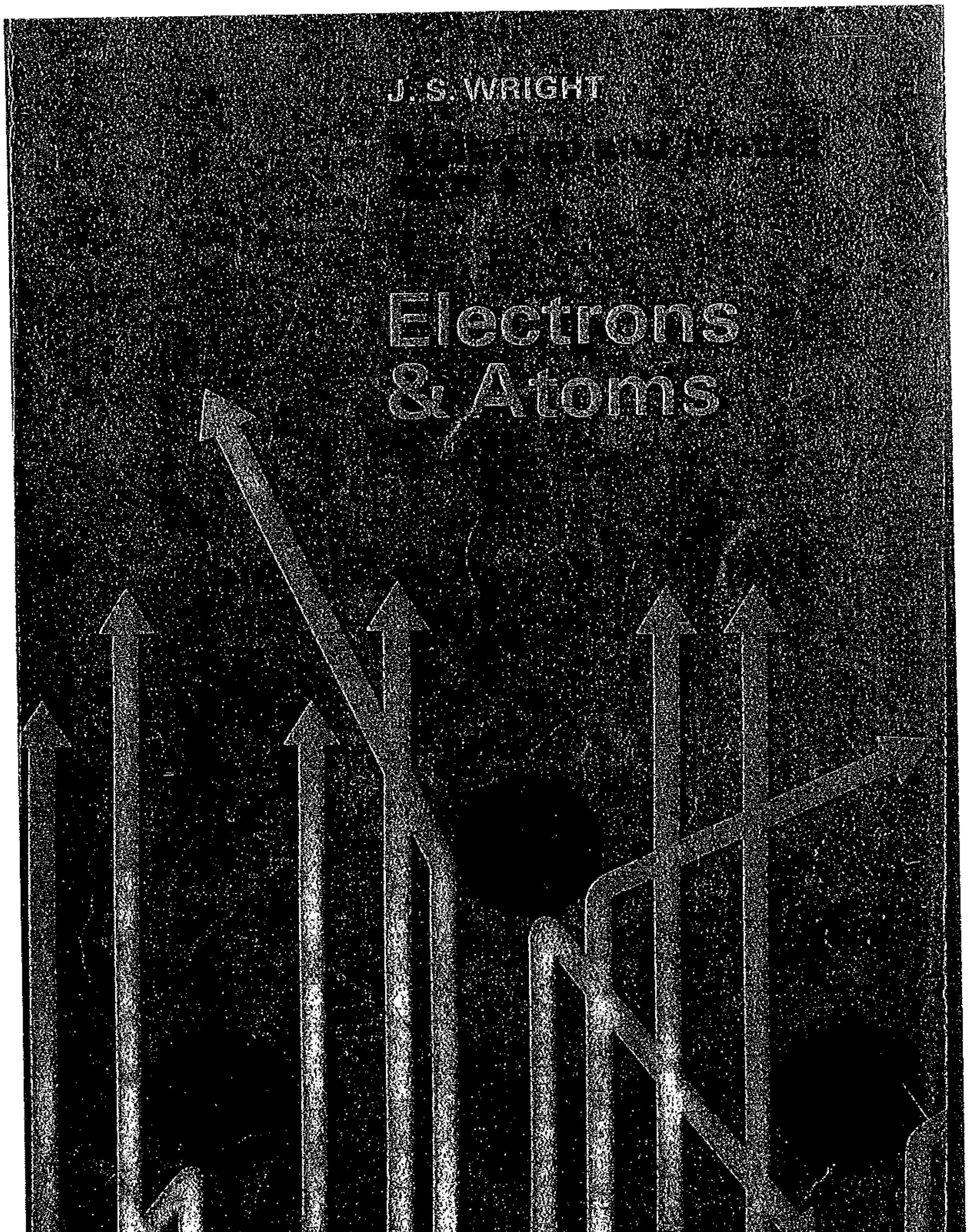
Electrons & Atoms

By J. S. Wright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Chapter 1

Electrostatics

1–1 INTRODUCTION

The importance of electricity to business, to industry, indeed to all facets of twentieth century living, is so obvious as to need no comment here. All of us use, and are served by, a multiplicity of electrical devices every day. In many cases we do not know how or why the devices work. Nor will it be our aim in this book to describe such devices and their operation. Rather, we will concentrate on that portion of the story of electricity which led not only to an understanding of the nature of electricity, but also to an understanding of atomic structure.

Most electrical devices are operated by electricity which flows through them, that is, by current electricity. Yet, until early in the nineteenth century, current electricity was unknown. Before that time, knowledge of electricity was limited to cases where the electricity remained essentially stationary or static, and practical applications of electricity were unknown. We know now, however, that the principles and relationships established in a study of static electricity are vital to an understanding of all electrical devices, and of molecular and atomic interactions.

1–2 OCCURRENCE OF STATIC ELECTRICITY

Examples of static electricity are common. A person walks across a thick-piled rug and touches some metal object. There is a spark and the person feels a tingle at his finger tip. A young lady combs her hair vigorously. There is a crackle and the hair flies up and clings to the comb. A child at a party rubs a balloon on his sleeve and then "sticks" the balloon to the wall. A man removes his sweater and, particularly if the sweater is of some synthetic material, finds that it tends to cling to his shirt. He may find also that he has difficulty in folding the sweater, for any two parts of it seem to repel one another.

All of the effects described in the preceding paragraph occur because of static electricity.

1–3 STATIC ELECTRICITY IN THE LABORATORY

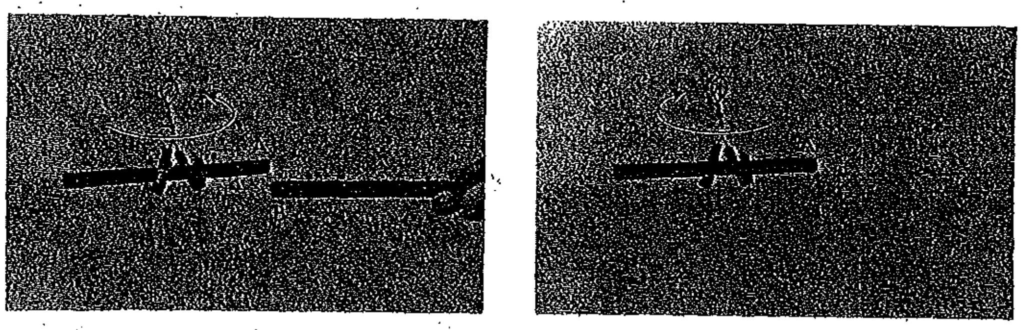
In the laboratory, we usually produce static electricity by rubbing materials together. The method dates back about 2500 years to the Greek physicist-astronomer, Thales, who found that rubbed amber became electrified and would attract small grains of sand. In fact the name, electricity, is derived from the Greek word, elektron, meaning amber.

The materials used most frequently in the laboratory are rods of lucite (or glass) and ebonite. If a lucite rod is rubbed with silk, nylon, or polyethylene, or if an ebonite rod is rubbed with fur, the rod acquires the ability to pick up sawdust, small pieces of paper, and other small objects. Since these particles are attracted to the rod and adhere to it, some force must be acting. This force is called an electric force, and the rod is said to bear an electric charge. Materials which exert no electric force are said to be uncharged or neutral.

1–4 TWO KINDS OF ELECTRIC CHARGE

Though the charge on the lucite rod and the charge on the ebonite rod, described above, are produced in similar ways, they are not necessarily alike. The phenomena may be investigated further as in the following experiment.

An ebonite rod A rubbed with dry fur or flannel, is placed in a glass or rubber-covered support which is suspended by a string so that the rod is free to turn in a horizontal plane (Fig. 1.1a). A second ebonite rod B similarly prepared, is then brought close to A. Rod A turns away from B, indicating the existence of a repelling force between the two. If now a lucite (or glass) rod C rubbed with dry silk or nylon, is brought near A (Fig. 1.1b), A draws closer to C and will follow it as C is moved, thus indicating the existence of an attracting force between



(a) (b)

Fig. 1.1. Demonstrating the two kinds of electric charge.

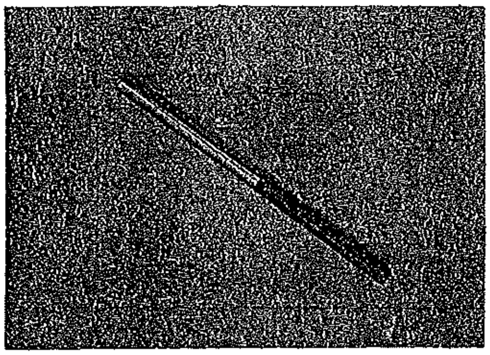


Fig. 1.2. A conducting rod with an insulating handle.

A and *C*. A second lucite rod *D* rubbed with silk, will repel the lucite rod *C* if the latter is suspended. The different effects of the charged rods *B* and *C* on the same rod *A* indicate that the charges on the lucite and ebonite are not of the same kind.

Experiments with a great many objects have revealed that when they are charged, they act either like the charged lucite or the charged ebonite. Thus, there would seem to be two and only two kinds of electric charge. To distinguish between the two known kinds, the great American statesman, publisher, and scientist, Benjamin Franklin (1706-1790), suggested in 1747 that charges similar to those produced on glass (or lucite) rubbed with silk be called positive charges, and that charges similar to those produced on ebonite rubbed with fur be called negative charges.

The experiment just described shows that two positive charges or two negative charges (two like charges) repel one another, whereas one positive and one negative charge (two unlike charges) attract one another. This is the law of charges.

Almost all substances can be given electric charges; the most effective method is to rub the substance with some other material. However, electrification occurs to a lesser extent even if the two substances are merely brought into close contact and then separated. Furthermore, the type of charge which a given substance receives depends upon the other substance with which it is brought into contact. For example, ebonite becomes negatively charged if it is rubbed with fur, but positively charged if it is rubbed with sulphur. Also, the strength of the charge depends on the nature of the two substances.

1–5 CONDUCTORS AND INSULATORS

An ebonite rod, held in the hand as it is rubbed with fur, becomes charged; a rod of steel or copper, similarly held and rubbed, does not become charged. This should not be interpreted as proof that steel or copper cannot be charged by friction methods. A length of copper tubing fitted with an ebonite handle, as shown in Figure 1.2, becomes charged, if the rod is held by its ebonite handle and if the copper tube is rubbed with fur. The copper will then attract small bits of paper and sawdust. However, if the copper tube, after being rubbed with fur, is touched, no matter how briefly, with the finger, or is touched to a water pipe, its ability to attract small bits of paper or to repel another charged object disappears. In other words, its charge is lost and the copper tube is said to have been neutralized. This reaction of the charged copper tube to a brief touch with the finger is in sharp contrast to that of a charged ebonite rod. In the latter case, almost the whole surface area of the rod must be wiped by hand before all of the charge is removed.

Since, in the above experiment, charge flows readily from all portions of the copper to the point where it is touched, copper is said to be a good conductor of electricity. And since, in the case of the ebonite, charge does not flow to the contact point, ebonite is said to be a good insulator. Substances differ greatly in their ability to conduct electricity. A classification into three groups is presented in the following table.

GOOD CONDUCTORS	FAIR CONDUCTORS	GOOD INSULATORS
silver	wood	mica
copper	tap water	glass
aluminum	the human body	silk
nickel	earth	sulphur
iron	germanium	amber
platinum	carbon	rubber
mercury	solutions of	shellac
	acids,	lucite
	bases, salts	ebonite

In the first of the above groups are copper and aluminum, the materials most often used as the conductors in electric circuits. Materials which are only fair conductors are used to limit and control the rate of flow of charge in such circuits; they are called resistors.

1-6 THE ELEMENTARY CHARGE

Many different theories have been proposed to explain electric charges. One common assumption has been that all matter contains small particles of at least two types—one positive and the other negative. As you no doubt know, this has been found to be the case. The sub-atomic particle bearing the basic or elementary negative charge is the electron, and we shall, therefore, call the magnitude of this charge the elementary charge. In the atom, the nucleus around which the electrons travel is positively charged; the elementary positive charge is borne by protons in the nucleus. The number of elementary positive charges in the nucleus is normally equal to the number of elementary negative charges borne by the electrons. That is, the number of protons is equal to the number of electrons, since each bears an elementary charge.

We shall have to wait until later chapters to see how a knowledge of electrons and protons (and other sub-atomic particles) was developed, but we shall not hesitate to use some of this knowledge in the meantime. Since it is normally the electrons rather than the protons which move, we explain the basic electrostatic phenomena in terms of electrons.

1-7 POSITIVE AND NEGATIVE CHARGES EXPLAINED

When an ebonite rod is rubbed with fur, electrons from atoms in the fur transfer to atoms at the surface of the ebonite. Thus the ebonite rod obtains a surplus of electrons and becomes negatively charged. Negatively charged objects possess more than the normal number of electrons.

When a lucite rod is rubbed with silk, electrons transfer from the lucite to the silk. Thus the lucite rod (or any positively charged object) has some atoms with fewer electrons than protons, i.e., there is a deficit of electrons.

As a corollary, the fur should become positively charged and the silk negatively charged. The fur may be observed to repel a positively charged lucite rod; thus the predicted presence of a positive charge on the fur is verified. Similarly, the silk will repel a negatively charged ebonite rod and so is shown to be negatively charged. Furthermore, the strength of the negative charge on the ebonite should be equal to that of the positive charge on the fur, since the numerical value of the electron surplus for the ebonite will equal the numerical value of the electron deficit for the fur.

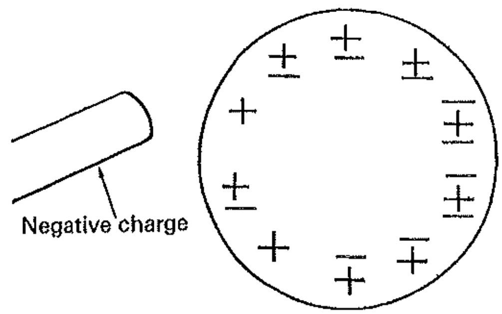


Fig. 1.3. The charge on the rod alters the distribution of electrons in the neutral object.

1–8 CONDUCTORS AND INSULATORS EXPLAINED

A conductor was previously defined as a substance through which charge can flow easily. A flow of charge in a solid conductor is due to the motion of electrons through it. The electrons move from atom to atom. In order for this flow of electrons to occur, the atoms of the conductor must be such that some of their electrons are loosely bound to the nuclei. For example, copper is a good conductor because in each atom of the copper one of the electrons is easily detached and drifts along from atom to atom. These drifting electrons are called free electrons. Conversely, insulators are materials in which all of the electrons are tightly bound to their nuclei; there are no free electrons.

1–9 ATTRACTION OF NEUTRAL OBJECTS EXPLAINED

The electrons in a neutral conductor, since they are negatively charged, will be affected by nearby charges, for example, by a charged ebonite rod (Fig. 1.3). Some electrons are repelled from the surface of the conductor near the negatively charged rod to the surface farthest from the rod. Thus, on the average, the attracting force between the rod and the protons in the conductor is greater than the repelling force between the rod and the electrons in the conductor. Therefore, there is a resultant force of attraction between the charged rod and the neutral conductor.

The charged ebonite rod, or any charged object, also attracts neutral insulators such as pieces of paper, sawdust, etc. The explanation for this attraction is similar to that given in the above paragraph for a neutral conductor. However, in the case of the insulator, it is likely that the displacement of the electrons occurs only within the atoms themselves.

1–10 THE METAL LEAF ELECTROSCOPE

Small bits of paper or pith, or a suspended rod, may be used as detectors of electrostatic charges, but the method is not very convenient and the agents are not very sensitive. A much better detector is the metal leaf electroscope, an instrument which can be used not only to detect the presence of charge but also to determine its kind and to compare the intensities of charges.

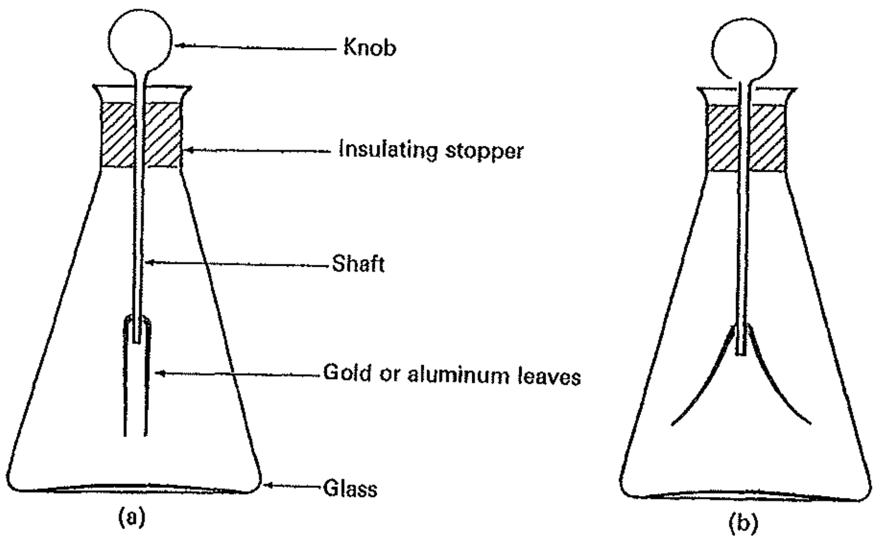


Fig. 1.4. The metal leaf electroscope.

The electroscope (Fig. 1.4*a*) consists essentially of a metal rod or shaft which is a good conductor. Near one end of the shaft are attached two narrow and very thin strips of metal leaf, gold or aluminum; at the other end is a metal knob. Since any slight air current would disturb the leaves, the unit is enclosed in a glass container, or in a metal container with glass windows front and back. In the latter case, the shaft must be insulated from the case by passing it through a cork or rubber stopper.

1–11 DETECTION, IDENTIFICATION, AND COMPARISON OF CHARGES

With the electroscope knob, shaft, and leaves electrically neutral, the leaves will hang vertically under the action of the force of gravity. If the knob is touched with a charged ebonite rod, some of the charge (electrons) will transfer from the rod to the knob. This charge will distribute itself throughout the electroscope since the shaft and leaves are conductors. Thus, the two leaves will bear the same kind of charge and a repelling force will exist between them. If this repelling force is great enough, it will overcome the force of gravity acting on the leaves and they will rise or deflect as shown in Figure 1.4(*b*). In a similar manner, a charged lucite rod may be used to place a positive charge on the electroscope and cause a deflection of the leaves.

To prevent the transfer of too great a quantity of charge, the charged rod should not be touched to the knob of a sensitive electroscope. Instead, a proof plane, which consists of a small metal disc with an ebonite handle, may be used. The proof plane is first touched to the conductor under test and is then touched to the knob of the electroscope. If the leaves diverge, the conductor bears a charge.

The electroscope can also be used to determine the kind of charge on an insulated conductor. Suppose the electroscope is charged negatively and a sample charge from the conductor under test is added. If the leaves show further divergence, the conductor bears a negative charge; if the leaves at first collapse, partially or completely, the conductor bears a positive charge.

Finally, the instrument can be used to compare the intensities of charges on different insulated conductors or on different parts of the surface of the same conductor. The more intense charge will cause the greater divergence of the leaves.

In the methods described above, the electroscope was charged by contact, i.e., the charged conductor, or the proof plane which had been in contact with the conductor, was placed in contact with the knob of the electroscope. If the conductor is negatively charged, the proof plane receives some of the surplus electrons and passes them on to the electroscope. Hence the electroscope, including the leaves, bears the same kind of charge as the conductor under test.

1–12 TEMPORARY INDUCED CHARGES

The leaves of an electroscope can be made to diverge by bringing a charged ebonite rod near the knob. If the rod is then removed from the vicinity of the knob, the leaves collapse. The divergence of the leaves indicates that, for a brief time at least, they bore a charge. Electrons did not transfer from the rod to the knob since these did not touch and since air is a poor conductor. This kind of charge, produced when there is no direct contact, is called an induced charge.

Electrostatic induction can be explained readily in terms of electron motion. Electrons are miniature negatively charged particles. The ebonite rod is negatively charged. Since like charges repel, free electrons in the knob of the electroscope are forced down to the leaves. Hence the leaves temporarily become negatively charged. The knob, having lost some of its electrons, should be positively charged. This reasoning can be verified as follows.

Two similar metal spheres, A and B, each on an insulating stand, are placed in contact (Fig. 1.5). A charged ebonite rod is then brought near the sphere A. If the theory holds, the negatively charged rod should repel electrons from sphere A to sphere B so that A will be charged positively and B negatively.

With the ebonite rod still in position, sphere B can be moved away from A. If the spheres, A and B, are now tested on

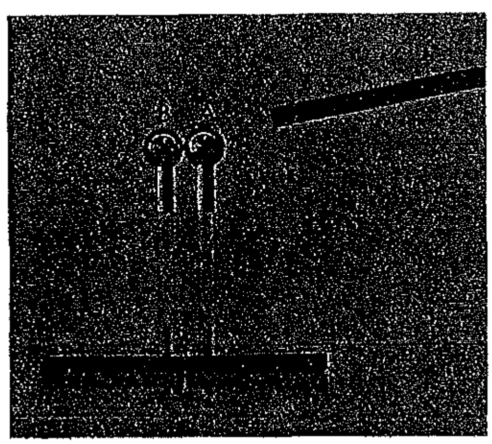


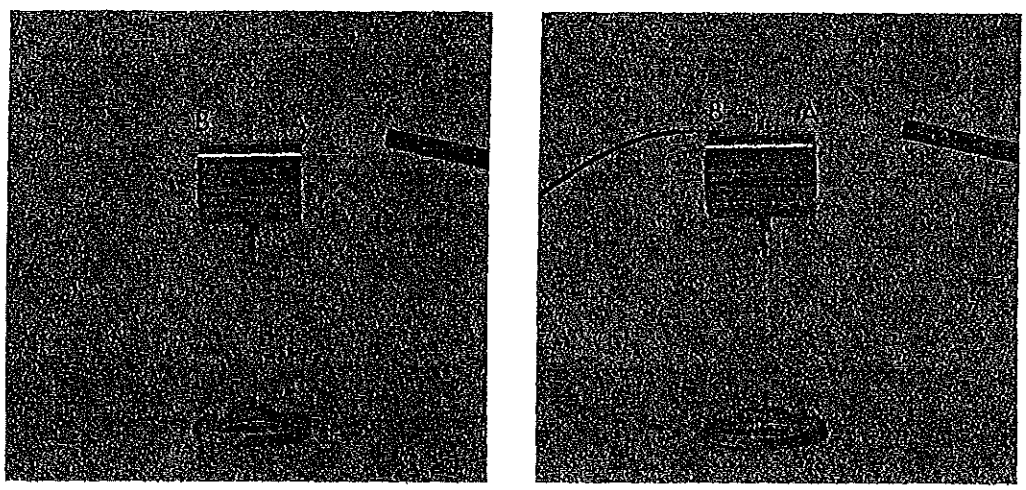
Fig. 1.5. The charged ebonite rod induces equal and opposite charges on the conductors A and B.

the electroscope, A is found to be charged positively and B negatively. If A is now allowed to touch B, testing shows that both are neutral. Thus the two temporary induced charges are equal in size but opposite in sign. This conclusion confirms the assumption made in Section 1–9 to explain the attraction of an uncharged object by a charged object.

1–13 PERMANENT INDUCED CHARGES

Except for the case when the two spheres A and B (Fig. 1.5) are separated while the charged rod is held near A, the induced charges described above have been temporary. In other words, when the charged rod is removed, the electrons in the conductors return to their normal positions. However, even in a single conductor like AB in Figure 1.6, the charge induced by the ebonite rod can be made permanent. If a grounded wire is connected to the end B of the conductor and then disconnected, and if the ebonite rod is then removed, a test on the electroscope will show that the conductor is charged positively over the whole of its surface. Apparently, while the brief contact was made with the earth, some or all of the electrons which the ebonite rod repelled to end B are further repelled to earth. When the contact with the earth is broken, the return of these electrons is cut off and the insulated conductor now has a permanent positive charge.

If a charged lucite rod is used in place of the ebonite, the conductor can be given a permanent negative charge. Thus, the permanent charge produced on an insulated conductor by induction and grounding, is opposite in kind to that of the charging agent.



(a) (b)

Fig. 1.6. Charging a conductor by induction and grounding. A charged rod is brought near the conductor as in *(a)*. The conductor is then grounded while the rod is in place, as in *(b)*. The ground connection must be removed before the rod is removed.

1–14 PROBLEMS

1. Suppose that a rod of material X, when rubbed with a material Y, attracted an ebonite rod which had been rubbed with fur, attracted a lucite rod which had been rubbed with polyethylene, and repelled another rod of material X, rubbed with material Y. What conclusion would have to be drawn?

