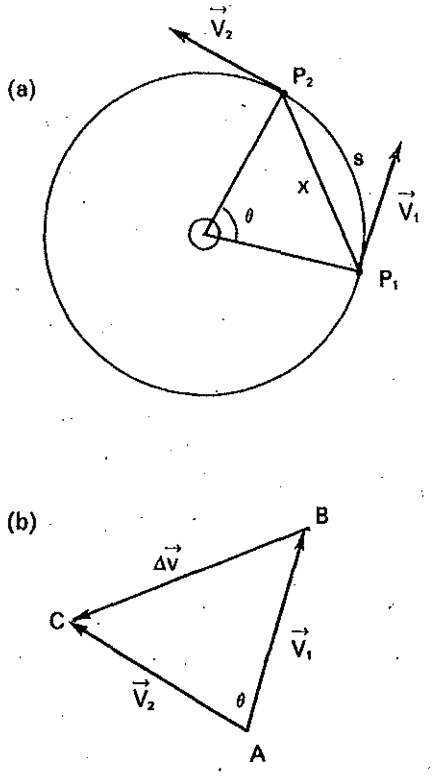


Fig. 5.6. *(a)* The velocity vectors at two points on a circular orbit. *(b)* Construction for determining the change in velocity.

Since each of the velocity vectors is perpendicular to the corresponding radius, the angle between the vectors is equal to the angle between the radii, i.e., $\angle BAC = \theta$.

Since $OP_1 = OP_2$ and $AB = AC$

$$\triangle OP_1P_2 \,|||\, \triangle ABC$$

$$\therefore \frac{OP_1}{AB} = \frac{P_1P_2}{BC}$$

$$\therefore \frac{r}{v} = \frac{x}{\Delta v}$$

where r is the radius of the circle and v is the constant magnitude of the velocity vector.

$$\Delta v = \frac{v}{r} \cdot x$$

and the magnitude of the average acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{v}{r} \cdot \frac{x}{\Delta t}$$

Now if $P_2 \rightarrow P_1$, $x \rightarrow s$, and the magnitude of the instantaneous acceleration at P_1 is given by the relationship

$$a = \frac{v}{r} \cdot \frac{s}{\Delta t}$$

But $\frac{s}{\Delta t}$ is the magnitude v of the constant velocity.

$$\therefore a = \frac{v}{r} \cdot v = \frac{v^2}{r}$$

The formula $a = \frac{v^2}{r}$ may be written in other forms. If T is the period of rotation and f is the frequency of rotation,

$$v = \frac{2\pi r}{T} = 2\pi r f$$

and $$a = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

Note also that as $P_2 \rightarrow P_1$, $\theta \rightarrow 0$, and that BC (Fig. 5.6*b*) is essentially perpendicular to AB. Therefore the vector Δv, and hence the acceleration vector a are directed toward the centre of the circle.

Let us now assume that Newton's Second Law, which we developed for straight line motion, holds also for circular motion. You will test the validity of this assumption in the Laboratory Exercise described in Section 5–8. If we use the Second Law formula, $F = ma$, for circular motion, we find that the magnitude of the centripetal force necessary to produce a centripetal acceleration $\frac{v^2}{r}$ is given by the formula $F_c = \frac{mv^2}{r}$. The force vector, like the acceleration vector, is directed toward the centre of rotation.

It should be realized that the actual force applied may not be equal to the centripetal force required to maintain circular motion. Under these circumstances, uniform circular motion does not occur. Two examples follow:

(*a*) Mud is thrown from a rotating bicycle wheel when the force of adhesion of the mud to the tire is less than the centripetal force required to cause the mud to follow the same circular path as the tire.

(b) For a space satellite circling the earth, the only force acting on the satellite is the gravitational force exerted by the earth. For a stable circular orbit, this gravitational force must be equal to the centripetal force required for that orbit.

5-8 LABORATORY EXERCISE: CENTRIPETAL FORCE

If Newton's second law holds for circular motion, then $F_c = \frac{mv^2}{r} = 4\pi^2 mrf^2$, where m is the mass in kg of the rotating object, r is the radius of rotation in metres, f is the frequency of rotation in revolutions per sec, and F_c is the centripetal force in newtons. You may test the validity of this formula with the apparatus shown in Figure 5.7.

The apparatus consists of a metal rod about one metre long, to which a spring balance calibrated in newtons is attached. One end of a nylon cord is attached to the balance. The cord passes through a polished glass tubing at the upper end of the rod, and the other end of the cord is attached to a rubber ball. The length of the cord between the glass tubing and the ball should be from 0.5 metre to 1.0 metre.

Hold the rod vertically, using both hands as shown. Practise whirling the ball in a horizontal circle with constant speed, so that the spring balance registers a constant force. When you have had sufficient practice, proceed to take measurements as follows.

Whirl the ball at constant speed and note the reading of the spring balance. Have your partner determine the time required for the ball to make 50 revolutions. At the same time you should note the position of the point of the balance hook with respect to the circular graduations on the rod. When your partner has finished timing the 50 revolutions, you may cease the whirling.

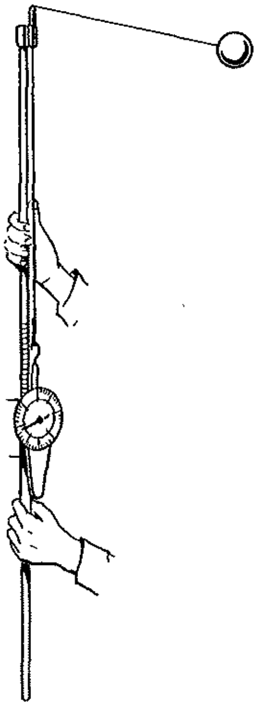


Fig. 5.7. Apparatus for measuring centripetal force.

The mass m of the ball may be determined by weighing the ball. The radius r of rotation of the ball is the distance from the glass tube to the centre of the ball, when the hook of the balance is in the position which you noted as the ball was being whirled. Measure this distance. Calculate the frequency f from the data which your partner recorded.

Compare the value of the product $4\pi^2mrf^2$ with the value of F_c which you read from the spring balance. Repeat the procedure several times. You may vary m by using balls (or rubber stoppers) of different sizes. You may vary r by adjusting the position of the balance on the metal rod. You may vary the frequency of rotation by whirling the ball at different speeds.

Within the limits of experimental error, is $F_c = 4\pi^2mrf^2$? Does Newton's second law hold for circular motion?

5–9 EARTH SATELLITES

The successful launching of an earth satellite is achieved by the use of multi-stage rockets. The prediction of the effect of the first stages must take into account the fact that the acceleration due to gravity changes significantly during the satellite's climb. The final stage is fired horizontally when the satellite reaches the desired height, and is designed to impart to the satellite the speed necessary to set it in a circular orbit about the earth.

Suppose we represent the satellite's orbital speed by v, and the radius of its orbit by R. Then the central acceleration is $\frac{v^2}{R}$, and this acceleration, if the orbit is to be circular, must be equal to g, the acceleration due to gravity at that altitude. That is,

$$\frac{v^2}{R} = g$$

The radius of the earth is about 6.4×10^6 m; therefore, at a height of 500 km, $R = 6.9 \times 10^6$ m. At this height $g = 8.4$ m/sec^2 approximately. Then $v = \sqrt{gR} = 7.6 \times 10^3$ m/sec. Under these conditions, then, the speed that must be imparted to the satellite by the rocket's final stage is about 18000 mi/hr.

We may calculate also the time required for the satellite to orbit the earth once. The distance travelled is the circumference $2\pi R$ of the orbit, approximately 43.3×10^6 m. The speed v we calculated as 7.6×10^3 m/sec. Therefore the time required is

$$\frac{43.3 \times 10^6 \text{ m}}{7.6 \times 10^3 \text{ m/sec}}$$

$$= 5.7 \times 10^3 \text{ sec} = 95 \text{ min.}$$

In practice, the correct combination of v and R is seldom achieved, and the orbit is elliptical rather than circular.

We need to be clear about one further phenomenon in connection with earth satellites. An astronaut in an orbiting space capsule is commonly said to be weightless, or to experience weightlessness. These terms do not mean that the force of gravity acting on him is zero; indeed it is the force of gravity which causes him to be centrally accelerated. If it were not for this force acting on him and on the capsule, both would travel in a straight line far out into space. Actually, as we saw in the calculations above, the acceleration due to gravity is about 8.4 m/sec^2, and therefore his weight is about $8.4 \div 9.8$, or about 0.86 of his weight on the surface of the earth.

The situation is that both the astronaut and the capsule are equally centrally accelerated and therefore the astronaut exerts no force on the materials upon which he is sitting or standing, and they exert no forces on him. Since we usually judge our weight by the magnitude of these forces, we say we are weightless if these forces are absent. Moreover, even though the astronaut may be moving through space at about 18000 mi/hr, he is not aware of this fact for he is not moving relative to the capsule. We considered relative motion briefly in Chapter 3; let us look more closely at it now.

5–10 FRAMES OF REFERENCE

Consider the sensations experienced by an astronaut in a space capsule during re-entry into the earth's atmosphere, during which time the capsule slows down rather quickly. If he tries to apply Newton's second law, he notes that with respect to, or in the frame of reference of, the capsule, he is not moving, but he feels a force acting on him. In the frame of reference of the capsule, then, Newton's second law does not apply. The reason is that the capsule is accelerating; Newton's second law does not hold in an accelerated frame of reference.

You may have noted a similar effect if you have ridden in a closed truck as it rounds a curve on a highway. Loose objects on the floor of the truck slide or roll across the floor; they are in motion relative to the truck with no force acting on them. If you wish to make their motion accord with Newton's second law, you must invent a force which you say is acting on them—a fictitious force. However, in an unaccelerated frame of reference, these fictitious forces are not necessary. Again, Newton's second law does not apply in an accelerated frame of reference.

You may wonder, then, if we should apply Newton's second law to the motions of objects on the surface of the earth. Surely the earth itself is rotating on its axis, and therefore constitutes an accelerated frame of reference in which Newton's second law is at least slightly invalid, and therefore requires a small fictitious force to restore its validity.

The classic experiment which indicates that the earth is indeed rotating was first performed by the French physicist Foucault. This experiment is performed with a pendulum consisting of a very heavy bob suspended by a wire 10 metres or more in length. If this pendulum is set vibrating, its inertia is great enough that it will continue to vibrate for several hours. As it vibrates, its plane of vibration continually rotates. The situation is most readily understood for a Foucault pendulum vibrating at the earth's geographic north or south pole. Here the plane of its vibration rotates 360° every 24 hours; perhaps it would be more reasonable to say that the plane of vibration remains fixed in space and that the earth rotates beneath it.

Foucault's experiment indicates that the earth does rotate, and that frames of reference attached to the earth are really accelerated frames in which Newton's laws are not valid. However, the effects of the earth's rotation are so small that they may be ignored except in the most precise experiments.

5-11 PROBLEMS

Assume, where necessary, that

$$g = 9.8 \text{ m/sec}^2 = 9.8 \text{ newtons/kg}$$

at or near the surface of the earth.