2. A suspended pith ball is attracted by a lucite rod which has been rubbed with polyethylene. What can be said about the state of charge of the ball?

3. Five rods have the following effects on one another. D attracts E and repels A. B attracts E and has no effect on C. A negatively charged ebonite rod attracts both D and C. What are the states of charge of the rods?

4. Describe several methods by means of which an insulated conductor can be charged.

5. Two identical conducting spheres, A and B, are placed some distance apart and are connected by a wire. They are then charged. A grounded conducting sphere C is brought near to A. The connecting wire is removed, and then C is removed. Will the charges on A and B now be equal? Explain.

6. A copper rod is suspended by an insulating fibre so that it is free to rotate in a horizontal plane. An ebonite rod is rubbed with fur and then brought close to the side of one end of the copper rod. (*a*) Describe what is observed. (*b*) Explain the observation in terms of electron motion. (*c*) How would the observation be changed if the ebonite rod were replaced by a glass rod rubbed with silk?

7. Explain why an initial repulsion is followed by an attraction when a small conductor bearing a large quantity of charge is brought slowly up to a large conductor bearing a small quantity of charge of the same sign.

8. Describe how an electroscope might be used to test the conducting properties of (*a*) a dry cotton thread, and (*b*) a wet cotton thread.

9. Why do electrostatic experiments usually work poorly on humid days?

10. You are given three insulated conductors A, B, and C. A is negatively charged, B and C are neutral. How can you charge B positively and C negatively, without altering the charge on A?

11. When an insulated conductor is charged permanently by induction and grounding, does it matter where the ground wire is connected to the conductor?

12. Describe how you would give an insulated conductor a permanent negative charge by induction and grounding. Explain the charging action in terms of electron motion.

13. Two insulated conducting spheres, A and B, are placed near, but not in contact with, one another. B is connected by a wire to an electroscope. State and explain the behaviour of the leaves of the electroscope when (*a*) A is given a positive charge, (*b*) the wire is then removed and A is removed, (*c*) the electroscope is touched momentarily, (*d*) the wire connection from B to the electroscope is restored.

14. A lucite rod with a strong positive charge is brought near the knob of a negatively charged electroscope. As the rod approaches the knob, the leaves collapse and then diverge again. Account for these observations in terms of electron motion. What is the final charge on the leaves?

1–15 SUMMARY

An object may become electrically charged when rubbed with a dissimilar material. Charged objects attract neutral objects.

There are two kinds of charge: positive and negative. Like charges repel each other, unlike charges attract each other.

Conductors transfer charge readily; insulators do not transfer charge. A resistor is a conductor, but not a good one. Most metals are good conductors.

Electrostatic phenomena are explained in terms of the motions of electrons in conductors. The electron carries one negative elementary charge.

The metal leaf electroscope may be used to detect, identify, and compare charges.

If a charged object is held near an insulated conductor, temporary equal and opposite charges are induced on the conductor. The charge on that portion of the conductor nearest the charged object is opposite in sign to the charge on the object.

An insulated conductor may be given a permanent charge by induction. The conductor is connected to ground, then disconnected, while the charged object is held nearby. When the charged object is removed, the conductor has a permanent charge, opposite in sign to the charge on the object.

Chapter 2

Electric Charge, Force, and Potential

2–1 THE CONSERVATION OF CHARGE

Quantitative measurements in electricity begin with measurement of quantities of charge. Basic to these measurements is the assumption—implied or expressed—that charge is conserved. If two insulated conductors, one charged and one neutral, for example, are connected by a conductor, the charged conductor will share its charge with the neutral conductor. However, in the transfer, charge is not created or destroyed; the total quantity of charge is the same after the transfer as before.

Such transfers of charge are the result of the motions of electrons carrying their elementary charges, and we automatically assume that electrons do not appear or disappear in the process. Many theories other than the electron theory have been advanced, but all have implied charge conservation and thus made clear what was meant by quantity of charge. The first question, then, is how to measure quantities of charge, and in what units. Since charged objects exert forces on one another, forces which depend on the quantities of charge on the objects, it is natural to attempt to measure charge in terms of these forces.

2–2 THE WORK OF COULOMB

The repelling force between two charged ebonite rods obviously depends on how strongly each is charged, and upon the distance between the rods. The greater the quantity of charge on either of the rods, or the closer the rods are to one another, the greater is the repelling force. The French physicist, Charles Coulomb (1736-1806) investigated the relationship between the force F, the charges Q and q, and the distance r. The basic portion of

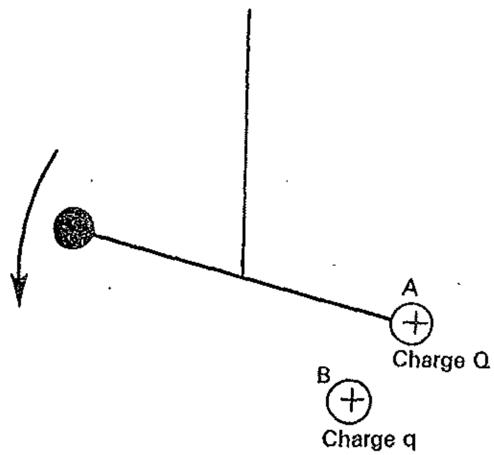


Fig. 2.1: The basic parts of Coulomb's torsion balance.

the apparatus he used is shown in Figure 2.1. He suspended a rod, on each end of which was a small sphere, by a long fibre. Such an arrangement is called a torsion balance, and Coulomb calibrated this balance so that he knew the force necessary to twist the fibre through any angle. The sphere A was then charged with a quantity of charge which we shall call Q. Another sphere B was given a like charge which we shall call q. Keeping q and Q constant, Coulomb determined the force of repulsion for various values of the distance r between the centres of A and B, and found that $F \propto \frac{1}{r^2}$.

Keeping r constant, Coulomb then allowed B to share its charge with an identical sphere, which was then removed. Thereby q was halved, and Coulomb found that F was halved also. Other variations of this procedure led to the conclusion that $F \propto q$, and also, by having A share its charge, that $F \propto Q$. Therefore the force of repulsion (or attraction) between two charged objects is directly proportional to each of their charges, and inversely proportional to the square of the distance between them. (If the spheres bearing the charges are not of negligible size compared to the distance between them, this distance must be measured from the centre of one to the centre of the other.) In formula form

$$F = \frac{kqQ}{r^2}$$

where k is a variation constant whose value depends on the units used for F, q, Q, and r, and also on the medium between the two charges.

This formula for electric force is called Coulomb's law; its similarity to Newton's law of gravitation is obvious. In fact Coulomb, aware of the law of gravitation, probably expected the law for electric forces to have this form. Since Coulomb's time, the validity of the relationship $F \propto \frac{1}{r^2}$ has been confirmed by experiments whose possible error is less than 1 part in 10^9.

2–3 UNITS OF CHARGE

As a result of Coulomb's law, various units of charge may be defined, depending on the units of force and distance used. The statcoulomb (electrostatic unit or esu) of charge is defined as that charge which at a distance of 1 cm from an identical charge in a vacuum repels it with a force of one dyne. For various reasons the coulomb, rather than the statcoulomb, is the most frequently used unit of charge. The coulomb is defined in terms of the magnetic effect which is produced by charge in motion; the exact definition cannot be given here.

1 coulomb = 3.0×10^9 statcoulombs, approximately.

The definition of the statcoulomb is such that the magnitude of the variation constant k in Coulomb's law is 1. The units for k may be found by solving the Coulomb law formula for k.

$$k = \frac{Fr^2}{qQ}$$

$$\therefore k = 1\frac{\text{dyne-(cm)}^2}{\text{(statcoulomb)}^2}$$

But 1 dyne = 10^{-5} newtons

1 cm = 10^{-2} m

and 1 statcoulomb =

$$\frac{1}{3.0 \times 10^9} \text{ coulombs, approximately.}$$

$$\therefore k = \frac{(10^{-5} \text{ newt})(10^{-2} \text{ m})^2 \times 9.0 \times 10^{18}}{\text{(coulomb)}^2}$$

$$= 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$$

That is, two charges, each of magnitude 1 coulomb, and 1 metre apart in a vacuum, repel each other with a force of 9.0×10^9 newtons.

2-4 ELECTRIC FIELDS OF FORCE

Any charged object exerts a force on other charges near it. The region in which this force is felt is called the electric field of force of the object. Since the force is inversely proportional to the square of the distance from the object, we may plot the force field by drawing force vectors as in Figure 2.2. Experimentally, the nature of electric force fields may be investigated by a method similar to the procedure in which iron filings are used to investigate magnetic fields. The charged objects are placed at the surface of a non-conducting liquid in an insulating container. Grass seeds sprinkled on the surface of the liquid reveal the nature of the field. Figure 2.3 shows photographs of five electric fields. Let us consider each in turn.

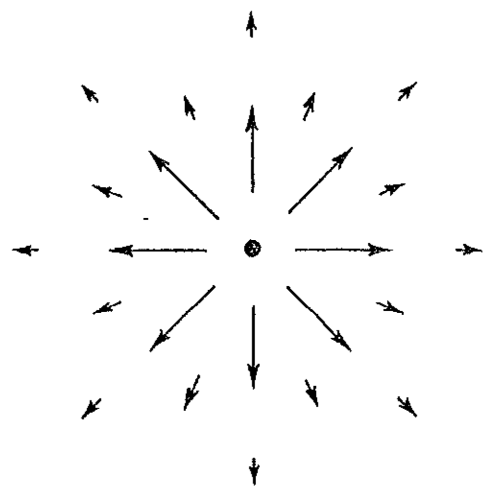


Fig. 2.2. An electric force field. The force is inversely proportional to the square of the distance.

Figure 2.3(a) shows the electric field around a vertical charged rod. The grass seeds align themselves in lines called electric lines of force. The direction of a line of force at any point in the field is the direction of the force vector at that point. This photograph verifies at least qualitatively the predictions made in Figure 2.2. A rough estimate of the magnitude of the electric force at any point in the field may be made by observing the concentration of the grass seeds in the vicinity of that point.

Figures 2.3(b) and (c) show, respectively, the fields due to the combined action of equal unlike and equal like charges. The force at any point in either of these fields is the resultant of two forces, one exerted by each of these

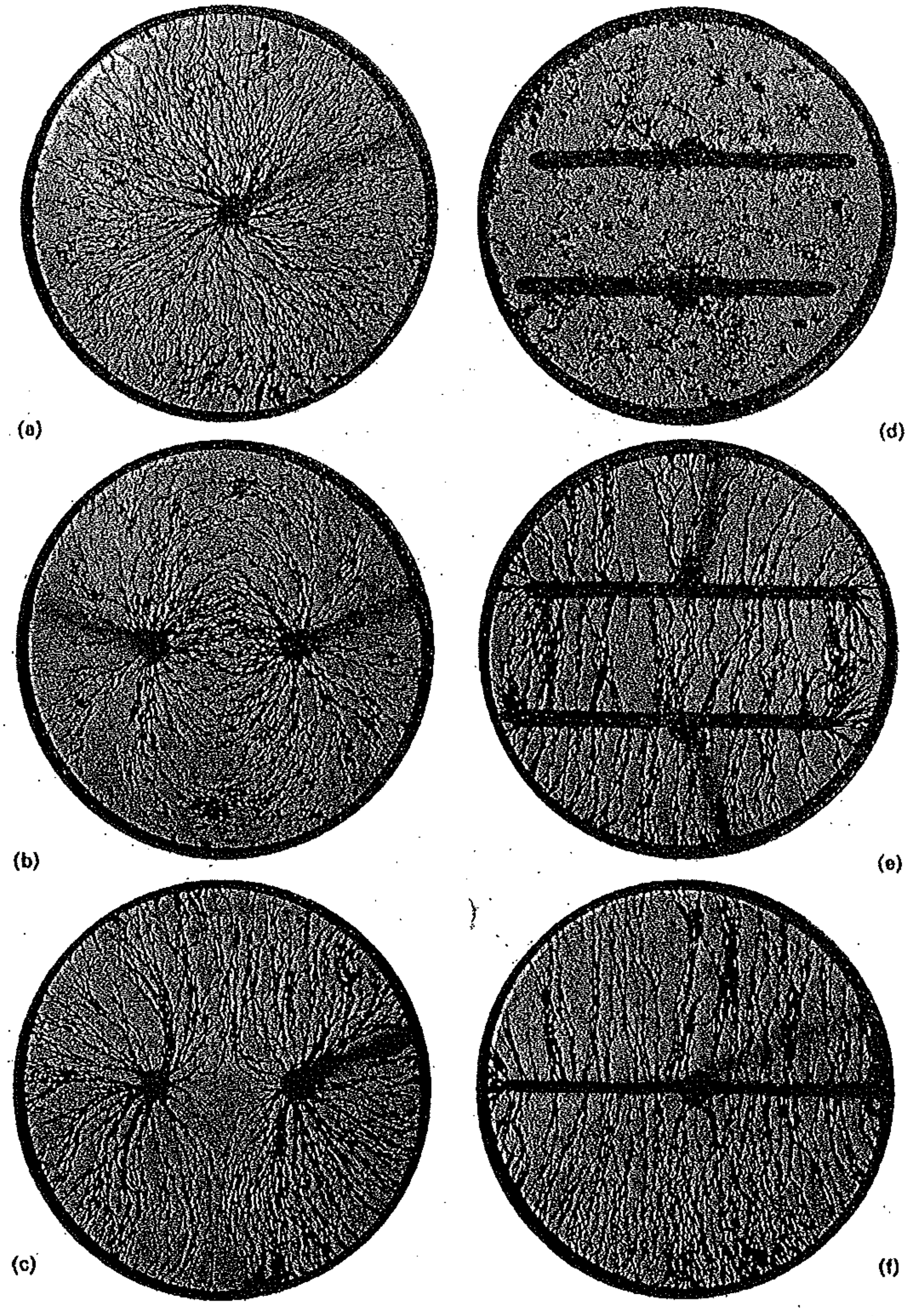


Fig. 2.3. Electric fields *(a)* about a point charge, *(b)* between two unlike point charges, *(c)* between two like point charges, *(d)* between uncharged parallel plates, *(e)* between parallel plates with unlike charges, *(f)* about a single charged plate.

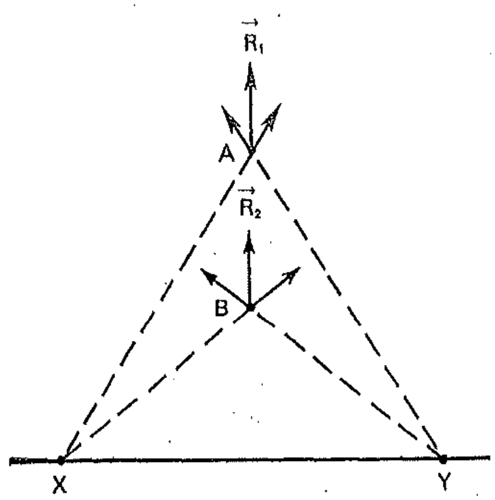


Fig. 2.4. $\vec{R_1}$ and $\vec{R_2}$ are very nearly equal.

charges. The nature of each of these fields might have been predicted from the law of charges.

Next we consider a very important field, that between two parallel charged plates. Figure 2.3(*d*) shows the positions of the grass seeds before the plates are charged; Figure 2.3(*e*) shows the positions of the grass seeds after the plates are charged. An examination of this latter photograph leads us to the conclusion that the resultant of the forces exerted by the two plates on a charge placed between them is uniform in the space between the plates, except near their edges. This might be expected, for, if a charge is moved from a position near the top plate toward the lower plate, the force exerted on it by the upper plate decreases, but the force exerted by the lower plate increases. Hence the magnitude of the resultant force remains constant. It is evident from the photograph that the direction of the resultant is perpendicular to the plates.

Figure 2.3(*f*) shows the field of force around a single charged plate. Again the lines of force are perpendicular to the plate except near the edges of the plate, and the magnitude of the electric force seems to be independent of distance from the plate. This conclusion does not seem correct at first glance, but consider Figure 2.4. Here we have simplified the situation by assuming that the charge on the plate may be assumed to be concentrated equally at two points X and Y on it. If a charge is moved from A to B toward the plate, the forces exerted by X and Y both increase, but the angle between them increases as well. Hence their resultant may very well remain constant, as Figure 2.3(*f*) seems to indicate.

2–5 A TEST OF COULOMB'S LAW

Figure 2.5 shows the electric field of charge placed on a hollow cylinder. Although the grass seeds outside the cylinder are affected by the field, those inside the

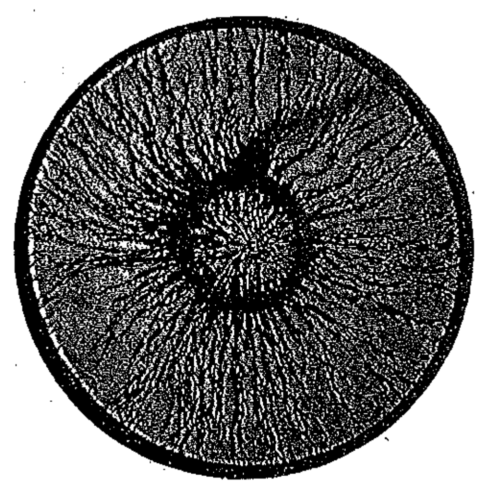


Fig. 2.5. The charge on the cylinder has no effect on the grass seeds inside the cylinder.

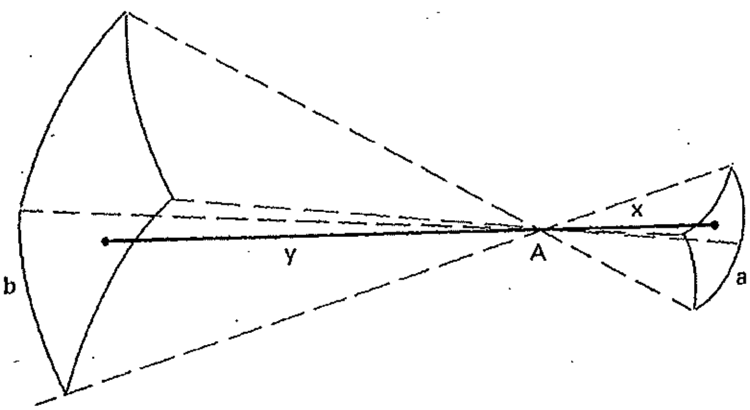


Fig. 2.6. Diagram for the analysis of the electric field inside a charged conductor.

cylinder do not appear to be affected at all. Apparently the resultant electric force acting at any point inside the cylinder is zero. This fact can be predicted with the help of Coulomb's law, and the correctness of the prediction indicates the validity of an inverse square law for electric force.

Consider a charge at a point A inside the cylinder (Fig. 2.6). Consider first the force exerted on it by the charge on a square of length and width a on the surface of the cylinder. This charge exerts on A a force F_1 to the left. However, F_1 is opposed by a force F_2 to the right, exerted by the charge on a square of length and width b, positioned on the opposite side of A as shown in Figure 2.6. Are F_1 and F_2 equal in magnitude?

According to Coulomb's law,

$$F_1 = k\frac{Qq_1}{x^2} \text{ and } F_2 = k\frac{Qq_2}{y^2}$$

where Q is the charge at A, q_1 and q_2 are the charges on the squares of sides a and b, respectively, and x and y are the distances from A to these squares, as shown in Figure 2.6.

$$\therefore \frac{F_1}{F_2} = \frac{q_1 y^2}{q_2 x^2}$$

If we assume that the charge is distributed uniformly over the surface of the cylinder, then

$$\frac{q_1}{q_2} = \frac{a^2}{b^2}$$

From similar triangles, $\frac{a}{b} = \frac{x}{y}$

$$\text{and therefore } \frac{a^2}{b^2} = \frac{x^2}{y^2}$$

$$\text{Hence } \frac{q_1}{q_2} = \frac{x^2}{y^2} \text{ and } F_1 = F_2$$

The above proof is true for any pair of opposite portions of the surface; in all cases F_1 and F_2 are equal in magnitude and opposite in direction. Therefore there is no electric field inside a hollow charged conductor. Because of this fact, the charge on a conductor may be shielded from the effects of other charges by surrounding the conductor with a metallic shield. Moreover, as we have noted above, the fact that the field inside a conductor is zero is convincing evidence of the validity of Coulomb's law.

2-6 ELECTRIC FIELD INTENSITY

The Coulomb law formula

$$F = k\frac{qQ}{r^2}$$

enables us to compute the force F exerted on a charge q at a distance r from an object bearing a charge Q. The magnitude of F depends on the magnitude of the charge q. In drawing fields of force it is convenient to have a vector whose magnitude is independent of q, and for this purpose field intensity vectors (also called field strength vectors) are used. The electric field intensity at a point is defined as the force per unit positive charge exerted on a small charge at that point, that is,

$$\text{Field intensity} = \frac{F}{q} = \frac{kqQ}{qr^2} = \frac{kQ}{r^2}$$

Field intensity is a vector quantity; its direction is the same as that of the force vector. Field intensity is measured in units of newtons per coulomb.

2-7 POTENTIAL DIFFERENCE

Suppose that an insulated conductor bears a positive charge Q_1 (Fig. 2.7), and that a small quantity Q_2, say 4×10^{-6} coulombs, of positive charge is at A. To simplify the problem assume that Q_1 and Q_2 are point charges and that they are isolated from the effects of all other charged objects. Since the two charges are alike, there will be a force of repulsion between Q_1 and Q_2. As a result, work must be done to move Q_2 closer to Q_1, say to a position B, 80 cm from A. The work done can be shown to be independent of the path taken from A to B, just as the work necessary to raise an object from one elevation to another is independent of the path taken. The force of repulsion between Q_1 and Q_2 will of course increase as Q_2 moves closer to Q_1. Suppose that the average value of the force is 10^{-3} newtons. Then the work done in moving the 4×10^{-6} coulombs of positive charge from A to B is 10^{-3} newtons $\times$ 0.8 metres or 8×10^{-4} joules. In other words, work is done at the rate of $\frac{8 \times 10^{-4}}{4 \times 10^{-6}} = 200$ joules per coulomb.

As a result of the work done in transferring Q_2 from A to B (Fig. 2.7), Q_2 possesses 8×10^{-4} joules more electric potential energy at B than at A. The difference in electric potential energy, or in short, the potential difference between points A and B is always calculated per unit charge. Thus the potential difference between A and B in this example is 200 joules per coulomb. One joule per coulomb is called one volt; therefore, the potential difference (P.D.) between A and B is 200 volts.

The following definitions are implied in the above discussion:

(a) The P.D. between two points X and Y in an electric field is the work done

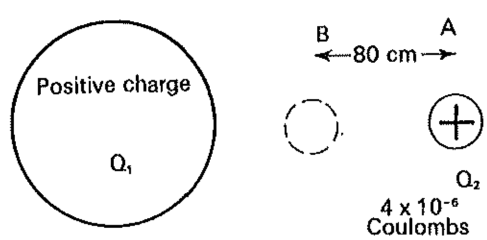


Fig. 2.7. Work must be done to move Q_2 from A to B.

per unit charge in moving positive charge from Y to X, or it is the electric potential energy gained per unit charge when positive charge is transferred from Y to X.

(b) One volt = 1 joule per coulomb. The P.D. between two points X and Y is 1 volt if work is done or energy is transformed at the rate of 1 joule per coulomb when positive charge is transferred from Y to X.

(c) If W joules of work is required to move a positive charge of Q coulombs from Y to X, then the work required per unit charge is $\frac{W}{Q}$, and thus

$$V = \frac{W}{Q} = \frac{E}{Q} \text{ and hence } E = W = QV$$

where V is the P.D. in volts between X and Y, and E is the number of joules of energy transformed.

2–8 WORKED EXAMPLE

The P.D. between two points in an electric field is 300 volts. How much work is required to move 5×10^{-9} coulombs of charge between these points? What average force is required if the points are 0.5 metres apart?

SOLUTION

$$W = QV$$
$$= 5 \times 10^{-9} \times 300 \text{ joules}$$
$$= 1.5 \times 10^{-6} \text{ joules}$$
$$F = \frac{W}{s} = \frac{1.5 \times 10^{-6}}{0.5} \text{ newtons}$$
$$= 3 \times 10^{-6} \text{ newtons}$$

2–9 ZERO OF POTENTIAL

The electric potential energy possessed by Q_2 (Fig. 2.7) was calculated to be 8×10^{-4} joules greater at B than at A. This statement in no way implies that the electric potential energy of Q_2 was zero at point A. Indeed, work must have been done at some time to bring Q_2 to point A. At what point would Q_2 have zero potential energy? A reasonable assumption, and one frequently made, is that its potential energy would be zero at an infinite distance from Q_1, for at this distance the repelling force exerted by Q_1 on it would be zero.

A somewhat similar argument provides a more practical choice for zero of electric potential energy. No electric forces are exerted by an isolated uncharged object; hence, an uncharged object may be considered to have zero electric potential energy. The earth may be regarded as a huge spherical conductor, so large that the addition to it of any likely quantity of charge leaves it still essentially uncharged, and at the same potential. Thus the earth or any conductor connected to earth may be reasonably assumed to be at zero potential.

Whether either of these choices of zero electric potential energy is correct is in practice immaterial. What matters to a man climbing a mountain is that the difference in elevation between the bottom and the top is 10,000 ft, even though a map may indicate that the respective elevations are 1000 ft and 11,000 ft above sea level. In electricity the similar vital information is that the difference in potential between two points is 10,000 volts, even though the respective potentials are 1000 volts and 11,000 volts above some arbitrarily chosen zero of potential, usually the earth.

The potential at a point A is defined to be the potential difference between that point and the point of zero potential, i.e., the work per unit charge necessary to bring positive charge to A from the point of zero potential.

Since work must be done to move positive charge towards a positively charged conductor, the conductor is said to be at a positive potential. Since work would have to be done to move positive charge away from a negatively charged conductor, this conductor is said to be at a negative potential.

2-10 DIRECTION OF MOTION OF CHARGE

Potential is traditionally and arbitrarily defined as above, i.e., in terms of positive charge. On such a basis, the discussion concerning Figure 2.7 has shown that the potential increases as the distance from the positive charge Q_1 decreases.

Suppose that a positive charge is released near a positively charged object (Fig. 2.8). It will be repelled by and move away from the positively charged object. A positive charge, if free to move in an electric field, will move towards lower potential. However, if a negative charge (Fig. 2.8) is released near a positively charged object, it will be attracted by and move towards the positively charged object. A negative charge, if free to move in an electric field, will move towards higher potential.

Since the charges which are free to move in conductors are most frequently the negative charges borne by free electrons, the following facts are worthy of note:

An electron flow will occur in a conductor AB and will be from B toward A if any of the following conditions exist:

(*a*) B is at a negative potential, while A is positive.

(*b*) B is at zero potential, while A is positive.

(*c*) B is at a negative potential while A is at zero potential.

(*d*) B is at a negative potential while A is at a higher negative potential (nearer zero).

(*e*) B is at a positive potential while A is at a higher positive potential.

In fact, an electron flow will occur in the conductor AB provided that a potential difference exists between A and B. Moreover, if there is no P.D., there will be no flow of charge. Conversely, if the charges in a conductor are at rest, all parts of the conductor must be at the same potential.

2-11 GRAVITATIONAL ANALOGY

The behaviour of electric charge in the electric field near a charged object is

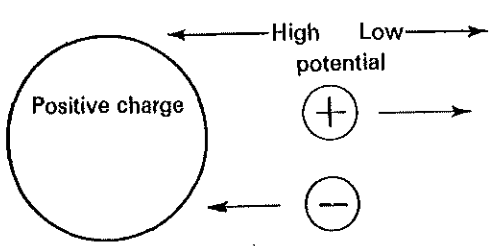


Fig. 2.8. Direction of motion of charge in an electric field.

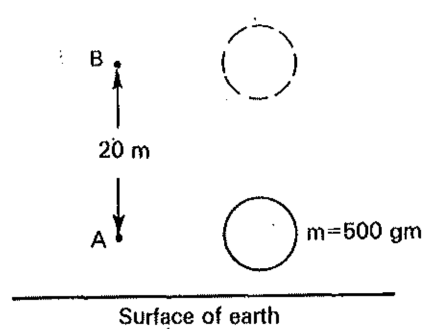


Fig. 2.9. Diagram to illustrate the similarities between a gravitational field and an electric field.

similar to the behaviour of matter in the gravitational field of the earth. Consider a 500-gm object at a position A above the surface of the earth (Fig. 2.9). The weight of the object constitutes a downward force acting on the object. The gravitational force, like the electric force, is inversely proportional to the square of the distance.

In the gravitational field, the gravitational force per unit mass is called the gravitational field strength and is given the symbol g. At the surface of the earth, the value of g is approximately 9.8 newtons/kg, and hence the acceleration due to gravity is approximately 9.8 m/sec^2. Similarly, in the electric field the electric force per unit charge, measured in newtons/coulomb, for example, is called the electric field strength (Sect. 2–6).

If AB (Fig. 2.9) is 20 m, and if $g = 9.8$ newtons/kg, the work required to raise the 500-gm mass from A to B is 98 joules. As a result, the gravitational potential energy of the mass is 98 joules greater at B than at A. This statement in no way implies that the gravitational potential energy of the object was zero at A. For the sake of convenience, the gravitational potential energy may be taken as zero at the surface of the earth. It is also common to consider the gravitational potential energy of an object to be zero at an infinite distance from the earth. Similar arbitrary choices of zero of potential are made in the electric field (Sect. 2–9).

In the gravitational field, the change in gravitational potential energy per unit mass, in joules per kg for example, is rarely calculated. However, in the electric field, the change in electric potential energy per unit charge, in joules per coulomb for example, is frequently calculated and is called potential difference.

If the 500-gm mass is released at B (Fig. 2.9), it will fall towards the earth, i.e., it will move towards a position of lower gravitational potential energy. The motion of mass which is free to move in the gravitational field is in the same direction as the motion of positive charge which is free to move in the electric field.

As the mass falls, its gravitational potential energy is converted to kinetic energy, and the law of conservation of energy applies. Similarly, electric potential energy may be converted to kinetic energy.

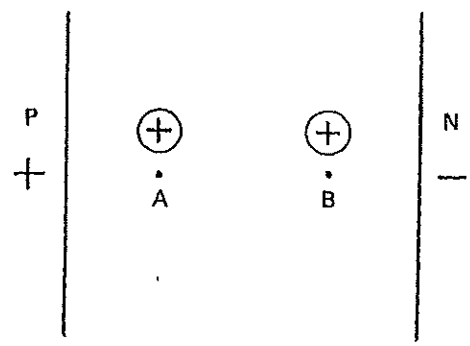


Fig. 2.10. If a positive charge is released at A, it accelerates as it moves toward B.

2–12 ACCELERATION OF CHARGE

When an object is released in the earth's gravitational field, it accelerates toward the earth. Let us consider the acceleration of charge in an electric field as shown in Figure 2.10. P and N are two parallel metal plates charged positively and negatively, respectively. A small light sphere, positively charged, is held at a position A between the plates. As we have already noted, the force exerted on this positively charged sphere by the plates remains constant when the sphere is permitted to move. As a result the sphere, when released, moves with uniform acceleration toward N, gaining speed and kinetic energy. We may predict its speed at any other point B as in the following example.

2-13 WORKED EXAMPLE

The charge on the sphere (Fig. 2.10) is 2×10^{-8} coulombs, the electric force exerted on it is 8×10^{-5} newtons, the mass of the sphere is 0.5 gm, and $AB =$ 4 cm. Calculate (*a*) the work done by the electric force on the sphere as the sphere moves from A to B, (*b*) the P.D. between A and B, (*c*) the horizontal component of the velocity of the sphere when it reaches B. Assume that there is no frictional resistance to the motion; i.e., that the work done by the force on the sphere (the electric potential energy lost by the charge) is equal to the kinetic energy gained by the sphere.

SOLUTION

(*a*)

$$W = Fs = 8 \times 10^{-5} \times 4 \times 10^{-2} \text{ joules}$$
$$= 32 \times 10^{-7} \text{ joules}$$

(*b*)

$$V = \frac{W}{Q} = \frac{32 \times 10^{-7}}{2 \times 10^{-8}} \text{ volts} = 160 \text{ volts}$$

(*c*)

$$E_K \text{ at } B = 32 \times 10^{-7} \text{ joules}$$
$$\text{i.e., } \tfrac{1}{2}mv^2 = 32 \times 10^{-7}$$
$$\tfrac{1}{2} \times 0.5 \times 10^{-3}\, v^2 = 32 \times 10^{-7}$$
$$v^2 = 128 \times 10^{-4}$$
$$\text{and } v = 11.3 \times 10^{-2} \text{ approx.}$$

Therefore the horizontal component of the velocity at B will be approximately 11.3×10^{-2} m/sec, or 11.3 cm/sec.

2-14 THE PROBLEM OF MEASURING *Q* AND *V*

So far in this chapter we have discussed the concepts of quantity of charge and potential difference; represented by the symbols Q and V. We have defined units, namely the coulomb and the volt, in which to measure Q and V, though admittedly the coulomb was only vaguely defined, and we shall have to leave it that way. However, in all of this, we have given no indication of how either Q or V might be measured.

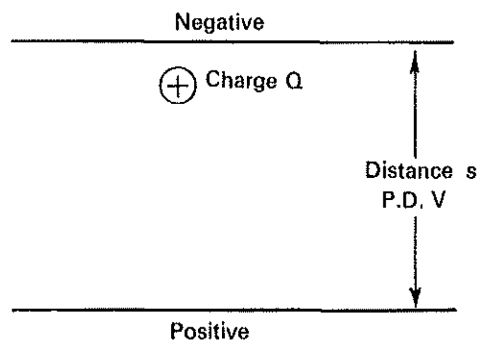


Fig. 2.11. *V* can be calculated if the force required to move *Q* from the negative plate to the positive plate can be measured.

The problem is not as complicated as it seems at first glance, for if Q can be measured, V can be calculated, and vice versa. Consider the situation shown in Figure 2.11, assuming that the effect of gravity is negligible. The small positive charge Q may be moved from the vicinity of the negative plate to the positive plate by a force F just sufficient to balance the electric force. The work W done by the force can be calculated from the formula $W = Fs$. But if V is the potential difference between the plates,

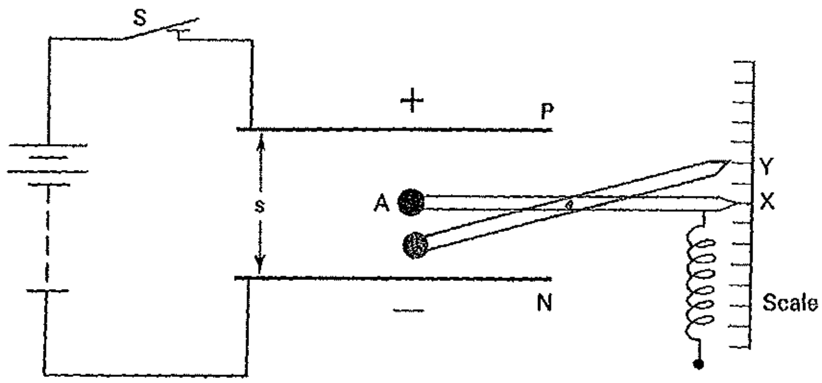


Fig. 2.12. Diagram of the main parts of an electrical balance.

$$W = QV$$
$$\therefore QV = Fs$$
$$\text{and } V = \frac{Fs}{Q}$$

F and s can be measured; then if we can measure Q we can calculate V.

The Coulomb's law formula

$$F = \frac{9.0 \times 10^{-9}\, qQ}{r^2}$$

may be applied in a measurement of quantity of charge. The two charges q and Q may be made equal by charge sharing as outlined in Section 2–2. Then a Coulomb type torsion balance may be used to measure F and r. The common magnitude of q and Q can then be calculated. If one of these charges is then placed in the apparatus whose very simple outline is shown in Figure 2.12, F and s may be measured and V calculated. Let us consider this apparatus in greater detail.

2–15 THE ELECTRIC BALANCE

The apparatus shown in Figure 2.12 is called an electrical balance. A battery, the positive and negative terminals of which are connected to P and N respectively, charges the plates P and N and establishes the electric field between the plates. As a result there is a potential difference V between them when the switch S is closed. If the sphere A is uncharged, the opposite end of its insulating arm is at X. Now a measured positive charge Q is placed on A. A moves down and the opposite end of the arm moves up to Y. The force necessary to produce the extension XY of the spring can be found readily, the distance s between the plates can be measured, Q is known and V can be calculated.

The use of a Coulomb torsion balance and then of an electrical balance for the determination of V may seem a laborious procedure, and it is. However, once such work is done, direct-reading voltmeters may be designed and calibrated. Further measurements are then very much simpler; in fact the electrical balance can then be used for determining Q, since V may be measured directly. We shall discuss this procedure in Chapter 4.

2–16 POTENTIAL AT A POINT

In Section 2–4 we investigated several electric fields qualitatively. Among them was the field about a point charge (Fig.

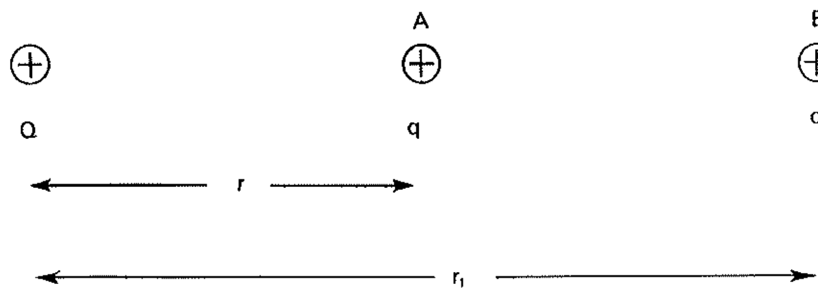


Fig. 2.13. Work must be done on the charge q in order to move it from A to B.

2.3a). Let us now analyse this field mathematically.

Suppose that a small positive charge q (Fig. 2.13) is placed at A, at a distance r from a positive charge Q. The charge Q exerts on q a force to the right, and if q is released at A, this force causes q to move to the right, losing electric potential energy as it moves. Suppose that its final position is B, at a distance r_1 from Q.

The potential energy lost by q as it moves from A to B is equal to the work done on q by the electric force. At A, the magnitude of this force is $\frac{kqQ}{r^2}$. If this force remains constant as q moves from A to B, then the work done by the electric force is given by

$$W = Fs$$
$$= \frac{kqQ}{r^2}(r_1 - r)$$

But the force does not remain constant, and r^2 is not the correct denominator to use here, nor is r_1^2. By means of mathematics beyond the scope of this book, it may be shown that the denominator should be rr_1.

$$\therefore\ W = \frac{kqQ(r_1 - r)}{rr_1}$$
$$= kqQ\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

If r_1 is infinitely large,

$$W = \frac{kqQ}{r}$$
$$\text{or } \Delta E = \frac{kqQ}{r}$$

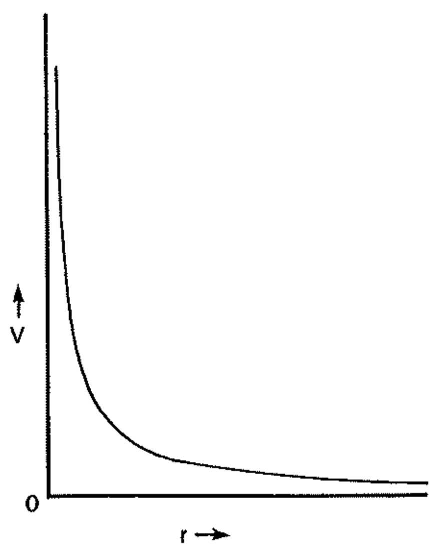


Fig. 2.14. Graph of the inverse relationship between V and r.

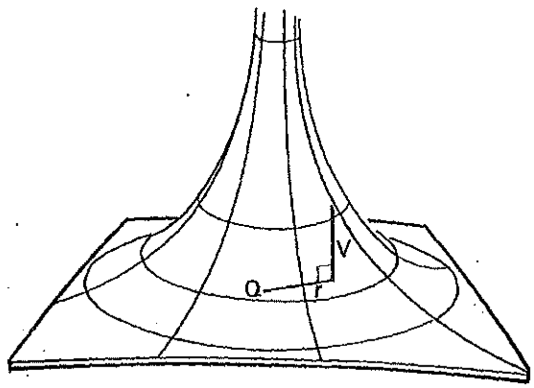


Fig. 2.15. If the graph in Figure 2.14 is rotated about its vertical axis, a potential hill is formed.

That is, the change in the electric potential energy of the charge q, as q is moved from a distance r to a point at infinite distance from Q, is $\frac{kqQ}{r}$. If we consider that electric potential energy is zero at infinite distance from Q (Sect. 2–9), then the potential energy at A (Fig. 2.13) must have been

$$E = \frac{kqQ}{r} \qquad (1)$$

We may go one step further and calculate for the charge q its energy per unit charge, that is, its potential V. In order to do this, we divide equation (1) by q, and obtain

$$V = \frac{kQ}{r} \qquad (2)$$

Equation (2) tells us that the potential at a point at a distance r from a charged object is inversely proportional to r. Figure 2.14 shows the graph of this inverse relationship between V and r. We shall make considerable use of this relationship between V and r in Chapter 5, particularly in the form shown in Figure 2.15. Here the graph has been rotated about its vertical (potential) axis to form what is called a potential hill. Figure 2.16 is a photograph of a model of a potential hill.

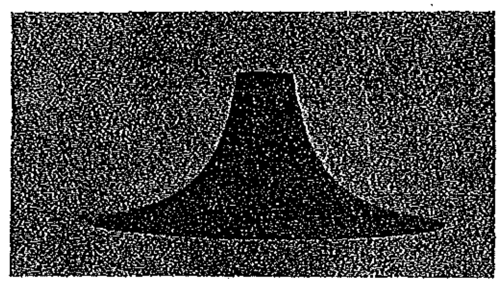


Fig. 2.16. A photograph of a model of a potential hill.

2–17 PROBLEMS

Where necessary, assume that
(*a*) in a vacuum, the constant k in Coulomb's law $= 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$
(*b*) the charge on the electron $= 1.60 \times 10^{-19}$ coulombs
(*c*) the mass of the electron $= 9.11 \times 10^{-31}$ kg.

1. Consider the relationship $F = \frac{kqQ}{r^2}$. What is the effect on F of (*a*) changing q

by a factor of 3, (*b*) changing Q by a factor of $\frac{1}{2}$, (*c*) changing r by a factor of 3, (*d*) making all of the above changes?

2. The force of repulsion between two small, positively charged objects, A and B, is 3.6×10^{-5} newtons when $AB = 0.10$ m. What is the force of repulsion if AB is increased to (*a*) 0.20 m, (*b*) 0.25 m, (*c*) 0.30 m?

3. Two charged objects, 2.0 cm apart, repel each other with a force of 3.0×10^{-5} newtons. What does the force become if (*a*) one of the charges is halved, (*b*) both of the charges are doubled, (*c*) the separation is increased to $2\sqrt{3}$ cm?

4. If each of two charges is tripled, while the distance between the charges remains the same, by what factor does the force between them change?

5. By what factor must the distance separating two charges be changed if the force between them is to be changed by a factor of 3?

6. Two small objects, each carrying a positive charge of 4.0×10^{-9} coulombs, are placed 2.0 cm apart in a vacuum. What is the magnitude of the force that each exerts on the other?

7. Calculate the force between charges of 5.0×10^{-8} coulombs and 1.0×10^{-7} coulombs, if they are 5.0 cm apart in a vacuum.

8. Calculate the force between charges of 4.0 microcoulombs and 6.0 microcoulombs when they are 8.0 cm apart in a vacuum.

9. Calculate the magnitude of the charge which will repel an equal charge with a force of 10 newtons when the charges are 10 cm apart in a vacuum.

10. At a certain point in an electric field, the magnitude of the field strength vector is 15 newtons/coulomb. Calculate the magnitude of the electric force exerted on a point charge of 3.0×10^{-7} coulombs placed at this point in the field.

11. A pithball bearing a negative charge of 0.50 microcoulombs is placed at a point in an electric field and is subject to an electric force of 1.0×10^{-4} newtons. What is the magnitude of the electric field intensity at that point?

12. At what distance from a charge of 6.0×10^{-6} coulombs is the field intensity equal to 3.0 newtons/coulomb?

13. The electric field strength mid-way between a pair of oppositely charged parallel plates is 3.0×10^{3} newtons/coulomb. What is the field strength half-way between this point and the positively charged plate?

14. When 4.0×10^{-3} coulombs of charge is transferred from A to B, 0.80 joules of work is done. What is the P.D. between A and B?

15. The potential difference between two plates is 1.2×10^{3} volts. If 0.12 joules of work is done in moving a charge from one plate to the other, what is the magnitude of the charge?

16. How much work is done when a charge of 0.005 coulombs is moved through a potential difference of 300 volts? If the distance the charge moves is 30 cm, calculate the average force required.

17. In a uniform electric field, the P.D. between two points 5.0 cm apart is 50 volts. Calculate the magnitude of the electric field strength.

18. Calculate the field intensity between 2 parallel plates 2.5 cm apart, if the P.D. between the plates is 200 volts.

19. Under normal atmospheric conditions the potential in the atmosphere near the earth's surface rises 80 volts for every metre increase in elevation. (*a*) In what direction will free electrons in the atmosphere move? (*b*) Calculate the loss in electric potential energy of a free electron which moves vertically through 10 metres in the atmosphere.

20. Two large, vertical, insulated metal plates, *A* and *B*, are placed 5.0 cm apart. A pith ball carrying a positive charge of 1.0×10^{-9} coulombs hangs on the end of a long silk fibre and is placed between the plates. The ball moves toward plate *A* under a force of 6.0×10^{-5} newtons. Calculate the P.D. in volts between the plates, and state which is at the higher potential.

21. If 5×10^{-2} joules of work is done in moving 0.01 coulombs of positive charge from earth to a positively charged conductor, what is the potential of the conductor?

22. A pith ball weighing 1.0×10^{-2} gm and bearing a positive charge of 4.0×10^{-7} coulombs is moved 50 cm from *A* to *B* through a potential difference of 8.0×10^{2} volts. The pith ball is released at *B* and "falls" back to *A*. Calculate (*a*) the work done in moving the pith ball from *A* to *B*, (*b*) the average force required, (*c*) the kinetic energy of the ball when it reaches *A* on returning from *B*, and (*d*) its speed at *A*.

23. A proton has a mass of 1.67×10^{-24} gm and a charge of 1.6×10^{-19} coulombs. If a proton, initially at rest, is acted upon by an electric field and moved through a potential difference of 10^{6} volts, what kinetic energy is imparted to the proton and what is its final speed?

24. An electron is accelerated from rest to a speed of 5.4×10^{8} cm/sec in a distance of 2.0 cm in a vacuum tube. Assuming that its mass remains constant, (*a*) what is the acceleration of the electron, (*b*) what average force is necessary to produce this acceleration, and (*c*) what is the potential difference through which the electron moves?

25. If the potential difference between an anode and a cathode is 8000 volts, what speed will free electrons, supplied at rest at the cathode, acquire in moving to the anode?

26. What must be the potential difference between a cathode and an anode to accelerate free electrons from rest to a speed of 3.0×10^{8} cm/sec?

27. Free electrons at rest are supplied at the cathode of an evacuated tube. The anode is at a potential of 240 volts above that of the cathode. Calculate the speed of the electrons as they reach the anode.

28. Consider the relationship $E = \frac{kqQ}{r}$. What is the proportional relationship between E and q, between E and Q, and between E and r?

29. For the relationship $V = \frac{kQ}{r}$, what is the effect on V of (*a*) changing Q by a factor of 10, (*b*) changing r by a factor of 0.2?

30. (*a*) Under what circumstances may the formula $E = \frac{kqQ}{r}$ be used to calculate the electric potential energy of a charge? (*b*) Use this formula to calculate the electric potential energy of a charge of 6×10^{-7} coulombs in the field of a charge of 5×10^{-5} coulombs, if the distance between the charges is (*i*) 10 cm, (*ii*) 20 cm, (*iii*) 30 cm.

31. (*a*) Under what circumstances may the formula $V = \frac{kQ}{r}$ be used to calculate the potential at a point in an electric field? (*b*) Use this formula to calculate the potential in the field of a charge of 8×10^{-7} coulombs, at distances of (*i*) 15 cm, (*ii*) 20 cm, and (*iii*) 30 cm.

2-18 SUMMARY

Charge may be transferred, but it is not created nor destroyed.

Coulomb's law states that the force between two charges is directly proportional to each of the charges, and inversely proportional to the square of the distance between them.

$$F = \frac{kqQ}{r^2}$$

$$k = 9.0 \times 10^9 \frac{\text{newton-(metre)}^2}{(\text{coulomb})^2}$$

The electric field between parallel, oppositely charged plates is uniform except near the edges of the plates.

Electric field intensity = electric force per unit charge.