1. What is the weight, at the surface of the earth, of (*a*) a ball of mass 0.05 kg, (*b*) a man of mass 100 kg, (*c*) a truck of mass 3.0×10^3 kg?
2. The weight of a boy at the surface of the earth is 588 newtons. (*a*) What is his mass? (*b*) What would be his weight at an elevation where the gravitational field was 8.0 newtons/kg? (*c*) What would his mass be, at the elevation given in (*b*)?
3. A wooden block, sliding along a horizontal floor, is acted upon by a force of friction equal to 10% of the weight of the block. The block comes to rest from a speed of x m/sec, in 4 sec. Find x.
4. An elevator having a mass of 1400 kg ascends with an acceleration of 0.50 m/sec². What is the tension in the cable supporting the elevator?
5. An 8-kilogram mass and a 12-kg mass are suspended from opposite ends of a string which passes over a pulley. What will be the acceleration of the masses when the system is released? What assumptions did you make in solving this problem?
6. Consider the relationship $s = \frac{1}{2}gt^2$. (*a*) What is the effect on s of changing t by a factor of 4? (*b*) By what factor must t change if s is to change by a factor of 3? (*c*) If s is plotted versus t, what sort of graph results? (*d*) How could you obtain a straight line graph from this relationship? Try to do so, assuming, for the sake of simplicity, that $g = 10$ m/sec².
7. Suppose that the net force applied to an object is equal to the weight of the object. What will be the object's acceleration?
8. A stone falls freely from rest. Using $g = 10$ m/sec², find (*a*) its speed at the end of each of the first five seconds, (*b*) its average speed during each of the first five seconds, (*c*) the distance it falls during each second, (*d*) its distance from the starting point after 1, 2, 3, 4, and 5 sec.
9. A stone which is dropped from a cliff strikes the ground in 5 seconds. With what speed does it strike the ground? How high is the cliff?
10. A stone dropped from the top of a tower hits the ground with a speed of 60 m/sec. Find the height of the tower and the time required for the stone to reach the ground.
11. A 45.0-gm golf ball is dropped from a height of 160 cm to a level solid concrete floor. It rebounds to a height of 90.0 cm. Calculate (*a*) the impulse given to the ball by its own weight, during its fall, and (*b*) the impulse given to the ball by the floor. State in each case the direction of the impulse.
12. After having fallen from rest for 2 seconds, a 2-kg mass strikes a pile of sand and penetrates it to a depth of 10 cm. Find the average force exerted by the sand on the mass.
13. A body of mass 2 kg falls freely from rest. Calculate the rate of change of its momentum.

14. A mass of 300 gm rests on a smooth table. From it two horizontal light strings run in opposite directions. Each string runs over a smooth pulley, and to the end of one string is attached a mass of 90 gm. To the end of the other string is attached a mass of 100 gm. How far will the masses move in 2 sec after being released?
15. From the top of a cliff 90 m above a lake, a stone of mass 1.5 kg is thrown horizontally with a speed of 10 m/sec. Air resistance in both the horizontal and vertical planes has the effect of a retarding force equal to 1% of the weight of the stone. When and where will the stone strike the water?
16. From a window 44.1 metres above ground level, a ball is thrown with a horizontal velocity of 5 metres per second. What time is required for its descent to the ground? How far horizontally does it go? Make a sketch showing its path. Calculate its resultant speed at the time of impact with the ground.
17. An object follows a circular path with a constant speed of 8.0 m/sec. It changes direction by 180° in 2.0 sec. Calculate (*a*) the magnitude of its change in velocity, (*b*) the magnitude of its average acceleration during the 2.0 sec.
18. Assume that, under the attractive force of the earth, the moon revolves about it in a circular path with constant speed. (*a*) Is the moon accelerated toward the earth? (*b*) If your answer in (*a*) is yes, account for the fact that the speed remains constant. (*c*) Why does the force exerted on the moon by the earth not cause the moon to move closer to the earth?
19. (*a*) A train goes round an unbanked railway curve; (*b*) an automobile goes round an unbanked highway curve; (*c*) a boy stands on a moving swing. In each of these three cases, state: upon what body, upon what part of the body, and in what direction the centripetal force acts.
20. A circular ring with a groove on the inside rests in a vertical position. A marble rolls in the groove at high speed so that it does not leave the groove. Show, in a diagram, the vertical forces which act on the marble when it is (*a*) at the lowest point in its path, (*b*) at the highest point. Label the forces, indicating what they are exerted by.
21. Show that the expression $\frac{v^2}{r}$ has the units of acceleration.
22. Consider the relationship $F_c = \frac{mv^2}{r}$. What is the effect on F_c of (*a*) changing m by a factor of 3, (*b*) changing v by a factor of $\frac{1}{2}$, (*c*) changing r by a factor of $\frac{1}{4}$? Interpret each of the changes in terms of vehicles rounding curves in a road.
23. A car of mass 1.5×10^3 kg travels around a circular curve at a speed of 25 m/sec. If the radius of the curve is 75 m, calculate the centripetal force acting on the car. What exerts this centripetal force?
24. A 1500-kg mass rotates at a constant speed of 12 m/sec in a circle of radius 200 m. Calculate the magnitude of the centripetal force acting on the mass.
25. A one-kilogram stone is whirled in a vertical circle at the end of a string 1.5 m long. The constant speed of the stone is 5 m/sec. What is the tension

in the string, (*a*) when the string is horizontal, (*b*) when the stone is at the top of the circle, (*c*) when the stone is at the bottom of the circle?

26. The moon is an earth satellite with a period of about $27\frac{1}{3}$ days. Its radius of rotation (the distance from the earth to the moon) is 3.8×10^5 km. (*a*) Calculate the magnitude of the moon's centripetal acceleration. (*b*) State the direction of the acceleration. (*c*) What force causes this acceleration? (*d*) How does this force compare with the similar force at the earth's surface?

5-12 SUMMARY

1. The magnitude of $\vec{g}$, the acceleration due to gravity, is independent of mass but is dependent on elevation.
2. The gravitational force (weight) per unit mass is g newtons/kg. That is,

$$F_g = mg$$

3. For a projectile,
   (*a*) the path is parabolic,
   (*b*) the horizontal and vertical components of motion may be considered separately.
4. Circular motion, even at constant speed, is accelerated motion, because the direction of the velocity vector is continually changing.
5. The following formulas apply for circular motion at constant speed.

$$a = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

   Both the centripetal acceleration and centripetal force are directed to the centre of the circle.

6. For an earth satellite in a stable circular orbit,

$$v = \sqrt{gR}$$

$$T = \frac{2\pi R}{v}$$

7. Newton's Second Law is not valid in accelerated frames of reference. In order to make the second law seem to apply in accelerated frames of reference, we invent fictitious forces.
8. The wide applicability of Newton's Second Law in unaccelerated frames of reference should now be evident. The cases we have investigated are summarized below.
   (*a*) If the force vector and the velocity vector have the same direction, the effect of the force is to increase the magnitude of the velocity vector, without changing its direction.
   (*b*) If the force and velocity vectors have opposite directions, the effect of the force is to decrease the magnitude of the velocity vector, without changing its direction.
   (*c*) If the force vector is perpendicular to the velocity vector, the effect of the force is to change the direction of the velocity vector without changing its magnitude. Circular motion results.
   (*d*) In all other cases, the effect of the force is to change both the direction and magnitude of the velocity vector. This is the case for projectile motion. In these cases, the motion is most readily analysed by considering components parallel to, and perpendicular to, the force vector.

In all cases, the vector law $\vec{F} = m\vec{a}$ applies.

Chapter 6

Universal Gravitation

6–1 INTRODUCTION

We noted in Chapter 4 that the orbital motion of the planets, apparently in the absence of any force acting on them, puzzled the early philosophers. We noted, too, their unusual explanation that celestial matter possessed properties which terrestrial matter lacked. It was not until the seventeenth and eighteenth centuries that the problems of celestial motion were solved in terms acceptable to us today. The names of two of the men involved—Galileo and Newton—are already familiar to us, but there were many more who contributed a great deal. Who these others were, what their contributions were, how they arrived not only at a kinematic description but at a dynamic solution for celestial motion is a very interesting and instructive story. As we shall see in this chapter, they eventually discovered that a force is responsible for planetary motion. They described in mathematical terms the magnitude of that force, and they put an end to the theory that celestial and terrestrial mechanics differ. Only a very brief outline of this story can be given here.

6–2 EARLY IDEAS ABOUT THE UNIVERSE

More than twenty centuries ago scientists had assembled considerable information concerning astronomy. They observed that the so-called fixed stars seemed to move on spherical shells with the earth at their common centre (see Fig. 6.1). But seven celestial bodies—the sun, the moon, Mars, Mercury, Venus, Jupiter and Saturn appeared to move among the stars. Moreover, the motion of the latter five seemed erratic, and the name planet (wanderer, in Greek) was applied to all seven. How could their motions be explained?

Early explanations made two basic assumptions, both of which seemed reasonable at the time—and for many years later. The first assumption was that the universe was geocentric (earth-centred); that the earth was stationary at the centre of the universe. The second

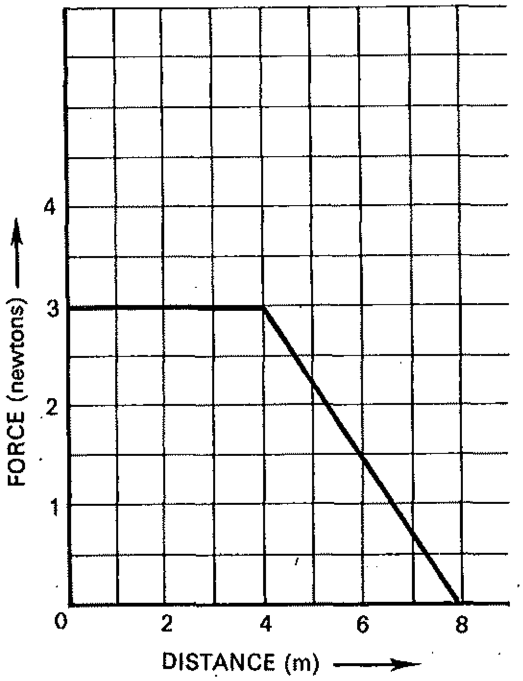


Fig. 8.11. For problem 14.

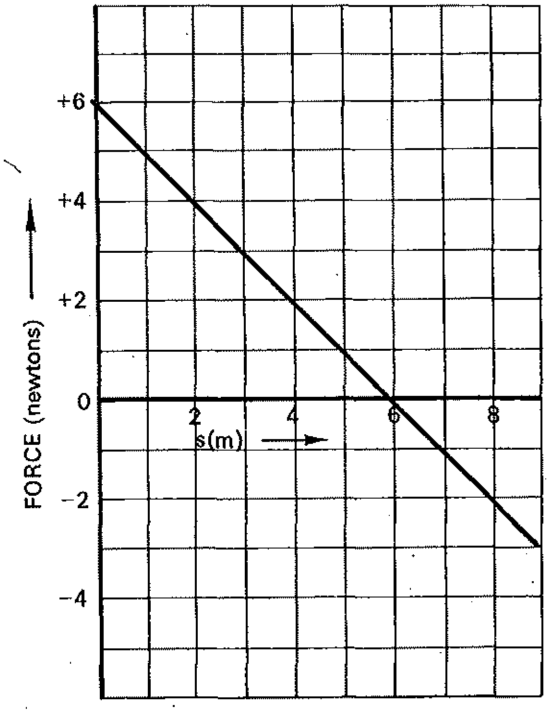


Fig. 8.12. For problem 15.

19. An object of mass 1.0 kg and speed 0.40 m/sec collides with a 3.0-kg object which is initially at rest. The forces of interaction depend only on the separation of the two objects. Calculate the velocity of each after the collision.

20. A neutron of mass 1.67 × 10^{-27} kg, travelling at a speed of 10^5 m/sec, collides with a stationary deuteron whose mass is 3.34 × 10^{-27} kg. The collision is elastic, and the particles do not stick together. Calculate the speed of each after collision.

21. Two spheres, *A* and *B*, are involved in a perfectly elastic, head-on, collision. The speed of *A* before collision is 10 m/sec; *B* is at rest. After collision *B* acquires a velocity of 16 m/sec. The mass of *A* is four times that of *B*. (*a*) What is the speed of *A* after impact? (*b*) What percentage of *A's* kinetic energy is transferred to *B*?

22. In Section 8–10, the following equation (equation 6) was developed:

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

(*a*) What is true of v_1 if (*i*) $m_1 > m_2$, (*ii*) $m_1 = m_2$, (*iii*) $m_1 < m_2$? (*b*) Check your mathematical predictions experimentally.

23. (*a*) Consider a head-on collision between a moving ball A and a stationary ball B of equal mass. Prove that, if the collision is elastic, A stops and B acquires a speed equal to the initial speed of A.
(*b*) Consider a glancing collision between a moving ball A and a stationary ball B of equal mass. Prove that, if the collision is elastic, the paths of A and B after collision are at right angles to each other.
24. A ball on the end of a string 40 cm long rotates in a horizontal circle with constant kinetic energy of 8 joules. (*a*) Calculate the centripetal force exerted by the string on the ball. (*b*) How much work does the centripetal force do on the ball during each revolution?
25. If the centripetal force acting on a rotating object did work on that object, what would be the effect on the energy possessed by the object? Is this actually the case? What conclusion must be drawn?

8–12 SUMMARY

1. Work = force × displacement

$$W = Fs \cos \theta$$

If $\vec{F}$ and $\vec{s}$ have the same direction, $\theta = 0$ and $\cos \theta = 1$.

Then $W = Fs$.

1 joule = 1 newton-metre.

2. The centripetal force does no work on a rotating object, and does not change the energy of the object.
3. The work done by the net force acting on an object is equal to the increase in kinetic energy of the object. That is,

$$Fs = \tfrac{1}{2}m(v^2 - u^2)$$

4. The area under a force-distance graph $= W = \Delta E_K$.
5. An interaction between two objects is elastic if the force of interaction depends only on the separation of the two objects.
6. In an elastic interaction, kinetic energy is conserved, in addition to momentum. That is, $\vec{\Delta p} = 0$ and $\Delta E_K = 0$.

Chapter 9

Potential Energy

9–1 INTRODUCTION

In Chapter 8 we noted that kinetic energy disappeared during the first stage of an elastic collision. During the second stage of the collision this kinetic energy is completely recovered, so that the total kinetic energy of the two objects is the same immediately after the collision as it was immediately before. However, we need to discuss more fully this disappearance and reappearance of kinetic energy. Does any or all of the energy really disappear; or is any or all of it temporarily transformed to another form?

9–2 STORED ENERGY

We shall try to answer these questions by considering again the slow elastic interaction which we considered first in Section 8–8. Figures 8.5 and 8.6 are reproduced here for your convenience. (See Figures 9.1 and 9.2.) We found in Section 8–8 that the kinetic energy of the car at a given position on the way in was the same as its kinetic energy at that position on the way out. The reason for the observed conservation of kinetic energy (before and after) is that the net force acting on the car depends on distance only, and not on the direction of the car's motion.

But kinetic energy is not conserved during the interaction; it becomes zero at the distance of closest approach (Fig. 9.1*f* or Fig. 9.2*a*). However, at this stage of the interaction, the magnetic force is able to, and is about to, do work on the car. That is, because the two magnets have been brought close together, energy has been stored in the system. This stored energy, or stored work, is called potential energy, and is given the symbol E_P. Because the interaction is elastic, the potential energy of the system, when the car is at the distance of closest approach, is equal to the kinetic energy lost by the car on the way in. Moreover, the potential energy lost by the system, as the car returns to its original position, is equal

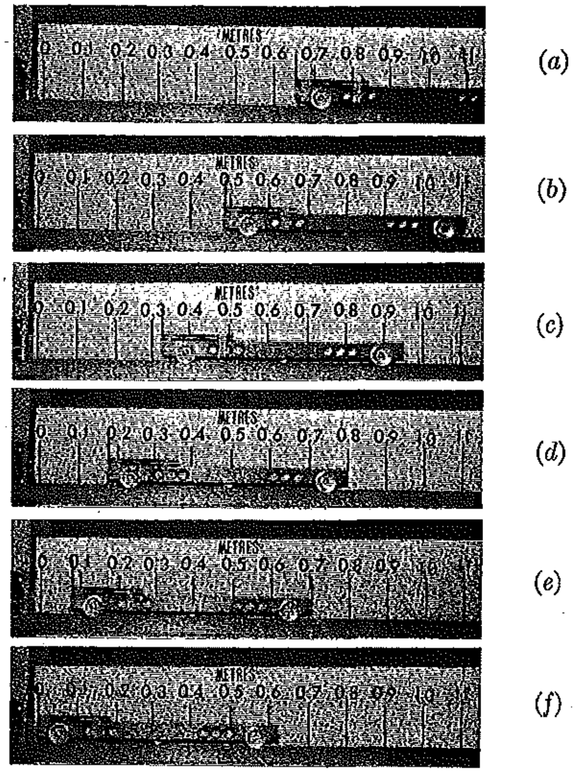
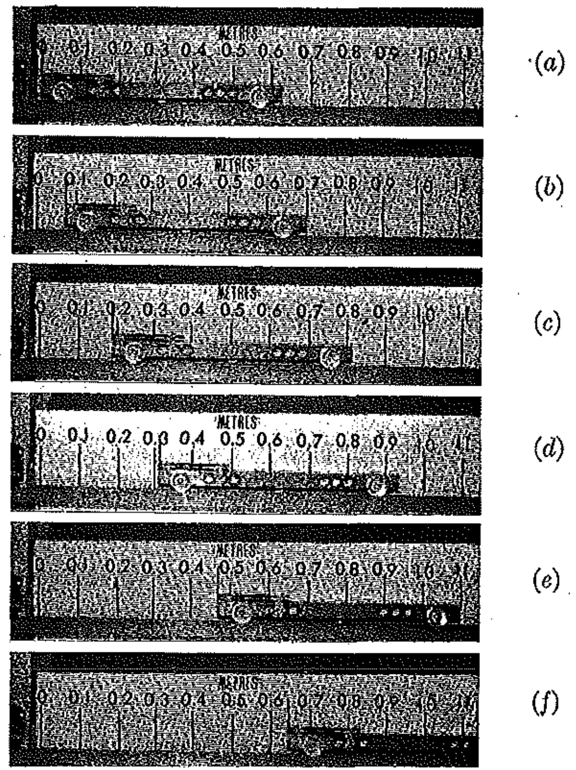


Fig. 9.1. Kinetic energy disappears during the first stage of a magnetic interaction.



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University of Western Ontario

Fig. 9.2. Kinetic energy reappears during the second stage of a magnetic interaction.

to the kinetic energy gained by the car on the way out.

Let us now examine the corresponding energy relationships when the car is at some intermediate position. At the stage shown in Figure 9.1*c*, the car has lost some kinetic energy and the system has gained some potential energy. Is the kinetic energy lost equal to the potential energy gained? Here again the answer depends on whether the interaction is elastic, that is, whether the force acting depends on separation only. If the collision is elastic,

$$\Delta E_K = -\Delta E_P$$
$$\text{or } \Delta E_K + \Delta E_P = 0$$
$$\text{or } E_K + E_P \text{ is constant.}$$

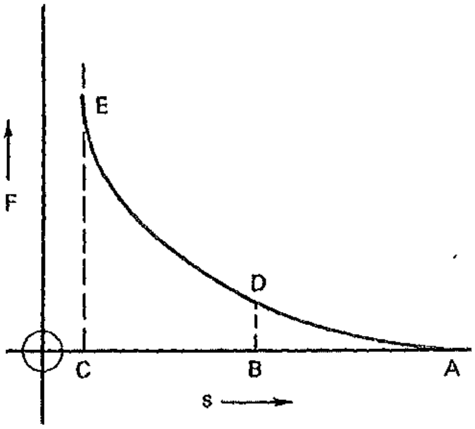


Fig. 9.3. Mechanical energy is conserved during an elastic interaction.

Any one of the three equations above is a mathematical statement of the law of conservation of mechanical energy. During elastic interactions—interactions not affected by internal or external frictional forces—the sum of the kinetic and potential energies remains constant. Any kinetic energy which disappears is converted entirely to potential energy and vice versa.

Conservation of mechanical energy for the magnetic interaction (Figs. 9.1 and 9.2) is shown graphically in Figure 9.3. When the car is at position *A* (at the limit of the range of interaction), the energy is entirely kinetic and is equal to the area of figure *ACE*. When the car is at position *C* (at the distance of closest approach), the energy is entirely potential and is equal to the area of figure *ACE*. When the car is at some intermediate position *B*, the energy is partly kinetic and partly potential. The kinetic energy at *B* is equal to the area of figure *BCED*; the potential energy at *B* is equal to the area of figure *ABD*.

9–3 GRAVITATIONAL POTENTIAL ENERGY

Suppose an object of mass *m* (Fig. 9.4) is elevated a distance Δh in the earth's

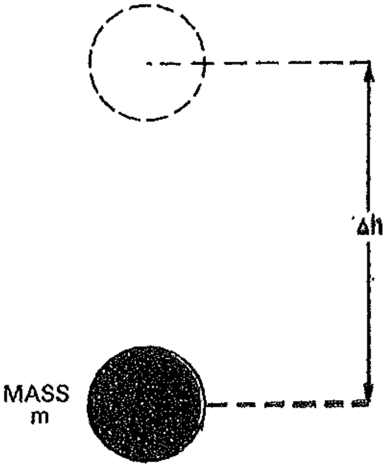


Fig. 9.4. When an object is elevated, its gravitational potential energy increases.