The P.D. between two points in an electric field is the work done or energy transformed per unit charge, when charge is transferred from one point to the other.

$$V = \frac{W}{Q} = \frac{E}{Q}$$

1 volt = 1 joule/coulomb

The work done, electric energy transformed, and kinetic energy gained when charged particles are accelerated from rest are connected by the relationships

$$E = W = QV = Fs = \tfrac{1}{2}mv^2$$

If the zero of potential is taken at an infinite distance from a charged object, then (1) the potential energy of a charge in the electric field of the object is given by the formula $E = \frac{kqQ}{r}$, and (2) the potential at a point in the field is given by the formula $V = \frac{kQ}{r}$.

Chapter 3

Charge in Motion

3–1 INTRODUCTION

Until early in the eighteenth century electricity was considered to be a static property of those substances which are now called insulators. The possibility of a transfer of this property (i.e., of charge) was not suspected. Then in 1731 Stephen Gray of England reported that he had succeeded in transferring charge over distances of several hundred feet. Subsequent similar experiments showed that charge could be transferred quite readily through those substances which are now called conductors.

In spite of these discoveries, several decades elapsed before worthwhile advances took place, because no method of producing a continuous flow of charge was known. Then in 1799 the Italian scientist, Alessandro Volta (1745-1827), built a device, later called a voltaic cell, which could maintain a constant potential difference between its terminals and therefore produce a continuous flow of charge in a conductor. In 1820 the Danish physicist, Hans Christian Oersted, discovered that a magnetic field existed about a conductor through which charge was flowing. Scientists then set as their goal the production of a flow of charge in a conductor by the use of a magnet. This proved to be a difficult problem until Michael Faraday reasoned that, since a moving charge produces a magnetic field, perhaps a moving magnet would produce a flowing charge. On October 17, 1831, he verified this deduction and produced a magnetically induced current by moving a magnet into and out of a coil of wire.

3–2 SOURCES OF CHARGE

As a result of the work of Volta and Faraday, batteries and generators are now our most important sources of electricity. However, the charges produced by batteries and generators are exactly the same

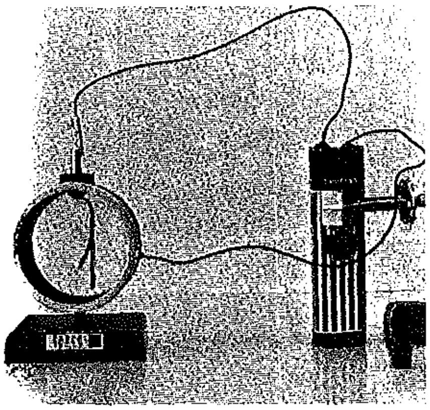


Fig. 3.1. The separation of the leaves of the electroscope indicates that the terminal of the battery bears a static charge.

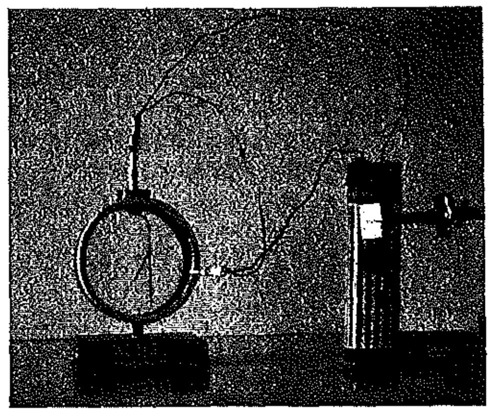


Fig. 3.2. The chemical action in a battery supplies charge continuously to the battery terminals. As a result, the leaves of the electroscope in this photograph remain separated, even though charge may transfer from the leaves to the case through a resistor. The resistor is to the right of the top of the electroscope.

as those previously encountered in electrostatics. If a battery or other source of electricity (of at least 90 volts DC) is connected to a sensitive electroscope as shown in Figure 3.1, the leaves of the electroscope diverge. The charge on the leaves may be identified as outlined in section 1–11. One of the battery terminals is found to bear a static positive charge, the other a static negative charge.

Apparently a chemical action within the battery separates the charges and causes them to accumulate at the terminals. Moreover, this action is continuous, as we may show by the circuit shown in Figure 3.2. The separation of the leaves of the electroscope remains constant even though charge may transfer slowly from the leaves to the case through the resistor. However, if the battery is disconnected, the electroscope quickly discharges.

3–3 THE P.D. OF A BATTERY

The potential difference between the plates P and N in Figure 2.12 is the P.D. between the terminals of the battery, because the work required to transfer charge from the plates to the terminals through the conducting wires is negligible. A battery is composed of two or more cells, connected usually in series; that is, with the positive terminal of one cell connected to the negative terminal of the next. Experiments with electrical balances (or with voltmeters) indicate that all similar cells have the same P.D.; for example the P.D. of a dry cell is about 1.5 volts. Moreover, the P.D. of a battery of cells connected in series is proportional to the number of cells. Thus the P.D. of a battery of 100 dry cells is 150 volts.

Apparently the chemical action in a cell forces charges to accumulate on the terminals. Thus chemical potential energy is used to increase the electric potential energy of the charges. The statement that the P.D. of a battery is 12 volts therefore means that each coulomb of charge supplied at the terminals possesses 12 joules of electric potential energy. The P.D. of a battery is the energy supplied by the battery to each unit of charge.

3–4 BASIC ELECTRIC CIRCUIT

We do not intend to give a detailed analysis of electric circuits in this text; we assume that you are sufficiently familiar with some basic facts concerning such circuits. The remainder of this chapter summarizes these facts briefly.

If the terminals of a battery are connected by a solid conductor, electrons flow from the negative terminal to the positive terminal through the conductor. In order to limit the number of electrons transferred in a given length of time, and to exercise some control over this transfer, a resistor (relatively poor conductor) must be included in the circuit (Fig. 3.3). Any electric appliance is a resistor, and therefore the comments which follow apply for any circuit operated by a battery.

3–5 ELECTRIC CURRENT

Just as a current of water, to the hydraulic engineer, means a rate of flow that may be expressed in gallons per second, so current of electricity means the rate of flow of charge. The standard unit of current is the ampere. 1 ampere = 1 coulomb per second. If 10 coulombs of charge are transferred from X to Y (Fig. 3.3) in 5 seconds, the rate of flow is $\frac{10}{5}$ coulombs per second, i.e., 2 coulombs per second or 2 amperes. Since 1 coulomb = 6.25×10^{18} elementary charges, then

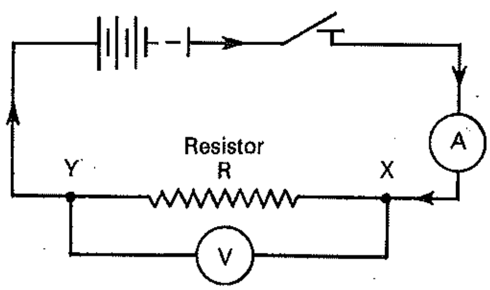


Fig. 3.3. The basic DC circuit.

1 ampere = 6.25×10^{18} elementary charges/second.

The symbol I is used to represent current; the following relationship is obvious: $I = \frac{Q}{t}$, or $Q = It$, when Q is the quantity of charge transferred in time t.

One of the early devices used to measure electric current was a special type of electrolytic cell called a silver voltameter (Fig. 3.4). An electrolytic cell contains two strips of metal, called electrodes, immersed in a liquid. The electrodes are connected to the terminals of a battery; the electrode connected to the positive terminal is called the anode and the electrode connected to the negative terminal is called the cathode. In the silver voltameter the anode is a strip of pure silver, the cathode is a strip of copper, and the liquid is a solution of silver nitrate.

When the switch S (Fig. 3.4) is closed, the ammeter indicates that the solution conducts electricity. One effect of this conduction is to cause a film of silver to be deposited or plated on the surface of the cathode. The mass of silver deposited can be determined by careful weighing of the cathode before and after the transfer of charge. Experiments have shown that

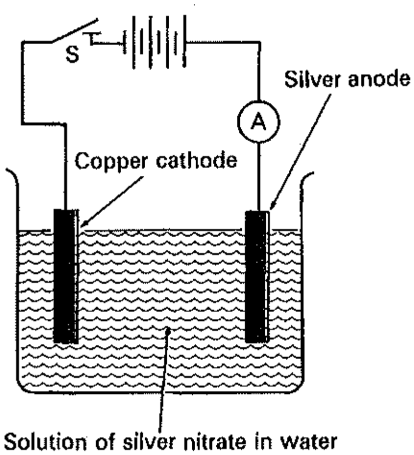


Fig. 3.4. A silver voltameter.

the mass of silver deposited is proportional to the current and also proportional to the time of flow.

An International Electric Congress, meeting first in 1881, and attended by many of the world's leading physicists, made a thorough study of this plating effect and the relationship between the ampere and the mass of silver deposited. Their recommendations were based on theoretical considerations beyond the scope of this book. Revisions were made in later years, and in 1908 the International Ampere was adopted. It was defined as that current which would deposit silver from a silver nitrate solution at the rate of 0.001118 gm per second.

A flow of electrons in a conductor produces a magnetic field in the region surrounding the conductor. This effect has been applied in the moving-coil type of ammeter in common use for measuring current. The silver voltameter may be used to calibrate the moving-coil type of instrument.

An ammeter is shown in Figure 3.3, connected between the negative terminal of the battery and the point X. From the ammeter reading, the quantity of charge transferred in a measured time can be calculated. The relationship $Q = It$ is used. Q is the quantity of charge in coulombs, I is the current in amperes, and t is the time in seconds.

3–6 ENERGY TRANSFORMATIONS

The resistor R (Fig. 3.3), though it conducts charge, offers some opposition to the flow of charge through it due to frictional resistance to electron motion. Two results of this opposition to the flow of charge are discussed below.

(*a*) Some of the electric energy possessed by this charge is expended as the charge flows through the resistor. The law of conservation of energy indicates that this electric energy is transformed to some other form of energy. If the resistor is an element in an electric stove, most of the electric energy is transformed into heat energy. If the resistor is the filament of an incandescent light bulb, both heat and light are produced. If the resistor is an electric motor, the energy produced is mainly in the form of kinetic energy of the armature.

(*b*) Because of this transformation of energy, there is a potential difference between X and Y. This P.D. is defined as the electric energy per unit charge transformed when charge is transferred from X to Y.

3–7 MEASUREMENT OF POTENTIAL DIFFERENCE

The measurement of the P.D. between X and Y (Fig. 3.3) is accomplished by connecting a voltmeter in parallel with the resistor R. The voltmeter is usually of the moving-coil type and consists essentially of a galvanometer coil in series with a high resistance conductor. To go from X to Y the charge has two possible routes, through the resistor R or through the voltmeter. The high resistance in the voltmeter is usually such that, with a P.D. of 100 volts between X and Y, not more than 1/1000 of an ampere will flow through the voltmeter. Hence, the current in the resistor R is not seriously altered. This small current of 1/1000 of an ampere will, however, deflect the galvanometer coil. The angle of deflection of the coil is approximately proportional to the current through the coil; this current is proportional to the current in the resistor R; and the current in R is proportional to the potential difference between X and Y.

3–8 ELECTRIC ENERGY AND POWER

If the formula $E = QV$ is used to calculate the electric energy transformed, and if Q is in coulombs and V is in volts, E is in joules. Since $Q = It$, the formula $E = QV$ may be written $E = VIt$, the units for E, V, I and t being respectively joules, volts, amperes, and seconds.

For example, referring again to Figure 3.3, if the ammeter reading is 2 amperes and the voltmeter reading is 10 volts, the electric energy transformed in the resistor in a 2 hour period $= E = VIt = 10 \times 2 \times 2 \times 3600$ joules, or 144000 joules.

Power is defined as the rate at which energy is transformed. For electric energy,

$$P = \frac{E}{t} = \frac{QV}{t} = \frac{VIt}{t} = VI$$

The unit of power is the watt.

One watt = one joule/sec.

Then, for the readings given in the example in the preceding paragraph, $P = VI = 10 \times 2$ watts $= 20$ watts. That is, electric energy is being transformed in the resistor at the rate of 20 joules per second.

3–9 CONDUCTION OF ELECTRIC CHARGES

The list of conductors in Section 1–5 contains only metals, all of which, with the exception of mercury, are solid at normal temperature. Conduction in metals consists of a transfer of electrons from the negative terminal of the source through the circuit to the positive terminal. Thus conduction in a solid metal is a relatively simple process. Conduction in liquids and gases is more complex.

Solutions of acids, bases and salts are listed in Section 1–5 as fair conductors. They do not conduct nearly as well as metals, but the fact that they do conduct is important. Not only is the conduction frequently accompanied by useful chemical changes, but these changes give us considerable insight into the structure of matter. Consider, for example, the silver voltameter circuit shown in Figure 3.4. When the switch is closed the ammeter indicates that the silver nitrate solution is a moderately good conductor. We have already noted that the effect of the conduction is that the mass of the anode decreases, and an equal amount of silver is deposited on the cathode. Not only is charge transferred, but matter (silver) is transferred as well. Apparently the silver nitrate in solution spontaneously dissociates into positively and negatively charged particles called ions. Each type of ion is then attracted to the electrode bearing the charge opposite to that on the ion. In general, conduction in a liquid consists of positive ions migrating to the cathode, and negative ions to the anode.

Under normal circumstances, most gases are very poor conductors. An electroscope will retain its charge for several hours or more, even though it is in contact with air. However, if a lighted match is held near the top of a charged electroscope, the charge is lost very quickly. Apparently the effect of the match is to ionize the air. The ions whose charges are opposite to that on the electroscope are attracted to the electroscope and neutralize its charge. An ionized gas, then, conducts electricity. In general, conduction in a gas consists of positive ions travelling in one direction, and negative ions, supplemented perhaps by free electrons, travelling in the other direction. As might be expected, a gas conducts better at reduced pressure, for then the moving ions undergo fewer collisions with gas molecules and with one another. Finally, if the gas is removed entirely, there are no ions present, and the conduction consists solely of a transfer of electrons.

3–10 THE BEGINNINGS OF A CHANGE OF OUTLOOK

In these first three chapters, we have described a number of physical phenomena, mainly in terms of the concepts prevalent at the end of the nineteenth century. These concepts were admirably suited to explaining physics as it was then understood. In fact, the concepts and their accompanying theories and models had been developed just for this purpose. The revolution which caused twentieth century physics to differ radically from nineteenth century physics, was prompted by new discoveries which had to be accompanied by new concepts and a radical change in outlook. Chief among the new concepts, and forcing a change in outlook, was the concept of quantization in all branches of science.

The guiding idea in quantization is that phenomena in science are not to be considered as proceeding smoothly, but as proceeding in steps. In most cases the steps are so small as to give the impression of smoothness, but close examination shows that the steps exist. For example, microscopic examination has shown that living matter is not infinitely divisible, but is composed of building blocks called cells. Moreover, matter in general is not infinitely divisible, but is composed of discrete entities called atoms. Cells and atoms, then, may be called the quanta (singular; quantum) of matter.

In the remaining four chapters of this book we will outline the experimental evidence which led to the conclusion that there is a quantum of electricity, and that there are quanta of radiant energy.

3–11 PROBLEMS

1. The P.D. of a storage cell of the type used in automobile electrical systems is approximately 2 volts. Most modern automobiles use a 12-volt battery. How are the cells in the battery connected? How many cells are there in the battery?

2. If the P.D. of a battery is 90 volts, each coulomb of charge supplied by the battery has 90 joules of electric energy. If the battery is connected in a circuit, charge is transferred through the circuit. What becomes of the energy possessed by the charge?

3. In 1 hour, 900 coulombs of charge are transferred through a circuit. Calculate the current in the circuit.

4. How long must a current of 0.5 amperes flow in a circuit in order to transfer 45 coulombs of charge?

5. A current of 5 amperes flows for 2 minutes. Calculate the quantity of charge transferred (*a*) in coulombs, and (*b*) in elementary charges.

M Ag = 108 gm

6. Assume that, in a silver voltameter, a current of 1 ampere deposits silver at the rate of 1.12×10^{-3} gm/sec. (*a*) How much charge is transferred along with each 1.12×10^{-3} gm of silver? (*b*) If silver is deposited at the rate of 2.80×10^{-3} gm/sec, what is the current? (*c*) How long must a current of 3 amperes flow to deposit 1.01 gm of silver?

7. The current in a resistor is 3.0 amperes, and the P.D. between the ends of the resistor is 40 volts. (*a*) How much electric energy is transformed by the resistor in 10 minutes? (*b*) What is the power of the resistor? (*c*) What becomes of the energy transformed by the resistor?

8. An electric light bulb is marked: 120 volts, 60 watts. (*a*) What is the current in the filament (*i*) when it is used in a 120-volt circuit, (*ii*) when it is used in a 240-volt circuit? (*b*) What is its power in the 240-volt circuit? (*c*) Account for your answer to (*b*) in terms of the relationship $P = VI$. (*d*) How much electric energy does the bulb transform in 1 hour in the 120-volt circuit? (*e*) What becomes of this energy?

9. The filament of a certain radio tube has a potential difference across it of 6.3 volts, when the current in the filament is 0.34 amps. Calculate the rate, in calories per sec, at which heat is being dissipated by the filament. (The mechanical equivalent of heat is 4.19 joules per calorie.)

10. The accelerating voltage in a TV picture tube is 20,000 volts. Calculate the work done on an electron which is accelerated by this voltage, assuming that the charge on the electron is 1.60×10^{-19} coulombs. If 3.0×10^{14} electrons are accelerated each second, calculate the power necessary.

11. An electric motor, operating in a 120-volt circuit, draws 4.0 amperes of current. It raises a 15-kg mass at a rate of 0.60 m/sec. (*a*) Calculate the rate at which the motor uses electric energy. (*b*) Calculate the rate at which the gravitational potential energy of the 15-kg mass increases. (*c*) Calculate the quantity of charge passing through the motor in 40 sec.

3–12 SUMMARY

Batteries and generators are devices for separating charges. The charges are exactly the same as those produced by rubbing objects together. The P.D. of a battery is the energy given by the battery to each unit of charge.

Current is the rate of flow of charge.

$$I = \frac{Q}{t}$$

$$1 \text{ ampere} = 1 \text{ coulomb/sec}$$

The function of a resistor in an electric circuit is to transform electric energy to some other form. The energy transformed by a resistor is given by the formula $E = VIt$, and the power of a resistor by the formula $P = VI$.

Conduction in solids is accomplished by electron transfer; conduction in liquids is accomplished by the transfer of positive and negative ions; conduction in gases is accomplished by the transfer of positive ions, negative ions, and electrons; conduction in a vacuum is accomplished by electron transfer.

Chapter 4

The Elementary Charge

4–1 INTRODUCTION

In Chapter 2 we discussed certain electrical concepts, units and measurements. We did not place any limits on the magnitudes of the forces, charges or potentials which we were considering but we likely assumed that they were of the order of magnitude of those associated with batteries. The electrical balance we discussed in Section 2–15 is a large one whose field is produced by a large number of cells in series. Now we turn to a consideration of very small charges, particularly the charge which we have already taken the liberty of calling the elementary charge.

4–2 CATHODE RAYS

The existence of atoms—small indivisible building blocks of matter—had been postulated since the days of ancient Greece. By 1890 experiments in chemistry had confirmed the facts that atoms existed and combined with one another to form molecules. Since 1890 many experiments have been performed which confirmed the existence of still smaller (sub-atomic) particles. The first of these particles to be discovered was the electron.

Some of the early evidence for the existence of the electron was obtained from experiments concerning conduction in gases at low pressures. Both Faraday and Coulomb, working independently, began the study of conduction in air at pressures lower than atmospheric. Other research physicists soon joined in the study, and in a period of about forty years (1855-1895) their investigations resulted in the discovery of the electron, the production of X-rays, and the information which is the basis for the amazingly successful electron theory of atomic structure.

The laboratory apparatus now used to illustrate the early studies of Faraday, Coulomb, and Sir William Crookes (1832-1919) is illustrated in Figure 4.1. It is

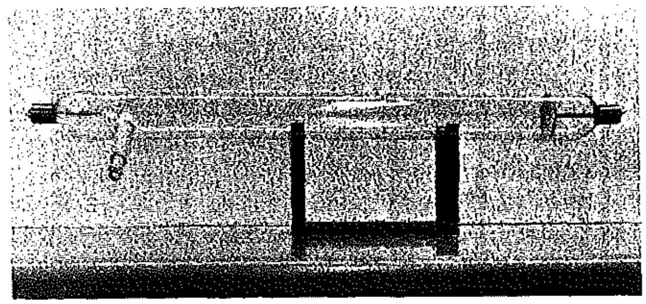


Fig. 4.1. Laboratory form of Crookes' tube.

called a Crookes' tube and consists of a hard glass tube about 15 inches long with one electrode at each end. A short side tube makes it possible to attach an air pump by means of which the air or gas pressure in the main tube can be altered. The electrodes, which may be aluminum, are usually a solid rod and a plane or concave disc. The rod electrode is connected to the positive terminal of a high-voltage induction coil; the disc electrode is connected to the negative terminal of the coil. Thus the rod electrode becomes the anode and the disc the cathode. The terminals of the induction coil should be equipped with needle points, their tips about an inch apart. The side tube is connected to a vacuum pump to which is attached a vacuum gauge.

If the coil is set in operation while the pressure within the tube is atmospheric, a violent discharge occurs between the needle points of the coil. If now the vacuum pump is set in operation and the gas pressure within the tube is steadily lowered, and if the room is darkened, a startlingly beautiful phenomenon is observed. When the pressure reaches about 1 cm of mercury, the discharge between the coil terminals stops. A purple glow appears in the tube on the surface of the cathode and a single sinuous purple streamer extends from the cathode to the anode. This streamer is neither straight nor steady. As the pressure is slowly lowered additional streamers appear until, at a pressure of $\frac{1}{2}$ cm of mercury, the light within the tube becomes diffused and the whole interior becomes luminous. When the pressure is further reduced to about $\frac{1}{2}$ mm of mercury a highly unusual pattern is seen (Fig. 4.2). A thin luminous layer, called the cathode glow, appears on the surface of the cathode. Adjacent to this cathode glow is a sharply defined, dark area which is called Crookes' dark space. Adjacent to this dark space is a diffused luminous area, the negative glow. Beyond this, towards the anode, is another

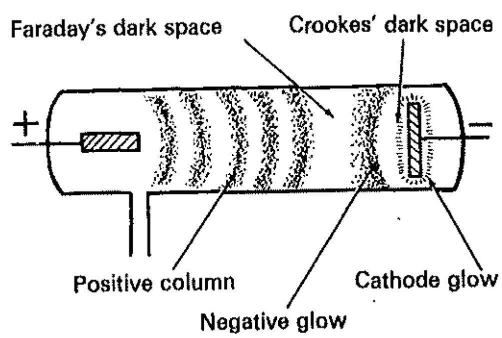


Fig. 4.2. Pattern caused by electrical discharge in a Crookes' tube when the air pressure is 0.5 mm of mercury.

dark, poorly defined area called Faraday's dark space. Between this space and the anode is a luminous area called the positive column. This column is not uniformly luminous but is usually alternately luminous and dark.

It is doubtful that Faraday, due to the limitations of his vacuum pumps, saw the pattern just described. Sir William Crookes continued the study with improved pumps, and as the pressure approached 1/100 mm of mercury he found that Crookes' dark space increased in length until it filled the whole tube. Coincident with this observation he noted that the glass of the tube, particularly in the neighbourhood of the cathode, fluoresced with a greenish colour. The German physicist, Julius Plücker (1801-1868), suggested that this fluorescence of the glass was due to some kind of ray being sent out by the nearby cathode; these rays had sufficient energy, when they struck the glass, to cause fluorescence and heat. Plücker called them cathode rays.

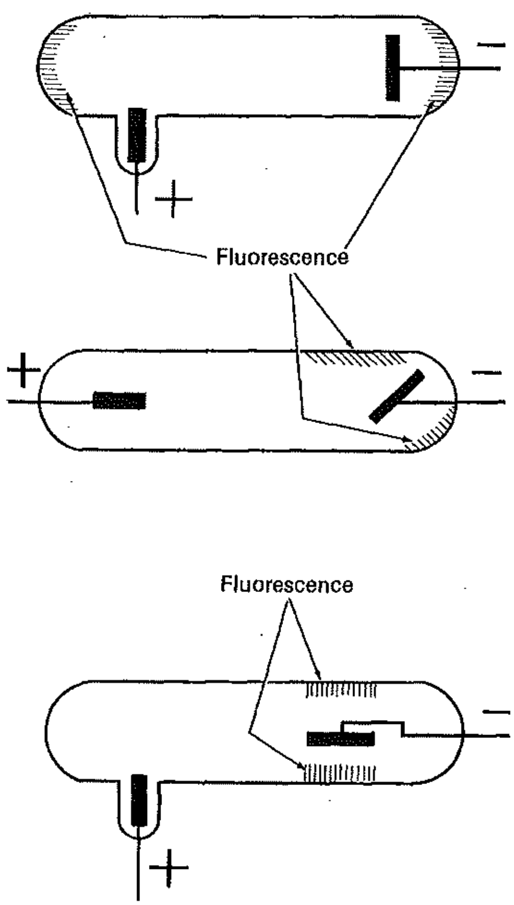


Fig. 4.3. The fluorescent areas of the glass are in directions perpendicular to the surfaces of the cathodes.