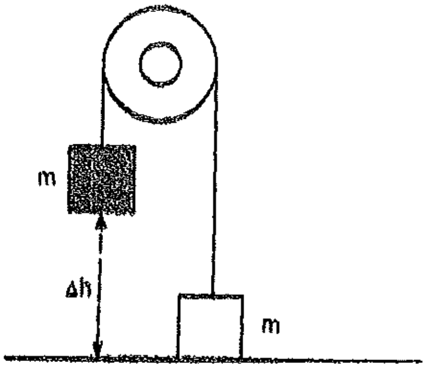


Fig. 9.5. The gravitational potential energy of an object gives it the ability to do work on a second object.

gravitational field. The force necessary to cause it to move upward at constant speed (that is, without any change in kinetic energy) is equal to the weight mg of the object. The work done on the object is then $mg\Delta h$, and because of this work that was done on it, the object is able to do work that it was unable to do before. If a string is attached to the object and passed over a frictionless pulley to a second object of mass m as shown in Figure 9.5, and if the first mass is given a slight downward push it can elevate the second mass through a distance Δh. Thus any object possesses potential energy because of its position in the earth's gravitational field. This energy is called gravitational potential energy and we give it the symbol E_G. The change ΔE_G in an object's gravitational potential energy as it undergoes a change Δh in height is given by the formula

$$\Delta E_G = mg\Delta h$$

As an object falls, its kinetic energy increases and its potential energy decreases. If the fall takes place in a vacuum, or if, for practical purposes, air resistance

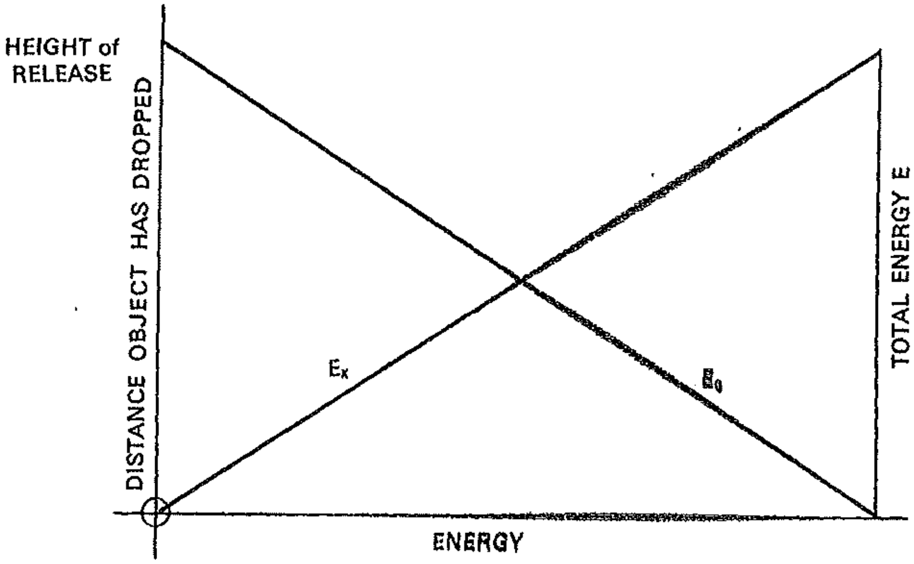


Fig. 9.6. The sum of the kinetic and gravitational potential energies of a falling object remains constant.

is negligible, mechanical energy is conserved, that is,

$$\Delta E_K + \Delta E_G = 0$$

In other words, the total energy of the object remains constant. We shall not go through all of the reasoning involved here; it is the same as that for the trolley discussed earlier in this chapter. Note that the force of gravity is essentially constant if Δh is small, and that its value does not depend on whether the object is moving up or down. Figure 9.6 shows the relationship between the kinetic energy E_K, the gravitational potential energy E_G and the total energy E.

9–4 WORKED EXAMPLE

A projectile of mass 20 kg is projected vertically upward with an initial speed of 50 m/sec. Find (*a*) its original kinetic energy, (*b*) its kinetic energy after 2 sec, (*c*) the change in its gravitational potential energy during these 2 sec.

Solution

(*a*)

$$\begin{aligned} E_K &= \tfrac{1}{2}mv^2 \\ &= \tfrac{1}{2} \times 20 \times 50^2 \text{ joules} \\ &= 2.5 \times 10^4 \text{ joules} \end{aligned}$$

(*b*) Using the formula $v = u + at$, and choosing the downward direction as the positive vector direction,

$$\begin{aligned} v &= (-50 + 9.8 \times 2) \text{ m/sec} \\ &= -30.4 \text{ m/sec} \end{aligned}$$

That is, the upward speed of the projectile at the end of 2 sec is 30.4 m/sec.

$$\begin{aligned} E_K &= \tfrac{1}{2} \times 20 \times 30.4^2 \text{ joules} \\ &= 9.2 \times 10^3 \text{ joules} \end{aligned}$$

(*c*)

$$\begin{aligned} \Delta E_K &= 9.2 \times 10^3 \text{ joules} \\ &\quad - 2.5 \times 10^4 \text{ joules} \\ &= -1.6 \times 10^4 \text{ joules} \end{aligned}$$

If mechanical energy is conserved,

$$\Delta E_G = -\Delta E_K = +1.6 \times 10^4 \text{ joules.}$$

That is, the increase in $E_G = 1.6 \times 10^4$ joules.

The increase in E_G may be calculated by another method.

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ \Delta h &= (50 \times 2 - \tfrac{1}{2} \times 9.8 \times 4) \text{ metres} \\ &= 80.4 \text{ m} \\ \Delta E_G &= \text{mg}\Delta h \\ &= 20 \times 9.8 \times 80.4 \text{ joules} \\ &= 1.6 \times 10^4 \text{ joules} \end{aligned}$$

9–5 LABORATORY EXERCISE: POTENTIAL ENERGY

(*a*) *The force-extension ratio for a spring.* The extension s of a spring depends on the magnitude of the force F used to stretch the spring. To determine the nature of the relationship between F and s, hang a spring from a support (Fig. 9.7*a*). Mark the position of the lower end of the spring. Now hang a 0.5 kg mass on the end of the spring and mark the new position of the lower end of the spring (Fig. 9.7*b*). The distance between the two markers is the extension s. The force in this case is the weight of the 0.5 kg mass, that is, 4.9 newtons.

Repeat the above procedure for several different masses, being careful not to stretch the spring too far. Draw a graph with s as abscissa and F as ordinate. Is the graph a straight line, within the limits of experimental error? If the graph is a straight line, what is the relationship between F and s? What is the slope of the graph? The slope—the constant value of $\frac{F}{s}$—is called the force constant, or force-extension ratio, of the spring, and is usually given the symbol k. The equation of the graph is then $F = ks$.

(*b*) *Potential energy stored in a spring.* When a spring is stretched, work is done, and potential energy is stored in the

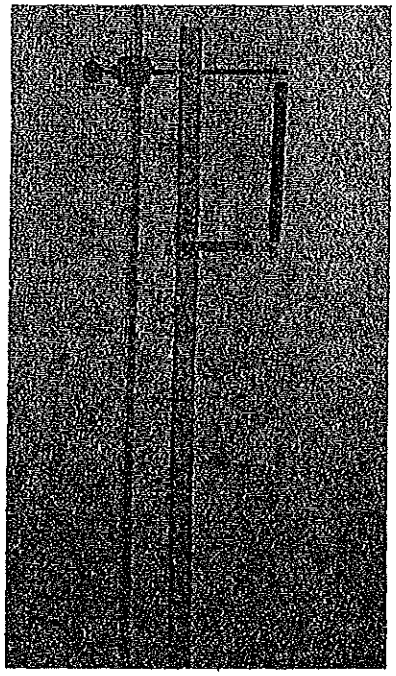


Fig. 9.7(a). An unloaded spring hanging vertically.

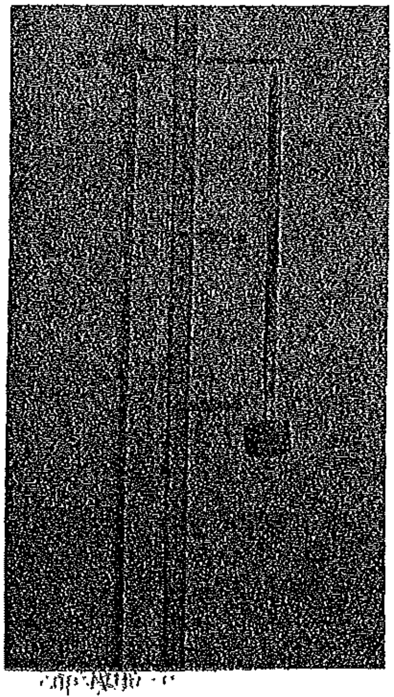


Fig. 9.7(b). The same spring stretched by a 0.5 kg mass.

spring. If the interaction is elastic, the work done by the force stretching the spring is equal to the potential energy stored in the spring. The potential energy may be calculated from the force-extension graph (Fig. 9.8). The potential energy stored at extension s_1 is the area of triangle OAB and is equal to $\frac{1}{2}OA \cdot AB$.

$$\tfrac{1}{2}OA \cdot AB = \tfrac{1}{2}s_1F_1$$

$$\text{But } F_1 = ks_1$$

$$\therefore\ E_P = \tfrac{1}{2}ks_1^2$$

Similarly the potential energy stored when the extension is s_2 is the area of triangle OCD and equals $\frac{1}{2}ks_2^2$.

The increase in potential energy as the extension increases from s_1 to s_2 is $\frac{1}{2}k(s_2^2 - s_1^2)$ and is equal to the area of figure $ABDC$.

Calculate the potential energy stored in the spring for extensions of 20, 25 and 30 cm, and the increase in potential energy as the extension increases from 20 cm to 30 cm. If your graph of force versus extension is not a straight line, the increase in potential energy must be found from the area of figure $ABDC$ on the graph, and not from the expression $\frac{1}{2}k(s_2^2 - s_1^2)$.

(c) *Changes in potential energy.* Hang a one-kilogram mass on the spring and support it with your hand (Fig. 9.9*a*) so that the extension is about 20 cm. Mark the position of the lower end of the spring. Release the mass, and mark the position of the lower end of the spring when the mass is at its lowest point (Fig. 9.9*b*). Several trials may be necessary. Calcu-

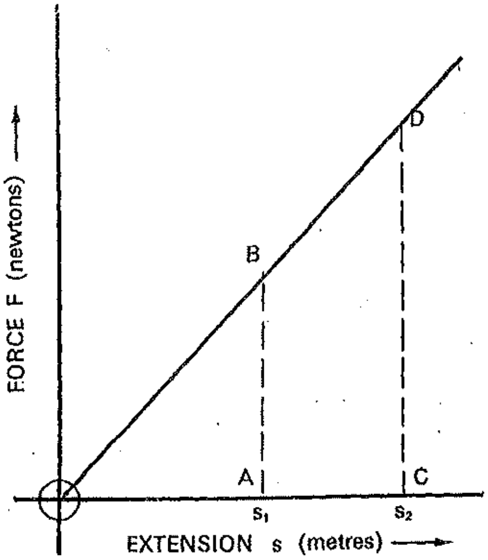


Fig. 9.8. Force-extension graph for a spring.

late the increase in the potential energy stored in the spring, and the loss of gravitational potential energy of the mass. Are the two quantities equal? Did you expect them to be equal? Is mechanical energy conserved in this interaction? Is it an elastic interaction?

9-6 CALCULATION OF ΔE_G WHEN Δh IS LARGE

The change in an object's gravitational potential energy cannot be calculated from the formula $\Delta E_G = mg\,\Delta h$ if Δh is so large that g varies appreciably. In such cases, a more general formula must be used; the development of this formula follows.

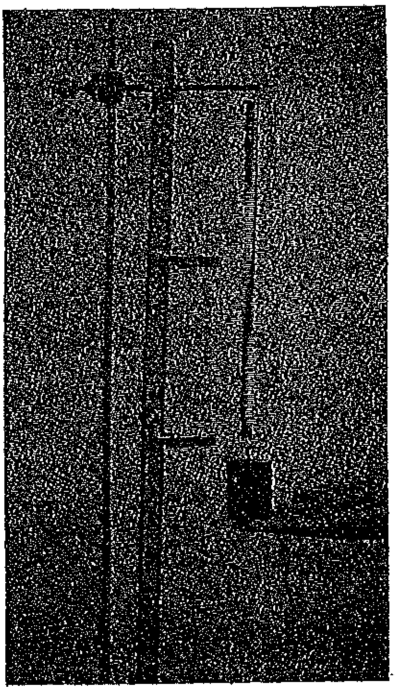


Fig. 9.9(a). The one-kilogram mass is supported by hand, limiting the extension of the spring to about 20 cm.