4–3 PROPERTIES OF CATHODE RAYS

Intensive investigation soon revealed additional properties of the cathode rays. It was found that the position of the areas of the glass showing the greatest fluorescence depended on the shape and position of the cathode, and that the position or form of the anode was immaterial. A study of Figure 4.3 which shows the fluorescent areas for different cathodes, indicates that the rays are emitted in a direction normal to the surface of the cathode. If a concave cathode is used (Fig. 4.4), and if a piece of tin

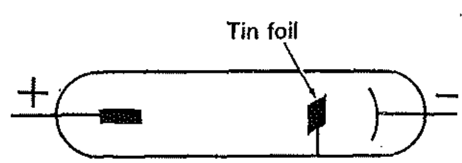


Fig. 4.4. Tube designed to demonstrate the heating effect of cathode rays.

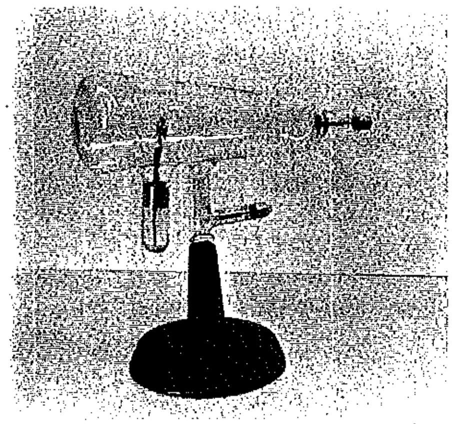


Fig. 4.5. A Crookes' tube used to demonstrate rectilinear propagation of cathode rays and their deflection by magnetic and electric fields.

foil is secured near the focus of the cathode, it is found that the foil becomes incandescent when the tube is operated. This discovery led Crookes to conclude that the rays are a stream of particles and that the heat is produced in the tin foil by the transformation of the kinetic energy of these particles.

Continuing the study of these rays, a tube as shown in Figure 4.5 was built. The cathode was a flat disc mounted at right angles to the central axis of the tube. A small hinged Maltese cross of sheet metal was mounted in the tube so that it could be placed in the path of the beam. Since a sharply defined shadow of the metal cross appeared on the end of the tube it was concluded that the rays travel in straight lines. It was discovered too that, if a permanent magnet was brought close to the tube, the shadow moved in a plane at right angles to the axis of the magnet, an action characteristic of a conductor in which charge is flowing. Also, a metal plate bearing an electrostatic charge deflected the beam towards the plate when it was positively charged and away from the plate when it was negatively charged. From these observations it was concluded that a cathode ray is a stream of negatively charged particles. In 1890 Dr. Johnston Stoney named these particles electrons.

4–4 THE CHARGE-MASS RATIO OF ELECTRONS

In the last decade of the nineteenth century, Sir J. J. Thomson (1856-1940), one of the greatest of English scientists, joined in the study of cathode rays. He fashioned a tube as shown in Figure 4.6. The anode consisted of a cylinder with a hole along its axis so that the electron stream would be a narrow one travelling along the central axis of the tube. The metal plates, A and B, each with a connecting wire, made it possible to place an electrostatic charge inside the tube close to the ray. Thomson hoped to determine the mass of the particles in the cathode ray, and the magnitude of their electric charge. With plates A and B uncharged, Thomson found that the tube face (the end of the tube) fluoresced at P. With plate A positive, the face fluoresced at P_1, and with plate A negative it

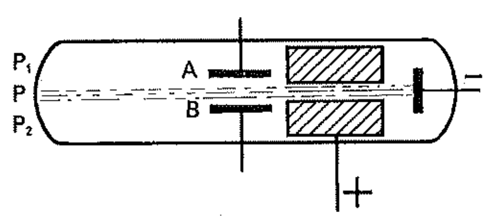


Fig. 4.6. A Crookes' tube of a type used by J. J. Thomson.

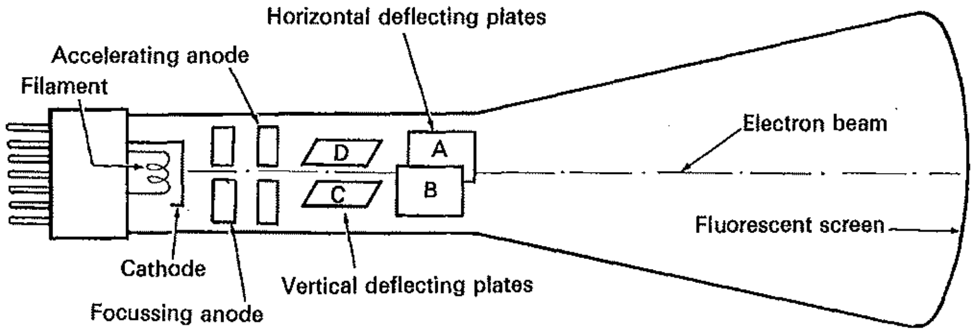


Fig. 4.7. The cathode ray tube in a cathode ray oscilloscope.

fluoresced at P_2, thus confirming Crookes' discovery that the particles bore a negative charge. Then Thomson applied magnetic and electric fields simultaneously, in such a way that the electric field tended to deflect the beam upward, and the magnetic field tended to deflect the beam downward. He adjusted the magnitudes of the two fields until their effects cancelled and the beam was undeflected. From his observations, he concluded that the electron had mass, and he was able to determine that the ratio of the charge to the mass was of the order of 10^{11} coulombs per kilogram. He was unable, however, to determine either the charge or the mass separately.

Since all of the observed properties of cathode rays are independent of the kind of metal in either the cathode or the anode, electrons seem to be constituent parts of all atoms.

4–5 THE CATHODE-RAY OSCILLOSCOPE

A Crookes' tube, in the form used by Thomson for his investigations (Fig. 4.6), constitutes a device for firing a stream of high-speed electrons at the end surface of a glass tube. The whole unit is called a cathode-ray tube. The fluorescence produced when the electrons strike the tube face is greatly increased if the inner surface of the tube face is coated with zinc orthosilicate, zinc sulphide, or calcium tungstate. Moreover, the rate of emission of electrons from the cathode is greatly increased if the cathode is heated by a filament. The filament is simply a small electric heating coil. Emission of electrons by a heated cathode is called thermionic emission.

The cathode and the associated anode in such a tube constitute an electron gun. This gun has many uses. It is the basic part of the cathode-ray tubes employed in radar sets, in television cameras and receivers, in the electron microscope, and in the cathode-ray oscilloscope.

In the cathode-ray oscilloscope (Fig. 4.7), the electron gun fires a cathode ray between horizontal and vertical deflecting plates. The path of the cathode ray can be changed quite readily by charges applied to the plates. The oscilloscope is very sensitive because of the fact that its moving part—an electron beam—has very little mass. Moreover, it responds practically instantaneously to charges or changes in charges on the deflecting plates. If, for example, the upper vertical deflecting plate D is connected to the positive terminal of a battery, and the lower plate C to the negative terminal, the beam is deflected upward and as a result the fluorescence is produced at a higher point on the fluorescent screen. The magnitude of the displacement of the spot depends on the charge on the plates. Thus the oscilloscope may be used to compare charges.

Perhaps the most important use of the oscilloscope is to show how charges change rapidly as time goes on. A circuit in the oscilloscope is used to charge the horizontal deflecting plates, A and B, relatively slowly, then to discharge them quickly. This charging and discharging process is repeated continuously and as a result the fluorescent spot "sweeps" horizontally across the screen and then returns quickly to its starting point. As it moves, changes in the charges applied to the vertical deflecting plates cause the spot to move up or down. Because of persistence of vision on the retina of the eye, and persistence of fluorescence of the spots on the screen, a pattern is traced on the screen (Fig. 4.8). This pattern is in effect a graph of the changes in charge on the vertical deflecting plates against time.

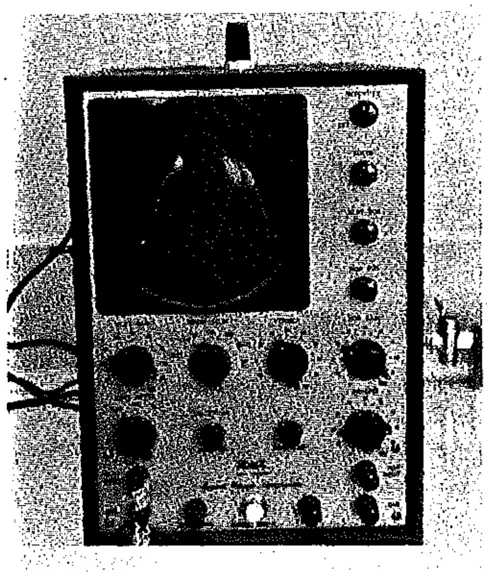


Fig. 4.8. Pattern produced on the screen of a cathode ray oscilloscope when the vertical deflecting plates are connected to 60 cycle alternating current.

4-6 X-RAYS

In November, 1895, Professor Wilhelm Konrad Röntgen (1845-1923), in his laboratory in Germany, was experimenting with a Crookes' tube. With the tube operating at very low gas pressure, he noted that certain minerals in the room fluoresced even when the tube was covered with black paper. To his amazement he discovered that the minerals would fluoresce, as well, in a room adjacent to that in which the tube was operating, even when the door to that room was closed. He concluded that a new and very penetrating kind of radiation was issuing from the tube. Using an algebraic symbol for the unknown, he called this radiation X-rays.

An X-ray tube with a heated cathode is shown in Figure 4.9. Electrons emitted by the cathode strike the tungsten target, which also serves as the anode of the tube. The bombardment of the target by high speed electrons causes X-rays to be radiated from the target.

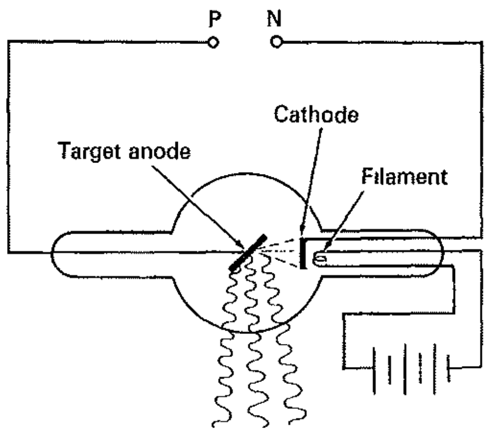


Fig. 4.9. An X-ray tube. *P* and *N* are the positive and negative terminals, respectively, of a DC source.

X-rays possess many of the same properties as does light. Unlike light, they penetrate very well materials composed of elements having low atomic weight, and therefore are commonly used in examining the bones of the body for fractures, and the internal organs of the body for disease or growths.

X-rays differ from light also in their ability to ionize gases through which they pass. This ionizing action may be demonstrated with the apparatus shown in Figure 4.10. A charged electroscope is placed near an X-ray tube, and the tube is then put in operation. The leaves of the electroscope fall, indicating that the X-rays ionize molecules in the air, thus increasing the conductivity of the air.

4–7 MILLIKAN'S EXPERIMENT

While the above properties of the cathode ray were known by 1895, it was another 15 years before Robert A. Millikan (1868-1953), at the University of Chicago, determined the magnitude of the charge on the electron. His experiments also established that the negative charge borne by an electron cannot be divided into smaller charges; the electron carries the smallest possible negative charge.

Figure 4.11(*a*) is a photograph of a modern version of Millikan's apparatus. The basic parts of the apparatus are two horizontal metal plates. They are visible in the central portion of Figure 4.11(*a*), and are shown in more detail in Figure 4.11(*b*). Figure 4.12 shows these plates and the electrical connections to them. The plates, *P* and *N* in Figure 4.12, were charged as shown, and small oil droplets were sprayed into the region between the plates, where they were observed through a microscope. Some of the oil drops became ionized during the spraying process; those which became negatively charged were attracted upward by the electric field. The potential difference between *P* and *N* was adjusted to cause some of these oil drops to remain stationary. Under these circumstances the electric force is equal to the gravitational force. The

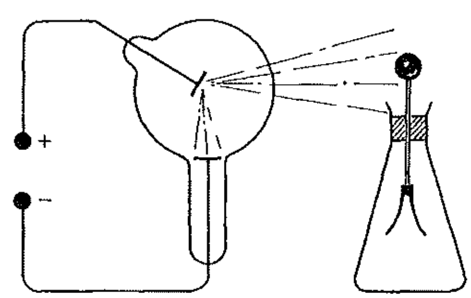
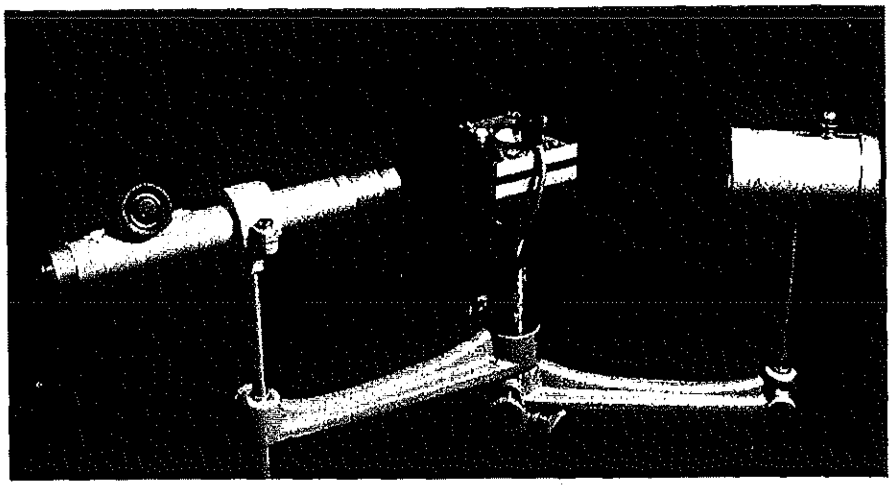
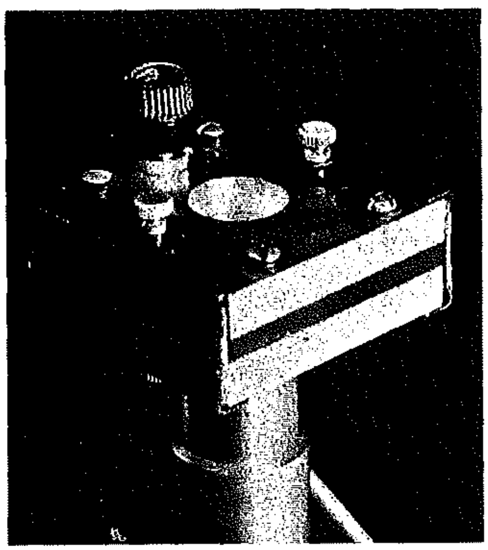


Fig. 4.10. An electroscope detects X-rays because the X-rays ionize the surrounding air.



(a)



(b)

Physics Department, University of Western Ontario

Fig. 4.11. *(a)* A modern version of Millikan's apparatus. *(b)* A close-up view of the charged plates.

weight of an oil drop was calculated from observations of its rate of fall through the air when the P.D. between P and N was zero. Thus the electric force was known since it was equal to the gravitational force. The formula

$$QV = Fs$$

was then applied just as we did in Chapter 2; V, F and s are known and Q can be calculated.

The results of a great many experiments showed that, within the limits of experimental error, Q was always some integral multiple of 1.60×10^{-19} coulombs. Millikan, assuming that the negatively charged drop had an integral number of surplus electrons, concluded that the charge on the electron was an indivisible and fundamental quantity of charge equal to 1.60×10^{-19} coulombs. We refer to it now as the elementary charge.

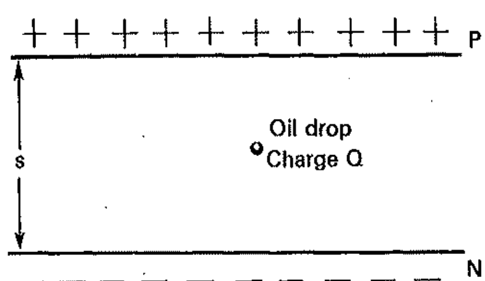


Fig. 4.12. The basic parts of the apparatus used by Millikan.

4–8 THE ELEMENTARY CHARGE

Electricity, then, comes in small bundles just as matter does; the charge on the electron is the "atom" of electricity. For many purposes in physics the charge on the electron—the elementary charge—is replacing the coulomb. Let us note some of the new units and relationships which result.

(*a*) Since the elementary charge is equal to 1.60×10^{-19} coulombs, then 1 coulomb $= \dfrac{1}{1.60 \times 10^{-19}}$ elementary charges $= 6.25 \times 10^{18}$ elementary charges. Therefore charges expressed in coulombs may be expressed in elementary charges if we multiply by 6.25×10^{18}.

(*b*) In the Coulomb's law formula

$$F = k\frac{qQ}{r^2}$$

and $k = 9.0 \times 10^9 \dfrac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$

If we change the coulombs to elementary charges, then

$$k = 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{\text{(elem. charge)}^2}$$

(*c*) A P.D. of 1 volt has been defined as 1 joule/coulomb. In terms of elementary charges, 1 volt $= 1.60 \times 10^{-19}$ joules/elementary charge.

(*d*) Reversing the definition of the volt,

1 coulomb-volt = 1 joule

Then 1 elementary charge-volt, more commonly called 1 electron-volt (abbreviated; ev), is a unit of energy as is the joule, and is equal to 1.60×10^{-19} joules. The electron-volt is widely used in atomic physics as an energy unit.

4–9 MEASURING THE MASS OF CHARGED PARTICLES

We mentioned earlier in this chapter that Sir J. J. Thomson was able to calculate the charge-mass ratio for the electron. Then, when Millikan determined the charge on the electron, the mass was calculated and found to be about 9×10^{-31} kg. Since the time of Millikan, other methods for independent measurement of the mass of the electron (and other charged particles) have been devised. One of these is described in the following paragraphs.

The basic part of the apparatus (Fig. 4.13) is essentially a diode-tube containing a cathode N and an anode P in an evacuated container. Let us consider first its use in measuring the mass of a hydrogen ion. A small amount of hydrogen is introduced into a box at the left of the charged plates. This hydrogen is ionized by a spark from an induction coil, and a few of the positive ions drift through a hole in the anode into the area between the plates. Here they are accelerated to the cathode, each acquiring a final speed v and kinetic energy $\frac{1}{2}mv^2$.

A hole drilled in the cathode allows some of the hydrogen ions to pass through the cathode into a tube about 0.5 metres long (Fig. 4.13). In the vacuum in this tube the ions travel at constant speed. We can calculate this speed if we use electronic apparatus (a cathode-ray oscilloscope, for example), to measure the time required for the ions to travel this 0.5 metres. Once the time of flight of the ions is known, their speed can be calculated. It remains then for us to calculate the mass of an ion.

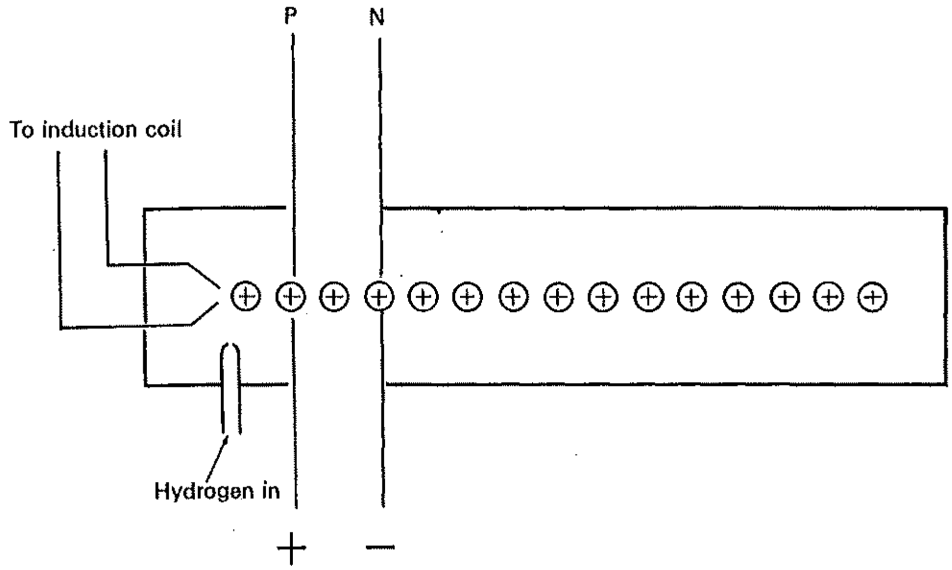


Fig. 4.13. Apparatus used to determine the speed, and hence the mass, of hydrogen ions and other charged particles.

As the ions are accelerated by the electric field between the anode and the cathode, they lose an amount of electric potential energy given by the formula $E = QV$ and gain an equal amount of kinetic energy .Thus

$$QV = \frac{1}{2} mv^2$$

where Q is the charge on each ion, V is the P.D. between the anode and cathode, m is the mass of each ion and v is the speed of each ion at the cathode, assuming that it was at rest at the anode.

We may not know the magnitude of the charge carried by a hydrogen ion, but let us assume that it is the elementary charge, equal in magnitude to the charge on an electron. As we will see later, this is a reasonable assumption. Experimental results such as the following are typical.

$$Q = 1.60 \times 10^{-19} \text{ coulomb}$$
$$V = 100 \text{ volts}$$
$$v = 1.36 \times 10^5 \text{ m/sec}$$

$$\therefore 1.60 \times 10^{-19} \times 100 = \frac{1}{2} \times m \times (1.36 \times 10^5)^2$$
$$m = 1.7 \times 10^{-27} \text{ kg.}$$

Calculations from experiments in chemistry indicate that the mass of a hydrogen atom is 1.7×10^{-27} kg and we would expect a hydrogen ion to have approximately the same mass as the hydrogen atom. Apparently our assumption that the charge on the ion was one positive elementary charge, was correct.

In this and other experiments a hydrogen ion is always found to have only one elementary charge and its mass is only very slightly less than that of a hydrogen atom. The hydrogen ion is called a proton. Many experiments indicate that the proton, like the electron, is a "building block" for matter and that it is present in all atoms. Even the results of the one experiment which we have just described seem to indicate that a hydrogen atom consists of a proton and an electron, the proton being much more massive than the electron.

The mass of the electron may be determined with this same apparatus if the oscilloscope is "fast" enough. The leads from the battery to the accelerating plates are reversed, and electrons are accelerated from what is now the cathode, to the anode. (The electrons are produced when the hydrogen is ionized, or may simply be obtained from a heated cathode in a vacuum tube.) The speed attained by the electrons is considerably higher than that of the protons under similar circumstances. The mass of the electron is now generally given as 9.11×10^{-31} kg, about $\frac{1}{2000}$ of that of the proton.

4-10 ELECTROLYSIS AND IONIC CHARGES

When we discussed the measurement of electric current, we described the special electrolytic cell called the silver voltameter (Fig. 3.4). The same type of cell may be used to investigate the conductivity of many liquids. If the electrodes are of platinum or carbon, so that they do not react with the liquid in the cell, the material deposited or liberated at the cathode depends on the nature of the liquid. If the liquid is a solution of silver nitrate, silver is deposited; if the liquid is a solution of copper sulphate, copper is deposited; and if the liquid is dilute hydrochloric acid, hydrogen gas is liberated. These materials are deposited or liberated because the corresponding ions are attracted to the cathode, give up their charges, and the materials are deposited or liberated in molecular form. If the necessary molecular and atomic weights are known, the charges carried by different ions can be compared.

The English scientist Michael Faraday investigated electroplating and electrolysis in detail. In 1833 he discovered that the charges carried by ions in solution were small integral multiples of the charge carried by the hydrogen ion. If we consider a hydrogen ion as being singly charged, then a silver ion is also singly charged, a copper ion doubly charged, an aluminum ion triply charged, etc. This was perhaps the first indication of the existence of an elementary charge. All of the charges carried by ions are now known to be multiples of the elementary charge determined by Millikan. In other words, there is only one elementary charge; ions have an integral surplus or deficit of electrons.

4-11 PROBLEMS

Where necessary, assume that
1 elementary charge $= 1.60 \times 10^{-19}$ coulombs
1 electron-volt $= 1.60 \times 10^{-19}$ joules
mass of electron $= 9.11 \times 10^{-31}$ kg
mass of proton $= 1.67 \times 10^{-27}$ kg
gravitational field strength $= 9.8$ newtons/kg

1. List the properties of cathode rays. Compare cathode rays and X-rays.
2. Summarize the experimental results which indicate that cathode rays (*a*) possess mass, (*b*) carry negative charge.

3. An electron is fired by an electron gun between the horizontal deflecting plates of a cathode ray tube. The initial speed of the electron is 3.0×10^8 cm/sec. A uniform force of 2.8×10^{-18} newtons acts on the electron when it is between the plates. The plates are 10 cm long. (*a*) Determine the amount of deflection which the electron undergoes as it passes between the plates. (*b*) Describe the path which the electron follows (*i*) between the plates, (*ii*) after emerging from between the plates.

4. A strong magnet is brought near an operating X-ray tube. The tube stops operating. Explain.

5. In a Millikan type experiment, the plates are horizontal and 2.5 cm apart. An oil droplet of mass 1.50×10^{-15} kg remains stationary between the plates when the P.D. is 460 volts, with the upper plate positive. (*a*) Calculate the magnitude of the electric field intensity between the plates. (*b*) Is the drop charged negatively or positively? (*c*) Calculate the magnitude of the charge on the drop. (*d*) How many surplus or missing electrons does the drop have?

6. An oil drop bearing a charge of 20 electrons and having a mass of 9.8×10^{-14} kg is suspended motionless by the electric field between two horizontal parallel plates 4.0 cm apart. (*a*) What is the P.D. between the plates? (*b*) What will happen to the drop when each of the following changes is made? (*i*) Five more elementary charges are placed on the drop. (*ii*) The mass of the drop becomes less because of the evaporation of some oil. (*iii*) The plates are moved closer together.

7. In a Millikan type experiment, a small plastic sphere carrying 2 negative elementary charges can be held motionless between two horizontal plates 3.0 mm apart by a P.D. of 270 volts. Calculate the mass of the sphere.

8. A conductor has a negative charge of 6.00×10^2 microcoulombs. How many surplus electrons does it have?

9. A pith ball carries a negative charge of 0.300 microcoulombs. How many surplus electrons does it have?

10. How many electrons must be removed from a neutral, insulated conductor in order to give the conductor a positive charge of 4.0×10^{-8} coulombs?

11. Calculate the quantity of charge on a conductor which has (*a*) 25.0×10^{12} surplus electrons, (*b*) 12.5×10^{14} too few electrons.

12. Express each of the following currents in elementary charges/sec: (*a*) 2.0 amperes, (*b*) 40 amperes, (*c*) 8 milliamperes.

13. Convert each of the following currents to amperes: (*a*) 6.25×10^{17} elementary charges/sec, (*b*) 12.5×10^{16} elementary charges/sec, (*c*) 5.0×10^{16} elementary charges/sec

14. If 6.4×10^{17} electrons pass between the electrodes of a vacuum tube in 4 seconds, what is the average current?

15. If the current in a conductor is 0.25 amperes, (*a*) how many coulombs of charge, and (*b*) how many elementary charges pass through the conductor in 2 minutes?

16. Protons are accelerated in a cyclotron until the current is measured as 1.6×10^{-3} amperes. How many protons pass a point in the cyclotron each second?

17. Convert each of the following energies to electron volts: (*a*) 6.0×10^{-17} joules, (*b*) 8.0 joules, (*c*) 2.0×10^{-5} joules.

18. Express each of the following energies in joules: (*a*) 2.0 ev, (*b*) 5.0×10^{7} ev, (*c*) 6.0×10^{-2} ev.

19. Charged particles move between an anode A and a cathode C. Calculate the current between the electrodes in each of the following situations. (*a*) 2.0×10^{14} protons pass from A to C each second; (*b*) 2.0×10^{14} protons pass from A to C each second, and at the same time 2.0×10^{14} electrons pass from C to A each second.

20. A singly-charged ion is accelerated from rest through a potential difference of 100 volts. A "time of flight" measurement (Sect. 4–9) shows that the ion, after having been accelerated, travels a distance of 0.50 m in 4.0×10^{-5} sec. Calculate the mass of the ion.

21. By what factor would the "time of flight" in Problem 20 be changed if (*a*) the ion is doubly charged, (*b*) the P.D. is 75 volts?

22. What is the speed, in m/sec, of a proton which has 3.4×10^{-17} joules of kinetic energy?

23. The P.D. between two plates, A and B, is 3.0×10^{2} volts. An ion having one elementary charge starts from rest at A and arrives at B with a speed of 2.0×10^{5} m/sec. Calculate (*a*) the kinetic energy gained by the ion, (*b*) the mass of the ion.

24. Two parallel electrodes in a vacuum tube are maintained at a constant P.D. of 90 volts. A proton (hydrogen ion) starts from rest at the anode and is accelerated to the cathode. (*a*) How much kinetic energy, in joules, is imparted to the proton? (*b*) If the same tube were used to accelerate alpha particles (doubly charged helium ions) how much kinetic energy, in ev, would be imparted to each alpha particle?

25. Protons having negligible speed are released at the anode of an evacuated tube. The anode and cathode are parallel, and are 0.10 m apart. The P.D. between the plates is 45 volts. (*a*) Calculate (*i*) the work done on each proton as it moves from anode to cathode, (*ii*) the magnitude of the electric force. (*b*) What would the P.D. need to be if the speed of the protons arriving at the cathode is to be double that in (*a*)?

26. Parallel conducting plates P and N in a vacuum tube are connected to a 100-volt battery. Positive ions are released at rest at the anode P. (*a*) Two positive ions have the same charge, but the mass of the second ion is double that of the first. What is the ratio of their speeds on arrival at N? (*b*) If a deuteron, consisting of one proton and one neutron, is accelerated from P to N, how much kinetic energy, in ev, does it acquire?

27. An electron is accelerated by a field of 1.82×10^{-26} newtons/elementary charge. Calculate its acceleration.

4–12 SUMMARY

Cathode rays possess negative charge, kinetic energy, and momentum. They are streams of electrons. They are used in many devices such as the cathode-ray oscilloscope and the X-ray tube. X-rays, as distinct from cathode rays, are a form of radiation.

Thomson determined the charge-mass ratio for electrons, and Millikan determined that 1 elementary charge equals 1.60×10^{-19} coulombs.

1 coulomb $= 6.25 \times 10^{18}$ elementary charges

1 volt $= 1.60 \times 10^{-19}$ joules/elementary charge

1 electron-volt $= 1.60 \times 10^{-19}$ joules

The constant in Coulomb's law $= k = 2.3 \times 10^{-28} \dfrac{\text{newton-(metres)}^2}{\text{(elem. charge)}^2}$

The mass of a charged particle may be obtained from a "time of flight" measurement, with the aid of the relationship

$$QV = \tfrac{1}{2}mv^2$$

Chapter 5

The Nuclear Atom

5-1 INTRODUCTION

The discovery of electrons and protons, the fact that their charges were equal and opposite, and the measurement of their masses, led to the conclusion that all atoms were composed of these two particles. It was obvious that, since atoms are electrically neutral, the number of electrons was equal to the number of protons, but no evidence of the relative positions of the protons and electrons within the atom was available until 1909. Before 1909 scientists assumed that an atom consisted of a globule of positively charged fluid in which the electrons were embedded like raisins in a bun or seeds in a watermelon. The electrons were sometimes assumed to vibrate about average positions in the fluid, but the basic assumption was that the atom was uniformly dense throughout the space it occupied.

This model of the atom (the so-called Thomson atom) failed to account for several facts already known, notably the occurrence of line spectra. However, a better model seemed impossible with the information available at the time. Startling new information concerning the atom became available in the years 1909-1911, and it is with this information and later developments that this chapter is concerned. Before discussing these events, however, we should review the data concerning the atom, which were available from chemical experiments and theory.

5-2 EARLY INFORMATION CONCERNING THE ATOM

The concept of an element as a pure substance which cannot be broken down into, or synthesized from, other pure substances was pretty well established by the beginning of the nineteenth century. About one-third of the elements we now know had been identified as elements, and, in addition, many substances which we now know to be compounds were then considered to be elements.

About the beginning of the 19th century the English scientist John Dalton (1766-1844) proposed a chemical atomic theory to explain the existence and chemical properties of elements. Dalton's basic assumptions were essentially as follows. (1) Matter is composed of indivisible atoms. (2) The atoms of different elements differ from one another, but all of the atoms of any one element are alike. (3) Atoms are not created nor destroyed during a chemical reaction; they are just rearranged. (4) A molecule of a compound contains a definite number of atoms of each of the elements which combine to form the compound.

The fourth assumption above explained very nicely two chemical laws which had been developed from experiment: the laws of definite and multiple proportions. When two elements unite to form one compound, they do so in definite proportions by weight; when they unite to form two or more compounds the weights of one element which combine with a given weight of the other element are in the ratio of small whole numbers. This is just what we would expect if elements take part in reactions in indivisible atomic units. These laws and Dalton's atomic theory led eventually to the calculation of atomic and molecular weights.

The French scientist, Joseph Gay-Lussac (1778-1850) noted that, when gases react under conditions of constant temperature and pressure, the volumes of the reactants and products are in the ratio of small whole numbers. This led the Italian physicist Amedeo Avogadro (1776-1856) to formulate the following hypothesis: Equal volumes of all gases, at the same temperature and pressure, contain equal numbers of molecules. The number of molecules in a given volume was eventually calculated by methods which we shall discuss very briefly later in this section.

The weights of atoms and molecules are small, and therefore we usually discuss, not their actual weights, but their relative weights. The basis of comparison is the atom of an isotope of carbon, $^{12}_{6}C$; the weight of this atom is arbitrarily taken as 12 exactly. We say that the atomic weight of this isotope of carbon is 12. The weight of any other atom or molecule, relative to this standard, is called the atomic or molecular weight of that element or compound. For example, since the weight of a nitrogen atom is $\frac{14}{12}$ times the weight of a carbon atom, the atomic weight of nitrogen is 14. Since the nitrogen molecule is diatomic, the molecular weight of nitrogen is 28. The gram-molecular weight (G.M.W.) of a substance is its molecular weight expressed in grams; the G.M.W. of nitrogen is therefore 28 gm.

The concept of gram-molecular weight has proven so useful in chemistry that gram-molecular weight is given a special name: the mole. Thus, when we speak of 3 moles of nitrogen, we mean 84 grams of nitrogen. All elements and compounds, whether in the solid, liquid or gaseous state, have the same number of molecules per mole. This number is called Avogadro's number.

Avogadro's number may be calculated if the mass of a molecule and the G.M.W. of the substance are known. Alternatively the charge transferred in an electrolytic cell per mole of an element liberated at an electrode, may be measured. If the charge on the ion of that element is known, the number of ions per mole may be calculated and Avogadro's number determined. As a result of these and other methods, Avogadro's number has been determined to be 6.025×10^{23}. That is, a mole of any pure substance contains 6.025×10^{23} molecules.

One use to which a knowledge of Avogadro's number was put was to determine the approximate radius of an atom. For example, the volume of a mole of a gaseous element is found to be 22.4 litres at standard temperature and pressure; this is the volume occupied by 6.025×10^{23} molecules. The volume occupied by one molecule can then be calculated; the volume occupied by one of the atoms in the molecule guessed at, and the radius calculated. All atoms seem to have a radius of an order of magnitude of 10^{-8} cm, i.e., 10^{-10} metres.

One more fact concerning atoms must be mentioned before we return to the discussion of the structure of the atom itself. Dmitri Mendeléeff (1834-1907) organized the elements into what is now called the periodic table of the elements. To each position in the periodic table a number is assigned. This number is called the atomic number (symbol Z) of the element occupying this position, and we shall see that it has a direct relationship to the structure of the atom.

5–3 HOW TO EXPLORE ATOMS

Physicists usually explore atoms by "firing" other small particles at the atoms and then observing what happens to the particles after they interact with the atoms. This procedure requires first of all a source of particles of relatively high speed, and some means of detecting and counting these particles after they have been "fired" at the atoms. Alpha particles were the first atomic "probes" used; they are produced by radioactive disintegration.

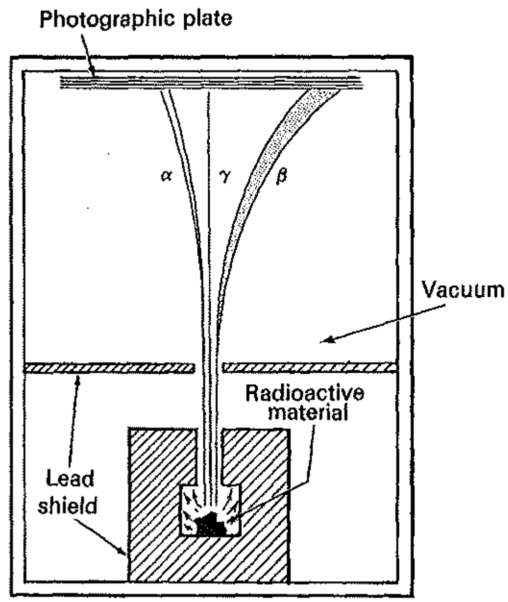
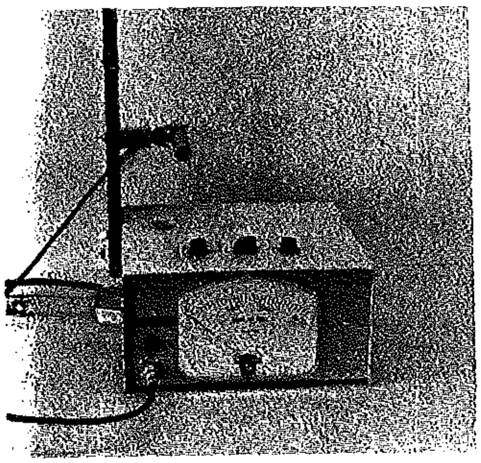


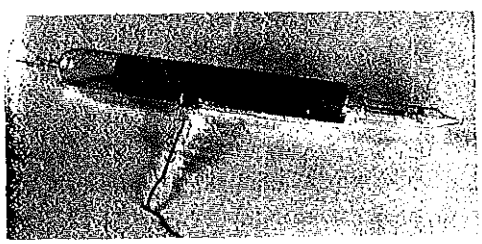
Fig. 5.1. The three types of radiation from a radioactive substance are separated by a magnetic field perpendicular to the plane of this diagram, and are detected by means of a photographic plate.

5–4 RADIOACTIVITY

In 1896 Becquerel, a French physicist, found that thorium and uranium emit radiant energy which can be detected by a photographic plate. In 1898 Madame and Pierre Curie isolated a few milligrams of radium, and eventually three types of radiation, alpha, beta and gamma rays, were detected. (See Figure 5.1.) Gamma (γ) rays are very penetrating rays similar to X-rays but having higher energy. Beta (β) rays are streams of electrons. Alpha (α) radiation is a stream of alpha particles which were eventually identified



(a)



(b)

Physics Department, University of Western Ontario

Fig. 5.2. Photographs of *(a)* a Geiger counter, and *(b)* a Geiger counter tube.

as ions of the element helium, each ion bearing two positive elementary charges.

The identification of alpha particles as doubly charged helium ions basically involved two phases. The magnitude of their deflection by a magnetic field (recall the method described in Section 4–4) indicated that their charge-mass ratio was the same as that for helium ions. Then a large number of alpha particles was collected in a discharge tube and the spectrum of the light produced by electrical discharge through the resulting gas was examined. The spectrum was that of helium.

Alpha particles can be stopped relatively easily by matter, but have sufficient energy to pass through thin metal foil. Their speed depends on the source; the speed of those emitted by the radioactive element polonium has been found to be 1.6×10^7 m/sec. The mass of an alpha particle is 6.6×10^{-27} kg.

5–5 DETECTION OF RADIATION

Several methods, some simple and some complex, have been devised for detecting different forms of radiation. As we have already noted (Sect. 4–6), X-rays ionize the gases through which they pass and as a result cause a charged metal-leaf electroscope to discharge.

One of the earliest detectors of alpha particles consisted of a zinc sulphide screen which fluoresces briefly when an alpha particle is incident on it; the brief flashes of light can be observed and counted with the aid of a microscope.

The Geiger counter (Fig. 5.2) is widely used in many branches of physics and chemistry. The basic principles of its construction are shown in Figure 5.3. A fine

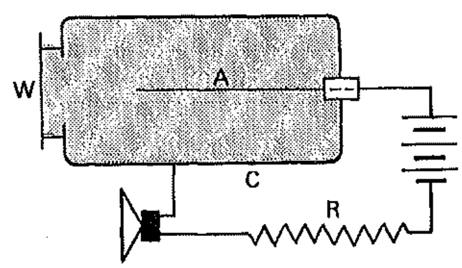


Fig. 5.3. The basic parts of a Geiger counter.

wire A is inserted into, and insulated from, a metal cylinder C. The cylinder contains gas at low pressure. The P.D. applied between A and C (with C negative), is slightly less than that necessary to produce an electrical discharge through the gas. An alpha particle entering the cylinder through the thin mica window W ionizes the gas in the cylinder, and electrons freed by this ionization are accelerated toward A. As these electrons travel through the gas, they collide with more gas molecules, freeing more electrons and so on. As a result there is a sudden surge of current which is amplified to produce a click in the loudspeaker. Because of the high resistance R in the circuit the current ceases very quickly, unless another alpha particle arrives to cause another click. The Geiger counter, therefore, may be used not only to detect, but also to count, alpha particles.

5-6 ALPHA PARTICLE DEFLECTION

We are now ready to discuss the experiments which were performed about 1910 under the direction of Ernest Rutherford. Lord Rutherford was a New Zealänder who spent some time at McGill University in Montreal and then, in 1907, moved to Manchester University in England. It was in Manchester that the experiments for which he is most famous were carried out, mainly by two of his assistants, Geiger and Marsden.

In these experiments, alpha particles were fired at the atoms in a thin sheet of gold foil. The arrangement of the apparatus is shown in Figure 5.4. Alpha particles from a radioactive source were directed at the gold foil, and were detected and counted on the other side of the foil

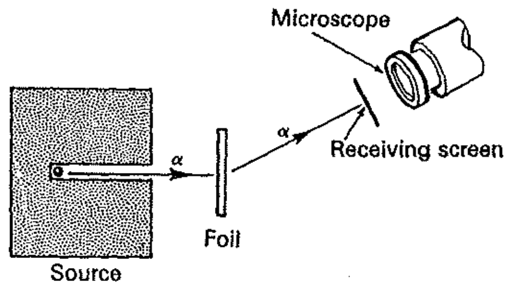


Fig. 5.4. Apparatus used by Rutherford in his experiment on the scattering of alpha particles. Only a very small percentage of the alpha particles were deflected to the extent shown in this drawing.

by viewing a zinc sulphide screen through a microscope. Geiger and Marsden spent hours in a darkened room noting and counting spots of light on the screen. No wonder Geiger later invented (and used) the Geiger counter.

Even the earliest observations with this apparatus indicated that the "raisin bun" model of the atom was incorrect. Although the gold foil was very thin, it consisted of several hundred layers of atoms—and yet most of the alpha particles went straight through the foil without being deflected at all. Most of the space occupied by an atom, then, is just space; the atom is not of uniform density throughout.

However, this information about the density of an atom was not the most startling conclusion to come from these experiments. Geiger and Marsden, by altering the position of the microscope and screen (Fig. 5.4), found that, although most of the alpha particles were deflected no more than one degree, if they were deflected at all, some were deflected through much larger angles. Indeed, about one alpha particle in 10,000 was deflected through more than 90°, and some were deflected as much as 180°,

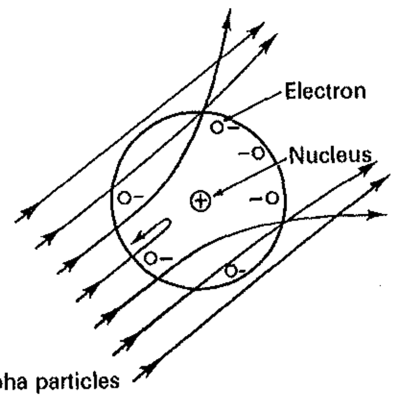


Fig. 5.5. Most alpha particles pass the nucleus with very little delfection; others are forced to turn completely around. The electrons have almost no effect on the alpha particles.

that is, they came straight back. It was in order to explain these large angle deflections that the Thomson model of the atom had to be discarded and a new model developed.

5–7 THE NUCLEAR ATOM

Rutherford realized that an atomic model which supposed that the mass of an atom was distributed thinly over its entire volume could not explain the fact that in some cases the alpha particles suffered very large deflections. He realized too that these deflections, although they occurred infrequently, were the ones which had to be explained. Consequently he assumed that an atom has a small positively charged nucleus, containing almost all of the mass of the atom and about which the electrons circle, somewhat as the planets circle the sun. Referring to Figure 5.5, let us see how this model explains the observations of Geiger and Marsden.

Since the volume of the nucleus is only a very small fraction of the volume of the atom, most of the alpha particles will not pass close enough to the nucleus to be affected by it. Moreover it is unlikely that even a series of encounters with electrons will have much effect on an alpha particle, for the alpha particle has a mass about 7500 times that of an electron. So most of the alpha particles will not encounter an obstruction and will not be deflected.

Now consider the situation which, though it will happen infrequently, is still a distinct possibility—an alpha particle travelling directly toward a nucleus (Fig. 5.5). Because both the alpha particle and the nucleus bear positive charges, the force of repulsion between the two causes the alpha particle to slow down, stop, and turn around. That is, it undergoes a deflection of 180°.

Alpha particles passing close to a nucleus but not travelling directly toward a nucleus will undergo an amount of deflection which depends on the aiming error. This situation is analyzed more fully in Figure 5.6. The aiming error b is the distance by which the alpha particle would miss the nucleus if it continued its original path; the scattering angle θ is the angle between the initial and final paths of the alpha particle. Obviously, as b increases, θ decreases. If we assume that Coulomb's inverse square law applies to the repulsion between the alpha particle and the nucleus (and Geiger and Marsden's results indicate that it does), the path of the alpha particle is a portion of an hyperbola.

If we use the idea of a potential hill which we discussed initially in Section 2–16, we can visualize the paths of alpha

5–10 THE ELECTROMAGNETIC SPECTRUM

The visible spectrum forms but a very small part of a much broader spectrum called the electromagnetic spectrum (Fig. 5.9). This spectrum contains many forms of radiation not detected by the eye: infrared and ultraviolet, X-rays, gamma rays, and radio waves. All of these components exhibit wave properties, notably diffraction and interference, and the manner of their transmission presents the same theoretical difficulties that we encounter in connection with light. Their common speed is the speed of light. Their other properties show some variation which is dependent upon their wave lengths or frequencies. However it is not with the properties or the resulting uses of electromagnetic waves that we are concerned here, but with an important principle in connection with their production.

5–11 ELECTROMAGNETIC WAVES

In 1864 James Clerk Maxwell (1831-1879), professor of physics at Cambridge University, published his famous electromagnetic theory. Although this theory is complex, it is based essentially on two facts which, due to the experimental work of Faraday and others, were well known at the time. (*a*) A moving electric charge produces a magnetic field. (*b*) A changing magnetic field causes electric charge to move. Maxwell, treating these facts mathematically, developed a set of equations which correlated the pieces of experimental information. In addition, the form of the equations caused Maxwell to conclude that a disturbance in an electric or magnetic field would have an effect throughout space, though not instantaneously. The disturbance would be propagated as a wave motion having a finite speed of 3×10^{10} cm/sec. The theory predicted the existence of types of radiation which were not discovered until later, for Maxwell predicted radiation of electromagnetic waves over a very broad range of frequencies.