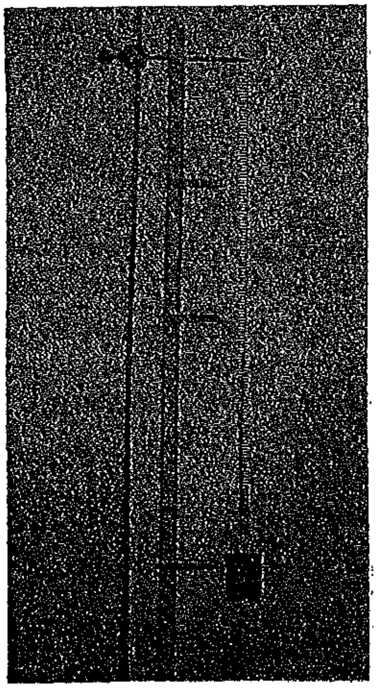


Fig. 9.9(b). When the mass is released, it falls to the position shown here.

We have seen in Chapter 6 that the gravitational force F_G exerted by the earth on an object of mass m at a distance r_1 from the centre of the earth is given by the formula

$$F_G = \frac{GmM}{r_1^2}$$

where M is the mass of the earth and G is the gravitation constant. If this gravitational force remains constant, the work necessary to elevate an object from a distance r_1 to a distance r_2 from the earth's centre (Fig. 9.10) is given by $W = Fs$.

$$\therefore\ W = \frac{GmM}{r_1^2}(r_2 - r_1)$$

But the force does not remain constant, and r_1^2 is not the correct denominator to use here, nor is r_2^2. By means of mathematics beyond the scope of this book, it may be shown that the denominator should be r_1r_2.

$$\therefore\ W = \frac{GmM(r_2 - r_1)}{r_1r_2}$$

or

$$W = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

But the work done against gravity is equal to the gravitational potential energy gained by the object.

$$\therefore\ \Delta E_G = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

9–7 ZERO OF GRAVITATIONAL POTENTIAL ENERGY

When does an object in the earth's gravitational field possess zero gravitational potential energy? This question does not really need to be answered, for a knowledge of the change in gravitational potential energy is all that is necessary in most cases. However, formulae and calculations are simplified if we make an arbitrary choice of the level at which E_G is zero. Two such choices are widely used.

(*a*) Heights of buildings are usually measured from ground level; that is, the height of the ground is taken as zero. Similarly, the gravitational potential energy of an object may be taken as zero at ground level, or at any other convenient level. If heights are measured from this level, the formula $\Delta E_G = mg\,\Delta h$ becomes $E_G = mgh$.

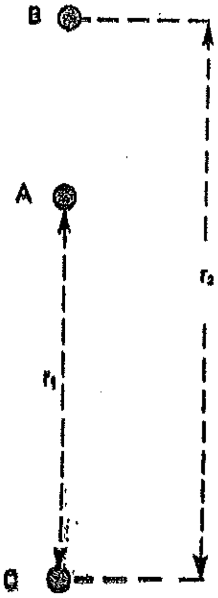


Fig. 9.10. Work must be done to elevate an object in the earth's gravitational field.

(*b*) Another choice is frequently made when discussing the motions of earth satellites. In this case, the value of E_G for an object is said to be zero when the object is at an infinite distance from the centre of the earth. Since the value of E_G increases as the distance from the centre of the earth increases, it follows that the value of E_G is negative at any finite distance from the earth's centre. Let us examine the situation mathematically.

Suppose an object is at a distance r from the centre of the earth, and is then removed to an infinite distance. Substituting in the formula

$$\Delta E_G = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

we obtain $\Delta E_G = GmM\left(\frac{1}{r} - 0\right)$

$$\therefore \Delta E_G = \frac{GmM}{r}$$

But the final value of E_G is zero, therefore the initial value of E_G must have been $-\frac{GmM}{r}$. (There is an easy analogy here. If the temperature increases 5 degrees to a final value of zero, then the initial temperature must have been -5 degrees). Therefore, assuming zero potential energy at an infinite distance from the earth, the potential energy at any finite distance r is given by the formula

$$E_G = -\frac{GmM}{r}$$

9–8 ESCAPE ENERGY AND ESCAPE VELOCITY

Suppose that we wish to launch an earth satellite which is meant to escape from the earth's gravitational field rather than to go into orbit. What minimum speed and kinetic energy must the satellite have? As it moves away from the earth, its kinetic energy decreases and its potential energy increases. If we ignore the effects of air resistance in the initial stages,

$$\Delta E_G = -\Delta E_K$$

But $\Delta E_G = -\frac{GmM}{r_e}$

where r_e is the earth's radius.

$$\therefore \Delta E_K = \frac{GmM}{r_e}$$

The minimum kinetic energy at launching must be $\frac{GmM}{r_e}$ for then the kinetic energy at infinite distance would just be zero. This works out to about 9.4×10^{10} joules for a 3000 pound satellite. This energy is called the escape energy of the satellite; it depends on the satellite's mass.

The escape velocity is the minimum initial speed (upward) which the satellite must have in order to escape. It is independent of mass, because

$$E_K = \tfrac{1}{2}mv^2 = \frac{GmM}{r}$$

and $\therefore v^2 = \frac{2GM}{r}$

The escape velocity works out to about 11.2 km/sec, or about 25000 mi/hr.

9–9 BINDING ENERGY

The total energy E of a satellite is the sum of its potential and kinetic energies.

$$E = \tfrac{1}{2}mv^2 - \frac{GmM}{r}$$

This total energy may be positive, zero, or negative. If the total energy is positive, the satellite can escape with kinetic energy to spare. If the total energy is zero, it can just escape. If the total energy is negative, the satellite cannot escape; it is bound to the earth.

Suppose that the total energy is -10^7 joules. If the satellite is to escape, its energy must be at least zero; that is, 10^7 joules of energy must be supplied to it. This 10^7 joules of energy is called the binding energy of the satellite. In general, for any object in the gravitational field of the earth,

Binding Energy $=$

$$-E = \frac{GmM}{r} - \tfrac{1}{2}mv^2.$$

9–10 PROBLEMS

Where necessary, use
$g = 9.8$ m/sec^2 at or near the earth's surface
$G = 6.67 \times 10^{-11}$ newton-metres^2/kg^2
mass of earth $= 6.0 \times 10^{24}$ kg
radius of earth $= 6.4 \times 10^6$ m

1. List as many systems as you can in which energy is stored and released later.
2. Consider the trolley apparatus shown in Figures 9.1 and 9.2. If the track is level, kinetic energy is not conserved. Explain.
3. In some areas electric motors are used to elevate water to reservoirs. Later, the water is released to turn generators to produce electricity. Discuss the procedure from the point of view of conservation of mechanical energy.
4. A book weighing 12 newtons is lifted 3.0 m. Calculate (*a*) the work done on the book, (*b*) the change in its potential energy.
5. A 60-gm mass projected vertically upward reaches its maximum height in 5 seconds. Calculate (*a*) the speed of projection, (*b*) the initial kinetic energy, (*c*) the maximum height, and (*d*) the gravitational potential energy at maximum height.
6. A boy on a sled starts at rest at the top of an icy hill. If the vertical height of the hill is 15 m, and if his speed at the bottom is 10 m/sec, what per cent of his initial potential energy was not converted into kinetic energy?
7. A hoist lifts a 3-kg stone to a height of 100 m and then drops it. What is the kinetic energy of the stone when it is half-way to the ground?
8. A stone of mass 0.20 kg is carried in a helicopter which is hovering 200 m above the ground. (*a*) What is the gravitational potential energy of the stone relative to the ground? (*b*) The stone is thrown vertically down with an initial speed of 7.0 m/sec. Calculate (*i*) its kinetic energy after it has fallen for 5 sec, (*ii*) its gravitational potential energy after it has fallen for 5 sec.
9. A pendulum consists of a 50-gm mass on the end of a string 60 cm long. The mass is pulled aside until the string makes an angle of 60° with the vertical, and is then released. What will be its maximum speed as it vibrates?
10. A box of sand of mass 10 kg hangs at the end of a long, light rope. When a bullet of mass 45 gm and moving horizontally strikes the box and remains buried in it, the box swings until it is 15 cm above its initial height. Calculate the initial speed of the bullet.
11. A 0.2-kg bullet travelling horizontally at 500 m/sec strikes and imbeds itself in a stationary wooden block suspended at the end of a long wire, causing the block to swing. If the mass of the block is 200 kg, calculate

(*a*) the speed of the block immediately after impact, (*b*) the maximum height to which the block rises as it swings.

12. A 1.0-kg object is projected up from the top of a cliff at an angle of 60° with the horizontal. If the cliff is 40 m high and the initial speed of projection of the object is 20 m/sec, calculate the magnitude of the velocity of the object when it is 10 m above the earth's surface at the base of the cliff.

13. The force-extension graph for a spring is shown in Figure 9.11. Calculate (*a*) the work that must be done to extend the spring (*i*) 0.2 m, (*ii*) 0.4 m, (*b*) the potential energy stored in the spring when the extension is (*i*) 0.2 m, (*ii*) 0.4 m.

14. The force-compression graph for a spring is shown in Figure 9.12. Calculate (*a*) the potential energy stored in the spring when it is compressed 0.1 m, (*b*) the work necessary to compress it 0.4 m, (*c*) the potential energy lost by the spring as its compression changes from 0.4 m to 0.2 m.

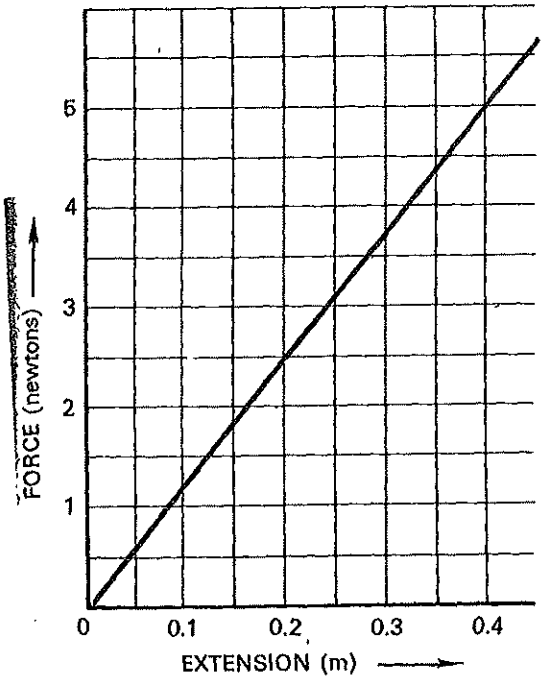


Fig. 9.11. For problem 13.