Physicists realized that, if Maxwell's predictions were correct, the oscillation (vibration) of charge in an electric circuit should cause electric energy to be radiated by means of an electromagnetic wave. The search for experimental verification

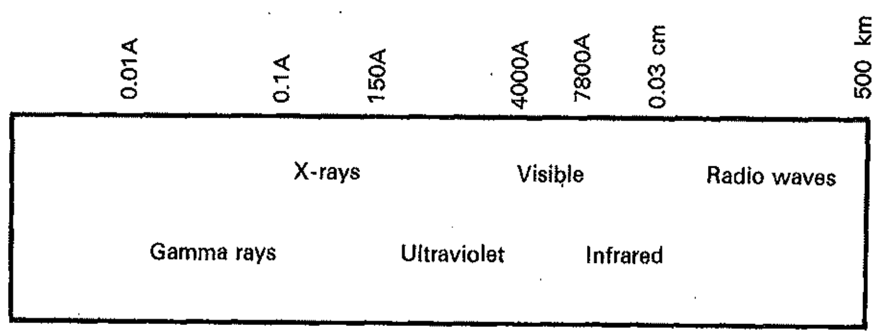


Fig. 5.9. The electromagnetic spectrum, and the approximate range of wave lengths for each component.

of the existence of these waves became a race between Heinrich Hertz (1857-1894) and Sir Oliver Lodge. In 1887 Hertz obtained evidence of the existence of such waves and because of this fact the name Hertzian waves is applied to all electromagnetic radiation having wave lengths longer than those of infra-red. They are also called electric waves or radio waves and they include those used in domestic radio as well as in short wave radio, television waves, and even those produced by 60-cycle alternating current.

We shall not attempt to tell here the story of the development of radio and television. Most of us are at least vaguely aware of the general procedures employed. Oscillating circuits connected to the transmitting antenna cause the electrons in the antenna to vibrate. As the electrons vibrate, they are continually undergoing acceleration. Maxwell predicted that this acceleration of charge should cause an electromagnetic wave to be radiated. This electromagnetic wave, travelling past the receiving antenna, causes the electrons in the receiving antenna to vibrate. The present wide use of radio and television indicates that Maxwell's predictions were correct. The important point for us to realize here is that the success of wireless methods of communication confirms Maxwell's prediction that an accelerated electric charge radiates energy.

5-12 CONTRADICTIONS IN RUTHERFORD'S ATOMIC MODEL

Rutherford supposed that electrons travel in orbits around the nucleus of an atom. In doing so, the electrons undergo a central acceleration which, according to classical electromagnetic theory, should cause them to radiate energy. As a result of this energy loss, the electrons should spiral in toward the nucleus and the atom should collapse into the nucleus in a matter of 2×10^{-11} sec. But atoms do not normally emit radiation except under excitation of some sort, and atoms are normally quite stable.

If the emission of light from an excited atom (for example, an atom of hydrogen in a discharge tube) is a result of the electrons spiralling toward the nucleus, the spectrum of the light emitted should be continuous. For as the radius of the orbit decreases, the number of revolutions per second (i.e., the frequency) should increase continuously. Yet this is not the case. The spectrum of atomic hydrogen is not a continuous spectrum, but a line spectrum.

At this stage then we have two unresolved problems.

(*a*) How is light transmitted, by waves or by particles? This was the major question introduced and left unresolved in Book I of this series.

(*b*) What modifications must be made in the Rutherford model of the atom to accord with the facts discussed above? The answers to these questions necessitated a radical change in the thinking of physicists early in this century. We shall attempt to give some indication of the answers in the next two chapters.

5–13 PROBLEMS

Where necessary, assume that

(*a*) the mass of an alpha particle $= 6.6 \times 10^{-27}$ kg

(*b*) 1 elementary charge $= 1.60 \times 10^{-19}$ coulombs

(*c*) the constant k in Coulomb's law

$$= 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{\text{(elementary charge)}^2}$$

$$= 9.0 \times 10^{9} \frac{\text{newton-(metre)}^2}{\text{(coulomb)}^2}$$

(*d*) the constant G in the law of gravitation

$$= 6.67 \times 10^{-11} \frac{\text{newton-(metre)}^2}{\text{(kg)}^2}$$

(*e*) the mass of the gold nucleus $= 3.3 \times 10^{-25}$ kg

(*f*) the mass of an electron $= 9.11 \times 10^{-31}$ kg

(*g*) the speed of electromagnetic waves in space $= 3.0 \times 10^{8}$ m/sec

1. One gram of radium emits approximately 3.7×10^{10} alpha particles/sec. Calculate the order of magnitude, in kg, of the mass of the helium which could be formed from the alpha particles emitted in one day.

2. What did Rutherford's experiment indicate about (*a*) the sign of the charge on the nucleus, (*b*) the magnitude of the charge on the nucleus, (*c*) the size of the nucleus?

3. Calculate the charge, in coulombs, on the nucleus of an atom of each of the following elements: (*a*) magnesium (atomic number 12), (*b*) copper (atomic number 29), (*c*) aluminum (atomic number 13), (*d*) gold (atomic number 79).

4. The radius of a gold atom is approximately 1.4×10^{-10} m and the radius of its nucleus is approximately 8.7×10^{-15} m. What percent of the atomic volume is in the nucleus?

5. What would be the effect on the trajectories of alpha particles of an atom in which the positive charge and mass were distributed over the whole volume of the atom?

6. What effect do the electrons in atoms have on alpha particles passing close to them? Justify your answer.

7. The nucleus of a gold atom carries 79 positive elementary charges. Calculate the force of repulsion between an alpha particle and a gold nucleus when the distance between them is 3.0×10^{-13} m (about 30 times the diameter of the nucleus).

8. Refer to problem 7. (*a*) Calculate the gravitational force of attraction between the alpha particle and the gold nucleus when they are 3.0×10^{-13} m apart. (*b*) What is the ratio of the electric force to the gravitational force at this distance? (*c*) What would the ratio be if the distance were three times as great? Does the gravitational force need to be taken into account in the mathematical analysis of alpha particle deflection?

9. Refer to problem 7. Calculate the electric potential energy of the alpha particle at a distance of 3.0×10^{-13} m from the gold nucleus.

10. An alpha particle with a speed of 2.0×10^{7} m/sec approaches a gold nucleus head-on. Assuming that the kinetic energy of the alpha particle is converted to electric potential energy, calculate the distance of closest approach of the alpha particle to the nucleus. What information does your answer give you about the dimensions of the nucleus?

11. An alpha particle approaches the nucleus of a cadmium atom (atomic number, $Z = 48$) with a kinetic energy of E joules. If the distance of closest approach is to be 6.0×10^{-14} m, calculate the value of E.

12. A neutron travelling at a speed of 1.0×10^{5} m/sec, collides head-on with a stationary deuteron whose mass is double that of the neutron. The collision is elastic, and the particles do not stick together. Calculate the speed of each after collision.

13. Summarize the electromagnetic spectrum by listing for each type of radiation (*a*) the speed in a vacuum, (*b*) the approximate range of wave lengths in a vacuum, (*c*) the approximate range of frequencies.

14. Calculate the wave length in a vacuum of electromagnetic waves of each of the following frequencies: (*a*) 2.0×10^{4} c/s, (*b*) 1.5×10^{8} c/s, (*c*) 6.0×10^{14} c/s, (*d*) 3.0×10^{18} c/s.

15. Calculate the frequency for an electromagnetic wave, if the wave length, in a vacuum, is (*a*) 4.0×10^{4} m, (*b*) 6.0×10^{3} A, (*c*) 2.0×10^{-2} A.

16. A radar signal directed toward the planet Venus returned to earth 10 seconds after transmission. What is the order of magnitude, in metres, of the distance from the earth to Venus?

17. Incident and reflected electromagnetic waves form a standing wave pattern in space. The average distance between nodes in the pattern is 1.8 m. Calculate the frequency of the waves. What type of waves are they?

18. Should sound waves be included in the list of electromagnetic waves? Justify your answer.

19. Do electric transmission lines, transmitting 60-cycle alternating current, radiate energy?

20. An electron in an X-ray tube has a kinetic energy of 7.2×10^{-16} joules just before it strikes the target. Calculate (*a*) its speed, and (*b*) the P.D. through which it has been accelerated.
21. Calculate the frequency of X-rays of wave length 1.5 A.
22. The electric current flowing through an X-ray tube is 0.24 milliamperes. Calculate the number of electrons striking the target each second.

5–14 SUMMARY

Thomson's model of the atom was a "raisin-bun" model, assuming that the density of the atom was uniform. Rutherford's experiments with alpha particle deflection showed that the Thomson model was incorrect. The mass of the atom was found by Rutherford to be concentrated in a dense central nucleus whose diameter is 10^{-3} or 10^{-4} of the diameter of the atom. The atomic diameter is of the order of 10^{-10} m, the nuclear diameter is of the order of 10^{-14} m.

Rutherford's atomic model, which assumed that electrons travelled in circular orbits about the nucleus, was not in accord with classical electromagnetic theory. The orbiting electrons do not radiate energy, as centrally accelerated charges should.

Chapter 6

The Dual Nature of Radiation and Matter

6–1 INTRODUCTION

The contradictions present in Rutherford's model of the atom seemed to indicate that some facts about matter could not be explained on the basis of nineteenth century physics. Some basic change in outlook seemed necessary. Moreover, the contradictions present in atomic models were paralleled by contradictions present in the wave model for light and other electromagnetic radiations. The first change of outlook, resulting in an extension of Newtonian mechanics but at the same time preserving the bases upon which it had been built, came as a result of experiments with light incident upon matter.

6–2 THE PHOTOELECTRIC EFFECT

About 1887 the German physicist, Heinrich Hertz, was investigating conduction in gases and the phenomenon of transfer of electric energy through them. Among other things, he found that a discharge between an anode and a cathode would take place more readily if the cathode was subjected to the ultraviolet radiation from a carbon arc lamp. Apparently the radiation made possible the emission of a greater number of electrons by the cathode. This type of electron emission is called photoelectric emission.

A more conclusive experiment than that of Hertz was devised by Thomson about 1900 and can be repeated with the equipment illustrated in Figure 6.1. A well cleaned zinc plate, about twenty centimetres square, is connected electrically to an electroscope. The plate is grounded to neutralize the system. The ground connection is removed and the radiation from a carbon arc lamp, without a lens or other glass intervening, is directed toward the

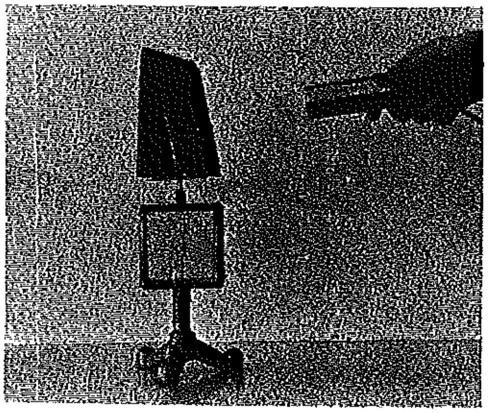


Fig. 6.1. The radiation from the ultraviolet source causes a negatively charged zinc plate to emit electrons.

zinc plate. The leaves do not show any response. The experiment is then repeated with the plate-electroscope system positively charged. Again, the leaves do not show any response. The experiment is repeated a third time with the zinc-electroscope system negatively charged. This time the leaves of the electroscope fall steadily, yet if the radiation is interrupted the fall of the leaves is arrested. This last test shows that electrons (negative charge) leave the zinc plate and that the radiation is the agent causing the emission.

6–3 THE MILLIKAN EXPERIMENT AND THE PHOTOELECTRIC EFFECT

The apparatus shown in Figure 6.2 may be used to show that light has some properties that are not characteristic of waves. The apparatus is similar to that used by Millikan. Charged oil drops "float" in the space between the plates; their motions may be observed through a microscope.

When the drops are illuminated by the light from an arc lamp (Fig. 6.2), the light energy incident on them may impart sufficient energy to electrons in the drops to cause photoelectric emission to occur. As a result a drop may be left sufficiently positively charged that it "jumps" upward toward the negative plate, because the upward electric force is then greater than the downward gravitational force.

Let us try to predict when these jumps occur, assuming that the light emitted by the arc and incident on the drops is transmitted by means of waves. A wave model for light assumes that the energy in the light beam is distributed uniformly along the wave front. Therefore all the drops in a given region should be affected in the same manner by the light, absorbing energy at the same rate. When each of the drops has absorbed sufficient energy to cause an electron to be emitted, all of the drops should jump simultaneously. Even in strong light, there should be a considerable and readily measurable time lag between the turning on of the light and the first jump or jumps. Moreover, the more intense the incident light, the sooner the drops should jump.

The experimental observations are entirely different from the results predicted above. Regardless of light intensity, the first jump in many cases is observed to occur almost immediately after the light is turned on. In other cases considerable time may elapse before a drop jumps, even in strong light. Experiments also indicate that the number of electrons ejected from the drops by strong light is greater than the number ejected by weak

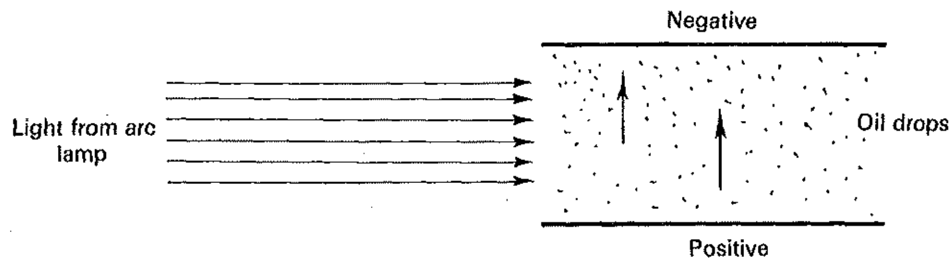


Fig. 6.2. A variation of Millikan's experiment. Light from the arc lamp, falling on the oil drops, may cause some of the oil drops to lose electrons and begin to drift upward.

light. Over a period of time the emission of electrons is uniform across the light beam. However, these are not the startling or important points. The important point is that, even in weak light, the first jump occurs much earlier than the wave model predicts.

6-4 PHOTONS

Since the wave model fails to explain random emission of electrons from oil drops, let us return to the particle model and see how well it fits the facts here. Ejection of an electron from one of the drops would occur when that drop is struck by a particle of light. The first strike causes the first drop to jump, and this may occur immediately after the light is turned on, whether the light is strong or weak. Moreover, even a dense beam of particles (a strong beam of light) might fail to make a direct hit on a drop for some time. Over a period of time, of course, the greater the number of particles in the beam, the greater will be the number of collisions, and the greater will be the number of electrons ejected.

A particle model seems to explain photoelectric emission in quite an accurate and convincing manner. In some ways light acts as if it came in bundles, just as matter is composed of atoms and electricity is composed of elementary charges. The bundles of light are called photons or quanta (singular: quantum). Many phenomena other than the photoelectric effect seem to indicate the existence of photons, for example, the action of light on a photographic film and the action of the television camera tube. But photons, if they exist, do not seem to solve the problem of how light is transmitted, for how, in terms of photons, can we explain interference of light?

6-5 INTERFERENCE EFFECTS IN EXTREMELY WEAK LIGHT

About 1910, Sir Geoffrey Taylor, at Cambridge University, set up an experiment to find whether interference resulted from the interaction of many photons, or whether single photons produced interference effects. The basic parts of the apparatus used in an experiment of the type carried out by Taylor, are shown in Figure 6.3. Light originating at a small source of low intensity passes through a very dark filter, thence through a single

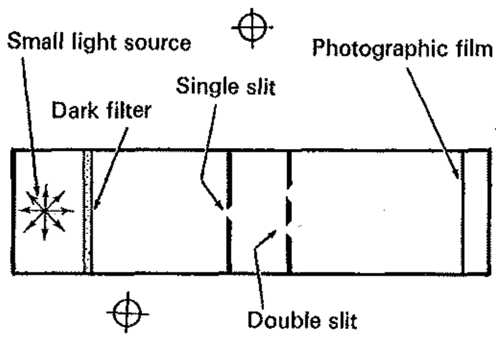


Fig. 6.3. Arrangement of apparatus for demonstrating interference effects in extremely weak light.

slit and a double slit, and falls on a photographic film. The complete apparatus is enclosed in a light-tight box. Taylor was able to calculate that there was never more than one photon of light in the box at a time; indeed the light reaching the photographic plate was so dim that a three month exposure was necessary. Yet the photograph showed the usual double slit interference pattern.

The above observations rule out the possibility of explaining interference on the assumption that photons reinforce or destroy one another. We might assume, then, that each photon splits into two halves, one half going through each slit. This assumption leads to the unlikely conclusion that by obstructing one of the slits we could obtain half-photons, each possessing half the energy of the original photon! This conclusion not only contradicts our basic ideas concerning photons, but is in fact never observed. The photon either arrives intact, or does not arrive at all. A photon never becomes divided, and yet, in order to produce interference phenomena with the apparatus shown in Figure 6.3, both slits must be open. Apparently "particles of light"—photons—possess wave properties. But how can they?

6–6 PROBABILITY AND THE PHOTON

In order to attempt to reconcile the particle and the wave aspects of photons we find it necessary to introduce some ideas concerning probability. We cannot treat the subject properly and fully here, but we can perhaps give some indication of how a series of random events can build up a regular pattern such as is associated with interference. If you toss a coin and record the occurrence of heads and tails, the first ten tosses may very well result in 8 heads and 2 tails. But if you persist for thousands of tosses, it is very probable that the number of heads will be very nearly equal to the number of tails.

Let us now try to apply this idea to the production of an interference pattern of light. Perhaps this attempt should not be made, for it is beginning to be obvious that light is not a wave and not a particle. Yet in our continued search for an analogy we continue to make the attempt. Apparently no single photon interferes with itself. Each photon arrives independently at the detector, and at a definite place. The probability of the arrival at some positions (the positions of the bright lines in the interference pattern) is high. The probability of photons arriving at other positions (the positions of the dark lines in the interference pattern) is low. At other positions the probability of arrival is between the maximum and minimum values. It is common to think of the probability of the photon's arrival at a particular point as being fixed by the wave which accompanies the photon. The situation seems completely paradoxical, yet the ideas of probability give us some vague notion of how a pattern can be built up from random events.

In spite of the difficulties inherent in the above model, there is no doubt about the existence of the photon. Further facts concerning photons and their properties can be obtained from quantitative measurements associated with the photoelectric effect.

6–7 THE PHOTOELECTRIC CELL

In applications of the photoelectric effect, the electron emitter (the cathode) is contained in a tube of glass or of metal with a glass window. The tube is either evacuated or filled with an inert gas at low pressure. An anode (plate) collects the emitted electrons. This device is called a photoelectric cell, an example of which is illustrated in Figure 6.4. The cathode is a piece of thin metal, semi-cylindrical in shape and coated with cesium, cesium oxide and silver. The anode is a straight metal wire placed in front of the cathode.

The circuit illustrated in Figure 6.5 may be used to demonstrate the action of the cell. If the semi-cylindrical electrode is connected to the negative terminal of the battery, and if light is incident on that electrode, the galvanometer indicates that current is flowing in the circuit. The strength of the current increases when the intensity of the light is increased.

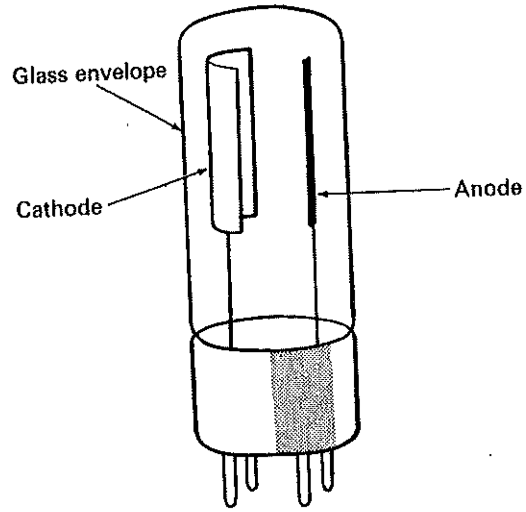


Fig. 6.4. A photoelectric cell.

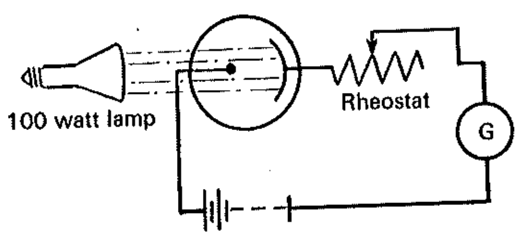


Fig. 6.5. Circuit to demonstrate the action of a photoelectric cell.

6–8 QUANTITATIVE FACTS ABOUT THE PHOTOELECTRIC EFFECT

The photoelectric cell described above employs a cathode coating which has been found to be suitable for many purposes. However, cesium-coated cathodes were not the first ones used, and experiments with other cathodes provide some startling information concerning the photoelectric effect.

(*a*) Suppose we use a cell having a copper cathode, connect it in a circuit as shown in Figure 6.5, and illuminate the cathode with light which contains a high proportion of ultraviolet (for example, the light from a carbon arc). The galvanometer reading indicates that the cathode is emitting electrons, and, as we expect, the greater the intensity of the light, the greater the number of electrons emitted.

Now suppose we use green light, (light of longer wave length and lower frequency than the ultraviolet). This time, no matter how intense the light, no electrons are emitted by the cathode. Apparently the wave length of the incident light must be less than a certain threshold value, otherwise the photoelectric effect does not occur. For a copper cathode, the threshold is about at the violet end of the visible spectrum. If we use a cathode coated with potassium, we find that it has a threshold in the green portion of the spectrum. That is, light of wave lengths longer than the green, (red, for example), does not cause photoelectric emission from potassium. But light of wave lengths shorter than the green, (blue, for example), cause photoelectric emission, and the number of electrons emitted depends on the intensity of the light incident on the cathode. Each material has a different photoelectric threshold; the cesium-coated type of cathode is used in most cells because it has a threshold in the infrared and is therefore sensitive to all of the components of white light.

(*b*) The second unexpected fact concerning photoelectric emission may be obtained with a circuit of the type shown in Figure 6.6. This circuit is essentially the same as that shown in Figure 6.5 except that the battery connections are reversed, and that the magnitude of the P.D. between the electrodes can be varied. A low positive potential is applied to the cesium-coated electrode. Light is allowed to fall on this electrode, and we find, perhaps contrary to our expectations, that

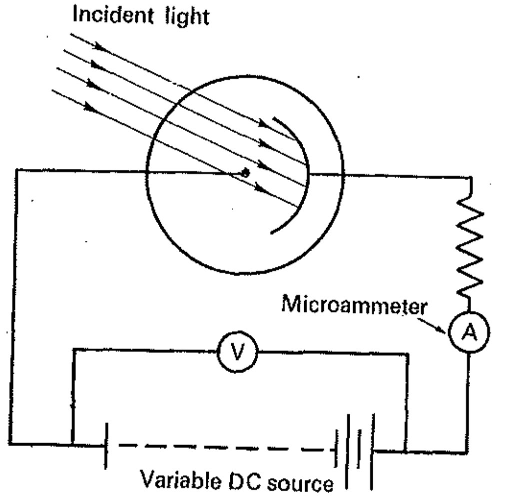


Fig. 6.6. Circuit for determining cut-off potentials.

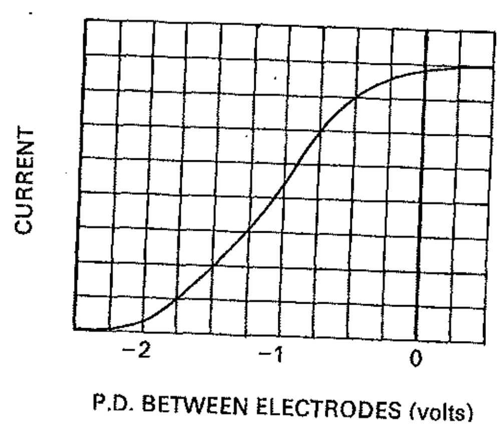


Fig. 6.7. As the receiving electrode becomes more negative the current decreases and is eventually cut off.

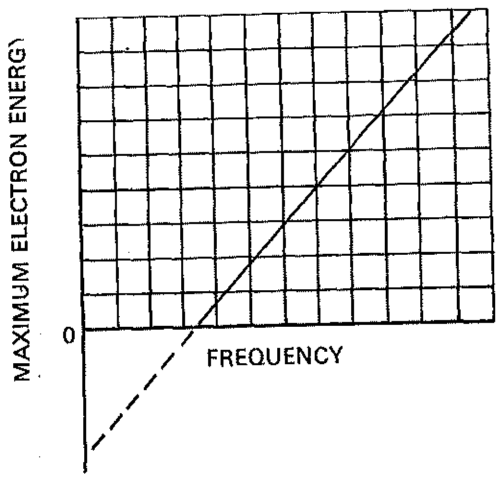


Fig. 6.8. The graph which results when the frequency of the incident radiation is plotted against the maximum kinetic energy of the emitted electrons.