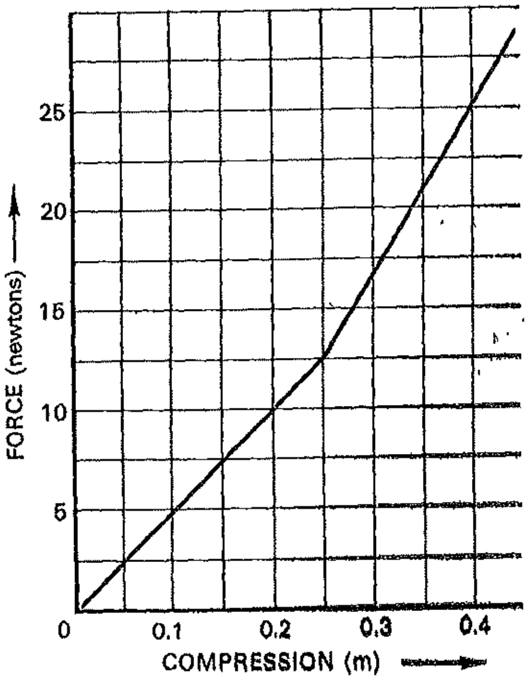


Fig. 9.12. For problem 14.

28. In Section 5-9 we showed that the speed v of a satellite in a circular orbit of radius r is given by the formula $v = \sqrt{gr}$. We also know that $g \propto \frac{1}{r^2}$. Show that the speed decreases as the radius of the orbit increases. What effect does an increase in the radius of the orbit have on the period of the satellite?
29. Show that the kinetic energy of a satellite in a stable circular orbit is exactly $\frac{1}{2}$ of its escape energy at the altitude of the orbit.

9-11 SUMMARY

1. Potential energy is stored energy.
2. During an elastic interaction, mechanical energy is conserved. That is, $\Delta E_K = \Delta E_P$.
3. If Δh is small, $\Delta E_G = mg\,\Delta h$. If Δh is large, $\Delta E_G = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.
4. If E_G is taken as zero at an infinite distance from the earth, then,
   (*a*) E_G (at distance r) $= -\frac{GmM}{r}$,
   (*b*) Escape energy of a satellite $= \frac{GmM}{r_e}$,
   (*c*) Escape velocity of a satellite $= \sqrt{\frac{2GM}{r_e}}$,
   (*d*) Total energy of a satellite $= \frac{mv^2}{2} - \frac{GmM}{r}$,
   (*e*) Binding energy of a satellite $= \frac{GmM}{r} - \frac{mv^2}{2}$.

Chapter 10

Conservation of Energy

10–1 INTRODUCTION

In an inelastic collision, the total kinetic energy after collision is less than the total kinetic energy before collision, and yet the potential energies of the colliding objects have not changed. Mechanical energy (kinetic plus potential energy) is not conserved.

As an object falls through air, it accelerates for a time, but eventually reaches a limiting constant speed. Thereafter as it descends it loses gravitational potential energy but does not gain kinetic energy. Again, mechanical energy is not conserved.

As a curling stone slides along a sheet of ice, it slows down and comes to rest. It loses kinetic energy but it does not gain potential energy, and again the total mechanical energy decreases.

The one common factor in all of these cases seems to be friction. The force of friction exerted by the ice on the curling stone, the force of friction exerted by the air on the falling object, and internal friction within colliding objects seem to be responsible for the energy losses. But what becomes of this lost energy? What is the effect of the work done by the forces of friction?

10–2 THE EFFECT OF FRICTION

The answer to the above questions is fairly obvious to anyone who has warmed his hands by rubbing them together, or started a fire by rubbing two sticks together. Friction is responsible for the production of heat.

In some cases, the amount of heat produced may be so small that it passes unnoticed. This is true for a curling stone sliding on ice, and for a stone falling through air for a short distance. Here the rate of loss of mechanical energy is low. On the other hand, the nose cone of a satellite re-entering the earth's atmosphere becomes very hot. In this case, the rate of loss of mechanical energy is high.

It may be, then, that the loss of mechanical energy in frictional interactions is balanced by the production of heat. As a result, we may be able, by considering heat as a form of energy, to say that energy is conserved in inelastic interactions. Before we come to this conclusion, we must show that the heat energy produced is proportional to the mechanical energy which disappears.

10–3 THE MECHANICAL EQUIVALENT OF HEAT

Prior to 1800, heat was not considered to be a form of energy, and units other than those used for measuring mechanical energy were adopted for measuring quantities of heat. The calorie, for example, is defined as the quantity of heat required to raise the temperature of one gram of water one centigrade (Celsius) degree. In the half century following 1800, experiments made it clear that heat could be considered as a form of energy.

In 1798 Count Benjamin Rumford (1742-1814) established that, in boring cannon, the amount of heat evolved had little relation to the quantity of shavings, the sharpness of the tools, or the kind of metal, but was proportional to the amount of mechanical work expended. The precise relationship between the heat produced and the mechanical work expended was not determined, however, until James Joule (1818-1899) performed a series of experiments between 1843 and 1850. In one experiment, water was churned by paddles and the rise in temperature of the water was compared with the mechanical work done in turning the paddles. In another experiment, mercury contained in an iron vessel was stirred with an iron paddle. In yet another experiment, heat was produced by rubbing two iron rings together under mercury. In all of these experiments, Joule found a constant ratio (within the limits of experimental error) between the heat produced and the mechanical work done. This constant ratio is called the mechanical equivalent of heat and is denoted by the symbol J. Thus $J = \frac{W}{H}$, where W is the mechanical work done and H is the heat produced.

The apparatus used by Joule in the water-churning experiment is illustrated in Figure 10.1. Paddles immersed in water in a calorimeter are turned when masses M_1 and M_2 descend and turn the spindle of the wheel. The mechanical work done is calculated by multiplying the sum of the weights of the masses M_1 and M_2 by the distance through which they fall. The heat produced is measured by multiplying the mass of the water plus the water equivalent of the calorimeter by the rise in temperature.

Since Joule's time, many experiments have been carried out to determine the value of the ratio $\frac{W}{H}$. The value commonly accepted now is 4.186 joules per calorie; that is 1 calorie of heat energy is equivalent to 4.186 joules of mechanical energy.

10–4 THE NATURE OF HEAT ENERGY

Experiments such as those performed by Joule indicate that heat may be considered as a form of energy. But what sort of energy is it—a new form or one related somehow to either the potential

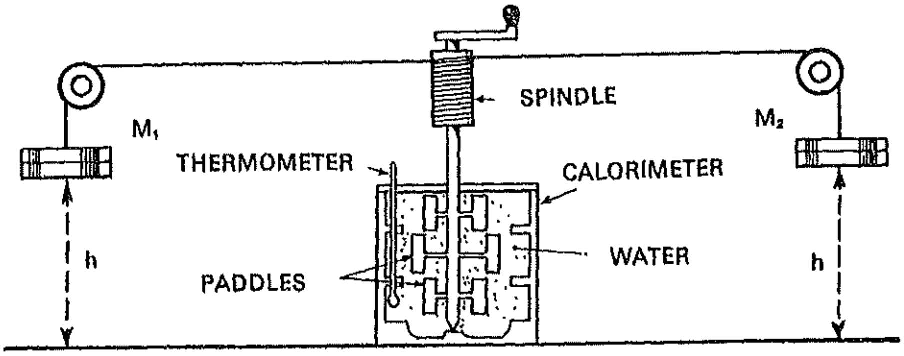


Fig. 10.1. The apparatus used in Joule's experiment to determine the mechanical equivalent of heat.

or kinetic energy with which we are already familiar? The molecular theory of matter, which also was developed in the nineteenth century, helped answer this question.

The molecular nature of matter became evident as the result of many experiments, particularly in the field of chemistry. A molecular model was constructed which pictured a gas as being composed of molecules in rapid motion, and separated from one another by distances which are large compared with the dimensions of the molecules themselves. This model provided explanations for many properties of gases. We cannot discuss all of these explanations here; we shall discuss only the energy of the molecules, for it is this energy which accounts for their heat content.

Molecular motion may be of several forms. (*a*) The molecule may be undergoing motion in a straight line and possess kinetic energy of translation. This kinetic energy is the same as the kinetic energy of moving objects which we considered in Chapter 8. The molecules of monatomic gases undergo translational motion only. (*b*) Polyatomic gas molecules (molecules composed of several atoms) may rotate and therefore possess rotational kinetic energy. (*c*) In polyatomic molecules, the atoms may vibrate within the molecule and therefore possess kinetic energy of vibration.

The kinetic energy of translation of a gas molecule can be shown to be proportional to the absolute temperature of the gas. This means that we may consider the temperature as a measure of the average kinetic energy of translation of the molecules. Rotational and vibratory motions do not affect the temperature.

It would seem, then, that the total heat content of a gas would be the sum of the average kinetic energies of translation of all the molecules. This is true for a monatomic gas, but for polyatomic gases the rotational and vibrational energies have to be considered as well. In addition, potential energy changes due to changes in the arrangements of the atoms in the molecules may have to be taken into consideration.

The higher the temperature of an object, the more rapidly its molecules move. If the rapidly moving molecules of a hot object or a hot portion of an object collide with the more slowly moving molecules of a colder object or a colder portion, some kinetic energy is transferred to the latter. Thus heat flow or conduction may be explained, in part at least. (There is considerable evidence of electron transfer as well.)

If heat energy is removed from a substance, by conduction or other means, the molecules slow down and the temperature drops. The average distance between molecules decreases, and the substance contracts or may even undergo a change of state, from a gas to a liquid, or from a liquid to a solid.

10–5 THE LAW OF CONSERVATION OF ENERGY

Having chosen to define heat as a form of energy, we are tempted to conclude that there is a law of conservation of energy which applies universally. Before we make such a conclusion, let us review the cases which we have considered in Chapters 8, 9 and 10.

(*a*) *Interactions free of friction.* Here we include elastic collisions, objects falling in a vacuum, and the motion of a pendulum. Friction may not be completely absent in all of these examples, but its effect is negligible, and therefore little heat is produced. The energy which we have to consider then is either kinetic or potential.

In an elastic collision, no potential energy is gained permanently by either of the colliding masses. Therefore, if there is a law of conservation of energy, kinetic energy should be conserved. This we found to be the case.

When an object changes elevation (and this includes the mass at the end of the suspension wire in a pendulum), its potential energy changes. However, we have found that its kinetic energy changes too, and that $\Delta E_K = -\Delta E_G$. We found a similar relationship when we considered the trolley in a magnetic field (Sect. 9–2). Again, a law of conservation of energy seems to be applicable.

(*b*) *Frictional interactions.* Here we include inelastic collisions and objects falling through air or other fluids. In fact we include any interaction in which friction reduces the total mechanical energy of the system of objects which we consider. The mechanical energy of such a system—often called the mechanical energy of bulk motion—is not conserved. If we are to insist that a law of conservation of energy applies here, we must look for internal energy which is stored in the system. We find it in the changed molecular energy, that is, as heat, which we decided to call a form of energy. When we conclude, as a result of experiment, that 1 calorie = 4.186 joules, we are really assuming that all of the mechanical energy lost is converted into heat energy. This assumption is not an unreasonable one. We feel convinced (though we cannot prove it), that energy should be conserved, and that no other recognizable forms of energy are produced.

As a result of countless experiments involving energy in many forms, it seems likely that energy is always conserved. Energy may be transformed from one form to another, but the total amount of energy after the transformation is the same as the total amount of energy before the transformation. The application of this principle to a scale which encompasses the whole universe is now under investigation. But we feel reasonably confident in applying it to smaller systems; in fact it has achieved the status of being one of the basic laws of science.

10–6 PROBLEMS

Assume, where necessary, that
1 calorie = 4.2 joules
$g = 9.8$ m/sec^2

1. A force of 0.5 newtons moves a 100-gm mass at a uniform speed of 50 cm/sec on a rough horizontal surface for 30 sec. Calculate (*a*) the force of friction, (*b*) the work done by the applied force, (*c*) the heat produced.
2. A body of mass 8 kg falls from a height of 80 m into a pile of sand. If all the kinetic energy at impact is transformed into heat energy, find the number of calories of heat produced.
3. Calculate the rate, in joules/sec, at which heat is being produced when an object of mass 5 kg falls through air at a constant terminal velocity of 100 m/sec.
4. A ball of mass 0.5 kg is dropped from a height of 250 m and strikes the ground with a speed of 40 m/sec. Calculate the heat produced as a result of the friction between the ball and the air.
5. A 3.6 gm bullet is fired horizontally through a 4.8-kg wooden block suspended by a long cord. The bullet emerges from the block with $\frac{1}{3}$ of the speed with which it enters, and the block starts to move at 12 cm/sec. Find (*a*) the speed with which the bullet enters the block, (*b*) the kinetic energy lost by the system as a result of the collision, (*c*) the heat produced.

10–7 SUMMARY

1. As a result of an inelastic interaction, kinetic energy is lost, and during an inelastic interaction, mechanical energy is lost. These losses can be accounted for by considering the heat produced to be a form of energy.
2. The heat produced as a result of an inelastic interaction is proportional to the mechanical energy lost.
   1 calorie = 4.19 joules
3. Heat may be considered as molecular mechanical energy.
4. It seems likely that energy is conserved in all interactions.

QUESTIONS FOR REVIEW

1. Define each of the new terms introduced in Chapters 1 to 10.

2. What is a law in Physics?

3. State each of the laws developed in Chapters 1 to 10. Indicate clearly the restricting or limiting conditions, if any, associated with each law.

4. Summarize the formulae developed in Chapters 1 to 10. For each formula, state (*a*) the meaning of each of the symbols used, (*b*) the restricting or limiting conditions, if any, associated with the formula, (*c*) the units associated with each symbol in the formula.

5. List the graphs for which either the slope, or the area under a part of the graph, has special significance, and state what significance it has.

6. What is the meaning of each of the following statements?
   (*a*) The speed of sound (under certain conditions) is constant at 1100 ft/sec.
   (*b*) The instantaneous speed of a car is 27 mi/hr.
   (*c*) The average speed for a trip was 37 mi/hr.
   (*d*) When the two cars collided, their relative speed was 120 mi/hr.
   (*e*) The acceleration of a falling object is constant at 9.8 m/sec^2.
   (*f*) The average acceleration of a scooter in the first 5 seconds after starting to move is 2.5 m/sec^2.
   (*g*) The instantaneous acceleration of a truck is 0.7 m/sec^2.
   (*h*) The central acceleration of a car rounding a curve in the highway is 1.6 m/sec^2 toward the centre of the curve.
   (*i*) The acceleration of an object is proportional to the net force acting on it.
   (*j*) The acceleration of an object is inversely proportional to its mass.
   (*k*) The gravitational attraction between two objects is inversely proportional to the square of the distance between them.
   (*l*) Motion in a circle at constant speed is accelerated motion.
   (*m*) Momentum is a vector quantity.
   (*n*) Work and energy are scalar quantities.
   (*o*) A collision between two steel balls is (almost) an elastic interaction.
   (*p*) A collision between two balls of putty is an inelastic interaction.
   (*q*) An object at rest in the gravitational field of the earth possesses potential energy.
   (*r*) The escape energy of a satellite is about 6.2 joules per kilogram.
   (*s*) The escape velocity of a satellite is about 11.2 km/sec.
   (*t*) The binding energy of a particular orbiting satellite is 10^6 joules.
   (*u*) The mechanical equivalent of heat is 4.2 joules per calorie.

7. Consider the relationship $F_G = \frac{GmM}{r^2}$. What is the effect on F of (*a*) doubling m, (*b*) tripling M, (*c*) halving r?

8. (*a*) We may show that the expression $\frac{mv^2}{2}$ has the units of work or energy as follows:

   The MKS units for $\frac{mv^2}{2}$ are kg-(m/sec)^2, or kg-m^2/sec^2 or (kg-m/sec^2)m, or newton-m, i.e., joules.

   (*b*) Show that the expression $\frac{mv^2}{r}$ has the units of force.

   (*c*) Show that the expression $\frac{v^2}{r}$ has the units of acceleration.

   (*d*) Show that the expression $\frac{GmM}{r}$ has the units of energy.

   (*e*) What units has G in the relationship $F_G = \frac{GmM}{r^2}$?

   (*f*) Show that g, the acceleration due to gravity, may be expressed in newtons/kg as well as in m/sec^2.

9. Show that (*a*) the expression $\frac{mv^2}{2}$ is not the correct expression for centripetal force, (*b*) the expression $\frac{mv^2}{r}$ is not the correct expression for kinetic energy, (*c*) the expression $\frac{GmM}{r}$ is not the correct expression for gravitational force, (*d*) the expression $\frac{GmM}{r^2}$ is not the correct expression for gravitational potential energy.

APPENDIX

1. FUNDAMENTAL UNITS IN THE METRIC SYSTEM

(*a*) LENGTH. The standard unit is the metre, defined as the distance, measured at 0° C, between two transverse lines engraved on a platinum-iridium bar kept in the International Bureau of Weights and Measures at Sèvres, France. Thirty copies of the standard metre have been distributed among other nations.

1 kilometre (km) = 1,000 metres (m)
1 hectometre (hm) = 100 metres
1 dekametre (Dm) = 10 metres
1 decimetre (dm) = 0.1 metres
1 centimetre (cm) = 0.01 metres
1 millimetre (mm) = 0.001 metres

For several years prior to 1962, a good deal of research was carried out with the aim of defining the standard metre in terms of the wave length of an easily reproducible line in the spectrum of some element. On January 1, 1962, the standard metre was redefined as a length equal to 1,650,763.83 wave lengths in a vacuum of a specified line in the orange portion of the spectrum of krypton 86.

The following additional units should be noted.

1 Angstrom unit (A) = 10^{-10} metres
1 micron (μ) = 10^{-6} metres.

(*b*) MASS. The standard unit is the kilogram, defined as the quantity of matter in a platinum-iridium cylinder kept at Sèvres. The mass of a cubic decimetre of pure water at 4° C is very nearly equal to one kilogram, so nearly equal that for practical purposes they may be considered identical.

1 kilogram (kg) = 1,000 grams (gm)

In stating the masses of atoms and molecules, the kilogram and gram are inconveniently large units. In these instances the atomic mass unit (amu) is frequently used. One atomic mass unit is defined as one-twelfth of the mass of a neutral atom of $^{12}_{6}C$ and is equal to 1.65980×10^{-24} gm.

(*c*) TIME. The standard unit is the mean solar day—the average length of the apparent solar day. An apparent solar day is the interval from the moment that the sun's centre is on a meridian until it next arrives on that meridian.

1 day = 24 hours (hr)
1 hour = 60 minutes (min)
1 minute = 60 seconds (sec)

(*d*) VOLUME. The standard unit is the litre, defined as the volume occupied, at 4° C, by one kilogram of pure water. This volume is 1.00027 cubic decimetres, or 1000.27 cubic centimetres. Hence 1 millilitre (ml) is 1.00027 cc. However, for most purposes the ml may be considered to be equivalent to one cc.

1 litre = 1.76 pints

PREFIXES

Commonly used prefixes and the corresponding powers of 10 are summarized below.

micromicro = 10^{-12}
micro = 10^{-6}
milli = 10^{-3}
centi = 10^{-2}
deci = 10^{-1}

deka = 10^{1}
hecto = 10^{2}
kilo = 10^{3}
mega = 10^{6}

2. SIGNIFICANT DIGITS

The precision of any measuring instrument is limited. For example, with an elementary type of laboratory balance, masses may be determined correct to the nearest tenth of a gram; whereas with more precise balances, masses may be determined to the nearest milligram. The mass of an object then might be found to be 32.2 gm with the first type of balance, or 32.197 gm with the second. In the first case the mass is not likely exactly 32.2 gm, but is known to be between 32.15 gm and 32.25 gm; the second, more precise measurement indicates that the mass is greater than 32.1965 gm but less than 32.1975 gm. The first measurement (32.2 gm) has three significant digits; the second measurement (32.197 gm) has 5 significant digits.

Normally all of the measurements made with a given measuring instrument will have the same precision. However, we may wish to perform calculations with numbers from different sources; we may, for example, want the total of the two masses, one of which is given as 27.4 gm and the other as 33.541 gm. The sum of these two masses cannot be quoted more precisely than 60.9 gm. In general, when we add or subtract measured numbers, the answer can contain no more decimal places than the least precise of the given numbers.

A different situation occurs, however, when we perform operations of multiplication and division with measured quantities. Suppose that, in order to determine the density of mercury, we find that the mass of some mercury is 276.5 gm and that its volume is 20.3 ml. Even though the first measurement has four significant digits, the density obtained by dividing 276.5 gm by 20.3 ml must be quoted only to three digits and is therefore 13.6 gm/ml. In general, when operations involving multiplication and division are performed with measured quantities, the answer obtained can have no more significant digits than the least precise number used in obtaining the answer.

The ideas outlined above apply to measured data, but do not necessarily apply to the numbers given in problems in the exercises in this text. The problems present an opportunity for practice in the application of fundamental principles; the numbers are in many cases chosen so that the necessary arithmetic is relatively easy.

3. TRIGONOMETRIC FUNCTIONS OF AN ANGLE

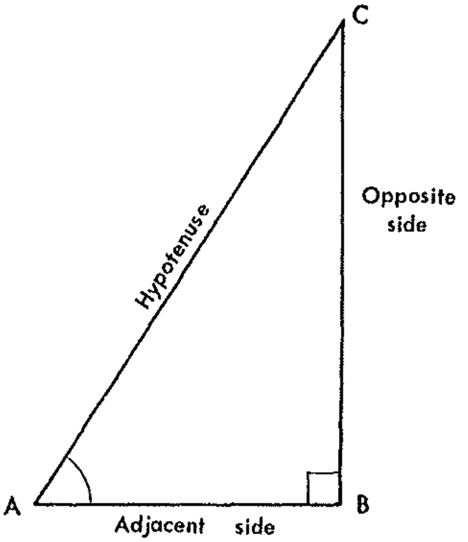
In addition to the mathematics involved in inverse relationships and the corresponding graphs outlined in Chapter 1, the student should be familiar with the meanings of the sine (*sin*), cosine (*cos*), tangent, (*tan*) and cotangent (*cot*) of an angle. In the right angled triangle *ABC* shown in the accompanying figure,

$$\sin A = \frac{BC}{AC}$$

$$\cos A = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$

$$\cot A = \frac{AB}{BC}$$



A right-angled triangle *ABC*. The sides *BC* and *AB* are named in terms of their positions relative to angle *A*.

The values of these ratios are independent of the lengths of the arms of the angle, and depend only on the size of the angle. Tables of values of these four trigonometric functions for angles between 0° and 90° follow.

INDEX

ANSWERS

Chapter 2—Section 2-14, page 20

1. (*a*) 68 km/hr
2. (*a*) (*i*) $37\frac{1}{2}$ mi (*ii*) $18\frac{3}{4}$ mi/hr
3. (*b*) Average speed = 20.6 cm/sec
4. (*a*) (*i*) 60 km/hr (*ii*) 40 km/hr
   (*b*) (*i*) 0.5 hr (*ii*) 1.5 hr
   (*c*) 1.0 hr
   (*d*) (*i*) 120 km (*ii*) 80 km
5. (*i*) 24 m; 18 m (*ii*) 6 m/sec; 4.5 m/sec
7. 0.24 m/sec; 0.72 m/sec; 1.92 m/sec^2
8. (*a*) 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 (*b*) 1.75 km/hr/sec
9. (*a*) 2 m/sec^2 10. 75.6
12. (*b*) 75 cm/sec (*c*) 78.0
13. 4 m/sec^2 14. 6.31 m
15. (*a*) (*i*) 2, 6, 10, and 14 m/sec (*ii*) 4 m/sec^2 (*iii*) 10 m/sec
16. Δv is (*a*) doubled (*b*) tripled
17. s changes by a factor of
    (*a*) 9 (*b*) 0.7
18. v changes by a factor of
    (*a*) 2 (*b*) $\sqrt{3}$
19. 1 m/sec^2; 10 m/sec 20. 10 m/sec
21. 30 m/sec; −2 m/sec^2 22. 140 m
23. 45 m/sec
24. 6 sec later, 36 m from the starting point
25. 6 m/sec^2; 8 m/sec

Chapter 3—Section 3-19, page 37

1. 15 mi/hr; 2 min
5. (*a*) (*i*) 9.4 cm (*ii*) 18.8 cm (*iii*) 37.7 cm
   (*b*) (*i*) 8.5 cm, 45° below horizontal to right (*ii*) 12 cm down (*iii*) zero
6. 3.5 mi north
10. (*a*) 23.2 m, 27° north of east (*b*) 10.6 m, 3° west of south
    (*c*) 10.6 m, 3° east of north
11. 4.7 m, 24° west of north
12. (*i*) 2 ft east (*ii*) 2 km west (*iii*) 5 m, 53.1° north of east

13. 50 m south-east 14. 141 m; 141 m
15. (*a*) 500 mi/hr; 700 ft/min (*b*) 100 mi; 8400 ft
16. (*a*) 13 km (*b*) 10.4 km/hr
17. (*a*) 1.57 cm/sec
(*b*) 1.57 cm/sec down; 1.57 cm/sec to the left
(*c*) 2.21 cm/sec to the centre
18. (*a*) 58.3 cm/sec (*b*) 74 cm/sec
19. 290 mi/hr
20. (*a*) 7.0 m/sec (*b*) 4.0 m/sec (*c*) 5.7 m/sec
21. 14 mi/hr, 15° east of north 22. 4° north of west, 802 km/hr
23. (*a*) 0.25 m/sec (*b*) −3 m/sec
24. (*a*) *EF* (*b*) *BC*
(*c*) *DF* (*d*) *D*
25. (*a*) 0.157 cm/sec
(*b*) 0.157 cm/sec to the left; 0.157 cm/sec up
(*c*) 0.221 cm/sec to the centre
(*d*) 0.015 cm/sec^2 to the centre
26. 2.5 mi/hr/sec, 37° south of east 27. 25 km/hr/sec west
29. 0.225 m/sec^3; −0.06 m/sec^3 30. 3 sec, 5 sec
31. 9.6 sec 33. (*a*) 20 m/sec
34. 31 ft/sec 35. 34.3 m
36. (*a*) (*i*) 14.7 m/sec up; 4.9 m/sec up; 4.9 m/sec down; 14.7 m/sec down; 24.5 m/sec down
(*ii*) 19.6 m up; 29.4 m up; 29.4 m up; 19.6 m up; zero
37. 0.64 sec; 192 m 38. 15 m/sec; 10 m/sec
39. (*a*) 80 m (*b*) 50 m/sec (*c*) 120 m
40. (*a*) 30 sec (*b*) 4.6 km

Chapter 4—Section 4-19, page 54

6. 22.4 newtons 7. 141 newtons north-west
8. 10 kg; 24 newtons 9. 1.5 newtons; 3.3 kg
10. 5 m/sec^2 11. 5 newtons
12. 4.9×10^{-2} newtons
13. (*a*) 0.5 m/sec^2 (*b*) 1.5 m/sec^2 (*c*) 0.15 m/sec^2
14. 2 kg 15. 6.4 newtons
16. 5.1×10^4 newtons
17. (*a*) 0.90 newton-sec; 14 newton-sec (*b*) 1.8 kg-m/sec; 24 kg-m/sec
18. 35 newton-sec; 35 kg-m/sec 19. 3.0 newton-sec
20. (*a*) 1.0 m/sec (*b*) 2.0 m/sec (*c*) 2.5 m/sec
21. 30 newton-sec
22. (*a*) 2.0 newton-sec (*b*) 20 newtons (*c*) 20 newtons

Chapter 5—Section 5-11, page 66

1. (*a*) 0.49 newtons (*b*) 9.8×10^2 newtons (*c*) 2.94×10^4 newtons
2. (*a*) 60 kg (*b*) 480 newtons (*c*) 60 kg
3. 3.9 4. 1.44×10^4 newtons 5. 1.96 m/sec^2
6. (*a*) *s* changes by a factor of 16
(*b*) *t* must change by a factor of $\sqrt{3}$ (*c*) a parabola
7. The acceleration due to gravity
8. (*a*) 10, 20, 30, 40 and 50 m/sec (*b*) 5, 15, 25, 35, and 45 m/sec
(*c*) 5, 15, 25, 35, and 45 m (*d*) 5, 20, 45, 80, and 125 m
9. 49 m; 122.5 m 10. 184 m; 6.1 sec
11. (*a*) 0.252 newton-sec down (*b*) 0.441 newton-sec up
12. 3.86×10^3 newtons 13. 19.6 newtons
14. 40 cm 15. 4.3 sec; ~~43~~ 42.09 m from foot of cliff
16. 3 sec; 15 m; 29.8 m/sec
17. (*a*) 16 m/sec (*b*) 8.0 m/sec^2
22. F_c changes by a factor of
(*a*) 3 (*b*) $\frac{1}{4}$ (*c*) 4
23. 1.25×10^4 newtons toward the centre
24. 1.08×10^3 newtons
25. (*a*) 16.7 newtons (*b*) 6.9 newtons (*c*) 26.5 newtons
26. (*a*) 2.7×10^{-3} m/sec^2

Chapter 6—Section 6-10, page 75

1. F_G changes by a factor of
(*a*) 4 (*b*) 0.75 (*c*) $\frac{1}{8}$
2. 6.67×10^{-9} newtons
3. (*a*) 6.67×10^{-10} newtons (*b*) 6.67×10^{-10} newtons
4. 4.1×10^{22} newtons 5. 2.4×10^{-9} newtons
6. 4×10^{-47} newtons; $10^{\cancel{-47}}$ -48 7. Approximately 3×10^6 m
8. 6×10^{24} kg 9. 4.9 m/sec^2
10. 24 m/sec^2 11. 1.8 hrs
12. 3.6×10^4 km

Chapter 7—Section 7-9, page 84

2. (*b*) 4.8 newton-sec (*c*) 4.8 kg-m/sec (*d*) 3.0 m/sec
3. 12.5 cm/sec 4. 100 cm/sec
5. 54.4 cm/sec 6. 7.1 m/sec
7. 2.0 m/sec 8. 30 gm and 90 gm

9. 1.5×10^7 m/sec toward the east
10. (*a*) 3 : 5 (*b*) 5 : 3
    (*c*) 3 : 5 (*d*) 1 : 1
11. (*a*) (*i*) 100 kg-m/sec (*ii*) 250 m/sec, 37° south of west
    (*b*) 60 kg-m/sec; 80 kg-m/sec; 100 kg-m/sec; 250 m/sec
12. (*a*) 40 cm/sec (*b*) 1.6×10^2 newtons
13. (*a*) 100 m/sec (*b*) 0.5 m/sec
14. $0.6\ u$ 15. 20 m/sec
16. (*a*) 10^5 newtons (*b*) 10^4 kg

Chapter 8—Section 8-11, page 95

1. (*a*) 392 joules (*b*) zero (*c*) zero
2. (*a*) 5 newtons (*b*) 0.5 m
3. 23 joules
4. (*a*) 60 joules (*b*) 170 joules (*c*) 60 joules
5. One division = 2 m
6. (*a*) 30 newtons (*b*) 300 joules (*c*) 600 joules
7. 12.5 joules; 12.5 joules 8. 2.5×10^3 joules
9. (*a*) 4.1×10^{-16} joules (*b*) 4.1×10^{-16} joules
10. 2.0×10^4 m/sec 11. E_K of $A = 0.8\ E_K$ of B
12. 0.04 newtons
13. (*a*) 24 joules (*b*) 9.8 m/sec
14. (*a*) 12 joules (*b*) 6.0 joules
    (*c*) 3.0 m/sec
15. (*a*) 10 joules; 3.1 m/sec (*b*) 16 joules; 4.0 m/sec
    (*c*) 18 joules; 4.2 m/sec (*d*) 16 joules; 4.0 m/sec
17. (*a*) 6.25×10^5 joules (*b*) 3.1×10^4 joules
18. 1.8 joules; 3.6 joules 19. -0.2 m/sec; 0.2 m/sec
20. -3.3×10^4 m/sec; 6.7×10^4 m/sec
21. (*a*) 6 m/sec (*b*) 64%
24. (*a*) 40 newtons (*b*) none

Chapter 9—Section 9-10, page 108

4. (*a*) 36 joules (*b*) 36 joules
5. (*a*) 49 m/sec (*b*) 72 joules
   (*c*) 1.2×10^2 m (*d*) 72 joules
6. 66% 7. 1.47×10^3 joules
8. (*a*) 392 joules (*b*) (*i*) 314 joules (*ii*) 83 joules

9. 2.4 m/sec
10. 3.8×10^2 m/sec
11. (*a*) 0.5 m/sec (*b*) 1.3 cm
12. 31 m/sec
13. (*a*) (*i*) 0.25 joules (*ii*) 1.0 joules
(*b*) (*i*) 0.25 joules (*ii*) 1.0 joules
14. (*a*) 0.25 joules (*b*) 4.4 joules
(*c*) 3.4 joules
15. (*a*) 16 : 1 (*b*) 4 : 1
17. Approximately 12 m/sec
18. (*a*) 0.71 m (*b*) (*i*) 14.1 m/sec (*ii*) 10.2 m
19. 60 joules
21. 25 m
22. (*a*) 900 km approximately (*b*) 5.6×10^{10} joules
(*c*) 1.1×10^4 m/sec
23. (*a*) 1.8×10^{10} joules (*b*) 1.8×10^{10} joules
(*c*) 3.7×10^{10} joules (*d*) 1.9×10^{10} joules
25. (*a*) -6.0×10^9 joules (*b*) (*i*) -3.0×10^9 joules
(*ii*) -4.5×10^9 joules (*iii*) -6.0×10^9 joules
26. 2.5×10^{10} joules
27. (*a*) no (*b*) 10^9 joules

Chapter 10—Section 10-6, page 117

1. (*a*) 0.5 newtons (*b*) 7.5 joules (*c*) 1.8 calories
2. 1.5×10^3
3. 4.9×10^3
4. 196 calories
5. (*a*) 240 m/sec (*b*) 92 joules (*c*) 22 calories