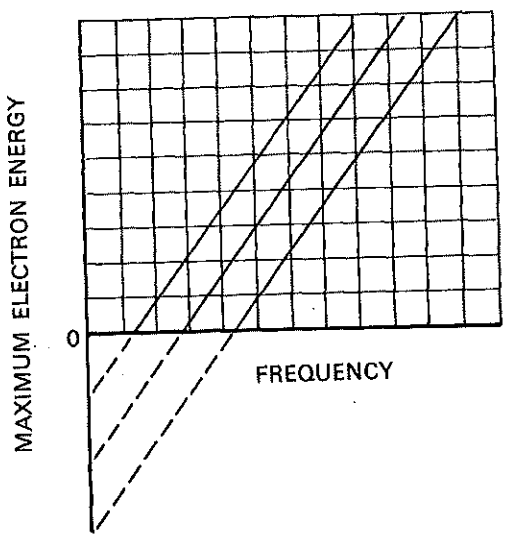


Fig. 6.9. The graphs of frequency versus maximum electron energy, for different surfaces, have equal slopes.

there is current in the circuit. If the positive potential of this electrode is increased, the current decreases and is eventually cut off (Fig. 6.7); the P.D. between the electrodes when the current ceases is called the cut-off potential. Apparently at this potential even the most energetic electrons emitted are attracted back to the cesium electrode. The startling fact here is that the cut-off potential is independent of the intensity of the incident light, but does depend on the frequency or wave length of the light. The intensity of the incident light does, however, affect the number of electrons emitted.

Knowing the cut-off potential V and the charge Q on the electron, we can use the relationship $E = QV$ to calculate the maximum kinetic energy of the emitted electrons. If we do this for light of various frequencies, we can plot a graph of frequency ν against maximum electron energy E (Fig. 6.8). (In most articles on modern physics, the Greek letter ν—pronounced nu—is used rather than the letter f to represent frequency.) Since this graph is a straight line, $E \propto \nu$, and since the graph does not pass through the origin, the equation of the graph is of the form

$$E = h\nu - B$$

The variation constant h is the slope of the graph and is called Planck's constant, after the German physicist Max Planck (1858-1947). B is the intercept of the extended graph on the energy axis.

(*c*) Experiments with different types of surfaces on the emitting electrode yield a set of graphs of E against ν as shown in Figure 6.9. These graphs constitute a family of parallel lines; that is, they have equal slopes but different intercepts on the energy axis. Planck's constant h then seems to be a universal constant whose

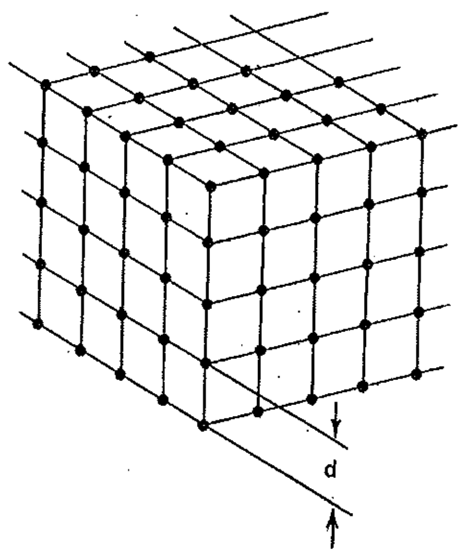


Fig. 6.11. In a crystal, successive planes of atoms are separated by a constant distance *d*. Therefore the crystal acts as a grating for particle diffraction.

electrons have proven to be the best for purposes of demonstration: they are easily obtainable; their speed and therefore their momentum can be controlled readily by altering the potential difference through which they are accelerated; and they produce excellent diffraction and interference patterns if a suitable grating can be found from which to reflect them or through which to pass them.

Mechanically ruled gratings or slits such as are used for visible light are too coarse to be of use for demonstrating particle diffraction. Various devices have been used; among the most satisfactory are crystals of various sorts. In 1912, Max von Laüe discovered that X-rays are regularly diffracted by crystals acting as diffraction gratings. By analysing the diffraction patterns von Laüe and others were able to infer what the structure of the crystal was, and to deduce the dis-

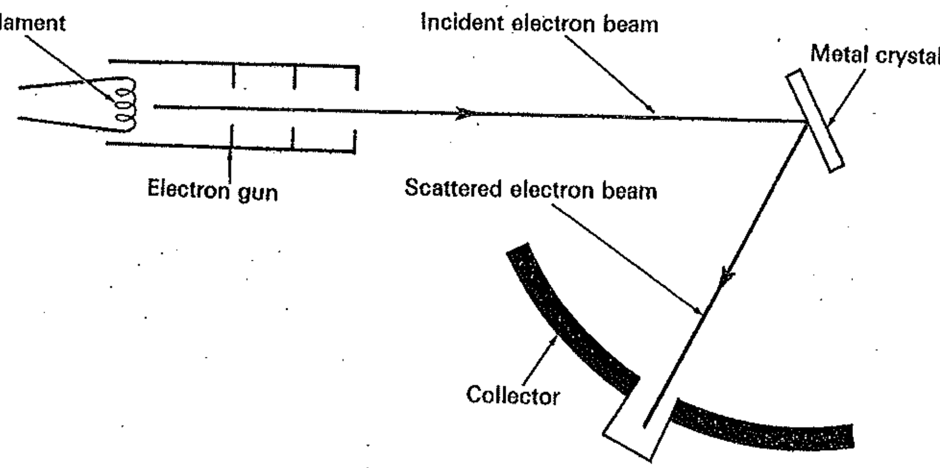


Fig. 6.12. The apparatus with which Davisson and Germer observed electron diffraction. The parts shown here were enclosed in a vacuum chamber.

tance between successive planes of atoms in it. This distance d (Fig. 6.11), corresponding to the distance between successive lines in an optical diffraction grating, happens to be of the right order of magnitude to demonstrate particle diffraction.

Electron diffraction and interference were first detected in 1927, by C. J. Davisson and L. H. Germer in the United States, and by G. P. Thomson in England. Figure 6.12 shows, in simplified form, the apparatus used by Davisson and Germer at the Bell Telephone Laboratories. Slow electrons of known energy were fired from an electron gun at a metal crystal. The reflected electrons were detected by a collector which could be moved around an arc of a circle having the crystal as its centre. Davisson and Germer, by moving the electron collector around, found that the electrons were selectively scattered in certain directions.

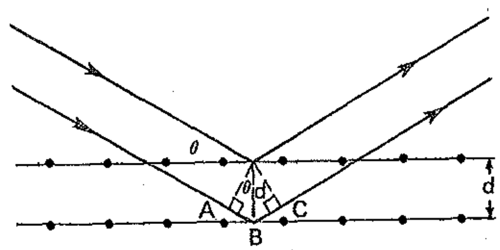
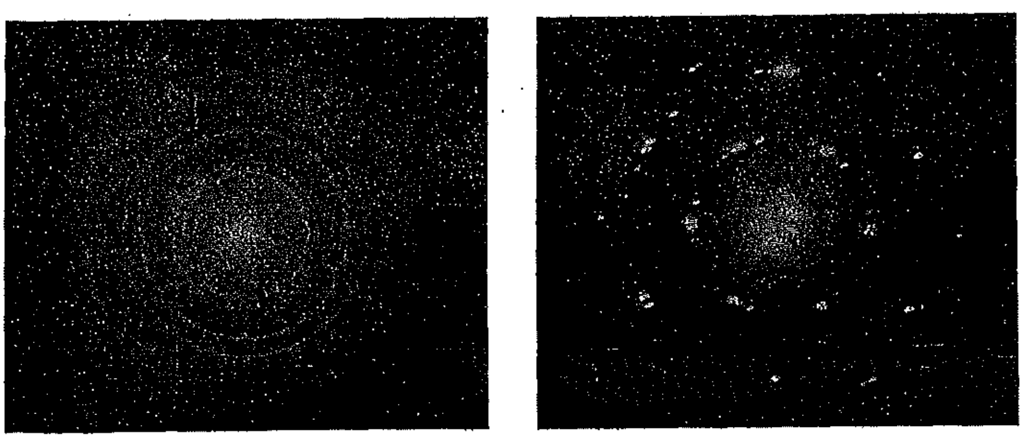


Fig. 6.13. The two reflected electron beams interfere constructively if $n\lambda = 2d \sin \theta$.

This selective scattering may be explained by assuming that the electrons have waves (deBroglie waves) associated with them. Some of the deBroglie waves associated with the electrons reflect from the first layer of the crystal, and some reflect from the second layer (Fig. 6.13). The extra distance travelled by those reflected from the second layer is ABC. If θ is the angle between the surface of the



(a)

(b)

M. J. Curry

Fig. 6.14. Two photographs of the diffraction and interference patterns produced when electrons pass through thin crystals.

13. A material having a work function of 0.5 ev is irradiated with light having a wave length of 6000 A. Calculate the maximum kinetic energy, in joules, of the emitted electrons.

14. A material having a work function of 2.0×10^{-19} joules/electron is irradiated with light of frequency 6.0×10^{14} c/s. Calculate the maximum kinetic energy of the emitted electrons.

15. In an experiment on the photoelectric effect, the light incident on the surface has a wave length of 4.0×10^{-7} m. The cut-off potential is found to be 1.0 volts. Calculate the work function of the surface.

16. The work function of potassium is 1.60 ev, of calcium is 3.33 ev, and of mercury is 4.50 ev. (*a*) Calculate the threshold frequency for each of the above elements. (*b*) Calculate the maximum kinetic energy of electrons emitted by each element, if the incident light is violet (frequency 7.5×10^{14} c/s).

17. Calculate the work function of nickel, in ev, from the following information. The cut-off potential is 1.2 volts when the wave length of the incident radiation is 2.0×10^3 A.

18. In a particular X-ray tube electrons are accelerated through a P.D. of 10^4 volts. As the electrons strike the target, they are decelerated and a few of them emit X-ray photons. Analysis of the X-ray spectrum shows that the spectrum is continuous, but that there is a definite minimum wave length. (*a*) Why is there a minimum wave length? (*b*) Calculate this minimum wave length.

19. Calculate the momentum of a photon of wave length 1.0×10^{-13} m.

20. The wave length of a photon is 5000 A. Calculate its momentum.

21. Calculate the momentum of a photon of frequency 3.0×10^{15} c/s.

22. The frequency of a photon is 7.4×10^{14} c/s. Calculate (*a*) its wave length in metres, (*b*) its energy in joules, (*c*) the magnitude of its momentum in kg-m/sec.

23. Show that the momentum p of a particle of mass m and kinetic energy E is given by the relationship $p = \sqrt{2mE}$.

24. Calculate the wave length of each of the following particles: (*a*) an electron having 20 ev of kinetic energy, (*b*) a 5.0×10^{-18} kg oil drop moving at 2.0 cm/sec, (*c*) a 1-ton car travelling at a speed of 30 mi/hr.

25. Calculate the deBroglie wave length for an electron (*a*) if its speed is 5.0×10^4 m/sec, (*b*) if its kinetic energy is (*i*) 1.0 ev, (*ii*) 2.0 ev, (*iii*) 4.0 ev.

26. What is the deBroglie wave length of a rifle bullet of mass 2.0 gm moving with a speed of 400 m/sec?

27. An electron is accelerated from rest through a P.D. of 4.8×10^2 volts. Calculate (*a*) its kinetic energy (*i*) in electron-volts, (*ii*) in joules; (*b*) its speed; (*c*) its wave length.

28. Compute the wave length in metres for (*a*) a 5.0-ev photon, (*b*) a 5.0-ev electron.

29. Compute the momentum of (*a*) a 5.0-ev photon, (*b*) a 5.0-ev electron.

6–16 SUMMARY

Both radiation and matter have both wave and particle characteristics.

The photoelectric effect can be explained fully only by assuming the existence of photons. For a photon,

$$E = h\nu$$

and for the photoelectric effect,

$$E = h\nu - B$$

Photons appear to be guided by a wave which determines the probability of a photon's arrival at a particular point.

The Compton effect indicates that a photon has momentum. For a photon

$$p = \frac{h}{\lambda}$$

Particle diffraction experiments indicate that particles have a wave associated with them. For a particle

$$\lambda = \frac{h}{p}$$

Chapter 7

Atomic Structure

7–1 DIFFICULTIES

In Chapter 5 we saw why, as a result of alpha particle scattering experiments, Rutherford concluded that an atom consists of a dense positively charged nucleus with electrons orbiting around it. But Rutherford's atomic model left at least four questions unanswered. (1) If accelerated charge radiates energy, why do the centrally accelerated electrons not radiate energy and spiral in to the nucleus in a relatively short interval of time? In other words, how can a Rutherford atom be stable? (2) Why are the spectra of incandescent gases, line spectra rather than continuous spectra? (3) What are the positions of the electrons relative to the nucleus? That is, what are the radii of the electron orbits? (4) Why are the orbital radii not infinitely variable? That is, why are all of the atoms of a given element of the same size?

Rutherford readily admitted the shortcomings of his atomic model, and it was one of his co-workers, Niels Bohr (1885-1962) who developed a more satisfactory model. Bohr, a Danish physicist, joined Rutherford and his group in Manchester in 1911, after having worked for some time with J. J. Thomson. He was influenced by the work and ideas of Thomson and Rutherford, and also by the still unverified quantum theory advanced by Planck and Einstein. In addition, he exhibited remarkably daring intuition and a willingness to break with some of the established traditions of classical physics.

7–2 BOHR'S MODEL OF THE HYDROGEN ATOM

Rutherford's model of the hydrogen atom (Fig. 7.1) consisted of a nucleus containing one proton around which a single electron travelled in an orbit of unspecified (and presumably continuously variable) radius. Bohr accepted this general picture, and elaborated on it. His

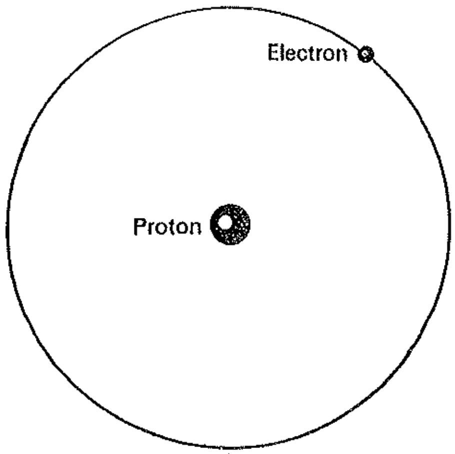


Fig. 7.1. Rutherford's model of the hydrogen atom.

novel and daring method of getting around the fact that an orbiting electron does not radiate energy was simply to accept it. In other words, he assumed that an orbiting electron normally does not radiate energy; that the laws which apply to electromagnetic radiation for large circuits do not apply to oscillators of atomic size. He assumed that the radius of rotation of the electron about the nucleus normally remains constant; the electron normally has a constant amount of energy and the atom is stable.

Perhaps the key word in the last sentence above is the word "normally". Hydrogen in a discharge tube certainly radiates energy and as it does so it produces a line spectrum. Bohr accounted for this radiation by assuming that the energy absorbed by a hydrogen atom as it was excited in a discharge tube caused the electron to move to an orbit of greater radius and higher energy. Later, the electron may return to its original orbit, reradiating the energy that it absorbed when excited. The energy ΔE lost by the electron upon its return to its original orbit is radiated as a photon whose energy is $\Delta E = h\nu$. Therefore the frequency ν of the radiation is given by the relationship

$$\nu = \frac{\Delta E}{h}$$

But the frequency ν can have only those values associated with the lines in the spectrum of atomic hydrogen (Fig. 7.2). Therefore ΔE can have only certain values, and as a result the electron orbits can have only certain radii.

Bohr's method of arriving at this atomic model was more detailed (and more quantitative) than that outlined above, but the conclusions were those we have given. Let us recapitulate, remembering that we are still referring to the simplest atom, that of hydrogen.

(*a*) The electron may orbit the nucleus in any one of a large number of orbits

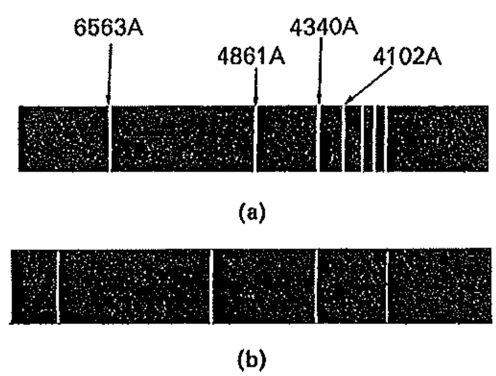


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Fig. 7.2. The bright line emission spectrum of atomic hydrogen. The drawing in *(a)* shows 4 lines in the visible portion, and three in the ultraviolet. The four lines in the visible portion can be seen in the photograph in *(b)*.

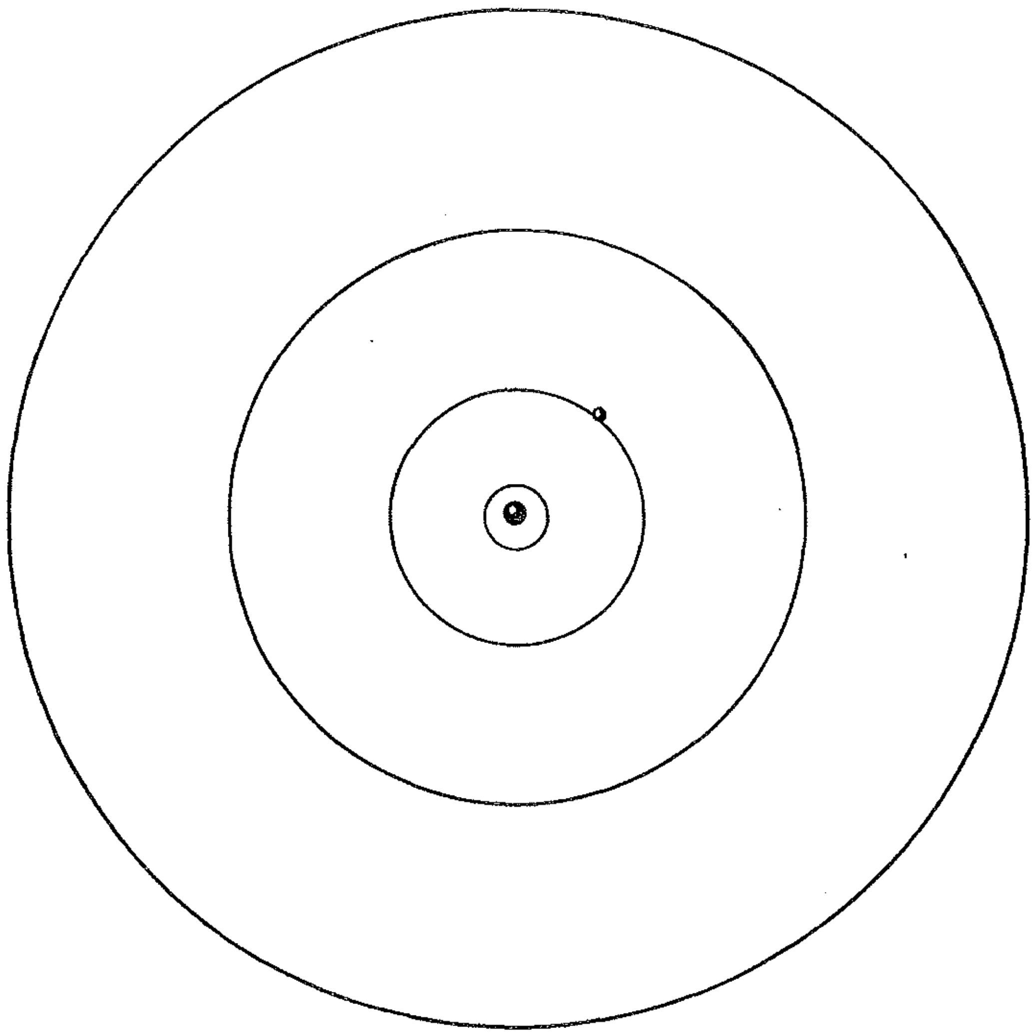


Fig. 7.3. Bohr's model of the hydrogen atom. Four of the permitted electron orbits are shown.

(Fig. 7.3). Bohr calculated that, if n is the number of the orbit and r the radius of the orbit, $r \propto n^2$. That is, the radii of the orbits, beginning with the smallest, are in the ratio 1 : 4 : 9 : 16 : 25—etc. More specifically, Bohr calculated that $r = 0.52 \times 10^{-10} \times n^2$ where r is the radius in metres. The radius of an unexcited hydrogen atom ($n = 1$) should therefore be 0.52×10^{-10} metres. This checked extremely well with the value of 0.5×10^{-10} metres calculated in other ways. This result seemed to indicate the validity of Bohr's assumptions. Moreover, it explained why all atoms of hydrogen are normally of the same size.

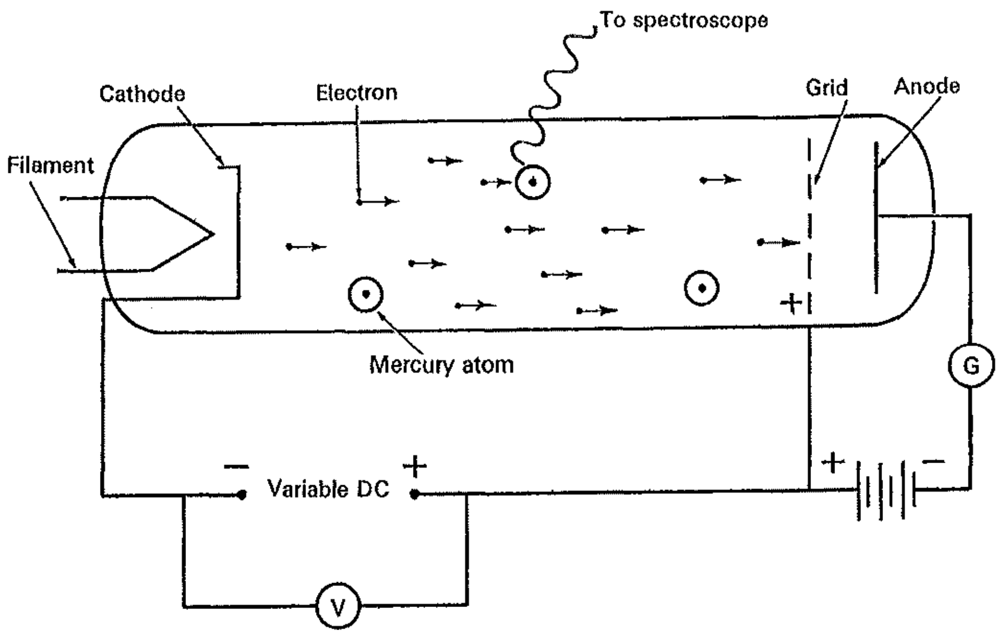


Fig. 7.4 (a). Arrangement of apparatus used in the Franck-Hertz experiment.

(*b*) Each orbit has associated with it a definite electron energy; the greater the radius of the orbit, the greater the energy. Since only certain orbital radii are permitted, the electron may possess only certain definite energies. This implies that an electron may accept energy only in certain definite amounts—those amounts necessary to raise it from its present energy level to some other permitted level. We shall see later how the Franck-Hertz experiments verified this prediction. Bohr's theory also predicted that the electron may radiate energy only in certain definite amounts—those amounts which it loses when "dropping" from an excited energy level to some lower energy level in an orbit of smaller radius. We have already had some indication of how this idea explains line spectra, and we shall examine the quantitative details of these spectra later.

7-3 THE FRANCK-HERTZ EXPERIMENT

James Franck and Gustav Hertz in Germany in 1914, and other physicists elsewhere in later years, carried out experiments which verified Bohr's assumption that atoms could accept energy only in sharply defined amounts.

Figure 7.4(*a*) is a simplified diagram of the apparatus used by Franck and Hertz for an experiment on the energy levels of mercury. Electrons are emitted by a heated cathode and given a known amount of kinetic energy by the potential difference applied between the cathode and the grid. The grid is a fine wire screen, and most of the electrons pass through it. These electrons may also have enough energy to overcome the small retarding potential difference between the grid and the anode. They reach the anode, and their flow through the circuit is recorded by the galvanometer G.

The tube contains mercury vapour at about 250°C. Some of the electrons collide with mercury atoms. If the kinetic energy of the electrons is low, the collisions are elastic, and the electrons continue with undiminished kinetic energy and eventually reach the anode. Under these circumstances, the anode current increases as the potential difference between grid and cathode is increased (Fig. 7.4*b*).

When the kinetic energy of the electrons is about 4.9 ev, the anode current drops sharply. Apparently collisions between electrons and mercury atoms remove energy from the electrons, with the result that they can no longer overcome the retarding voltage between grid and anode.

If the kinetic energy of the electrons is gradually increased further by increasing the accelerating voltage between cathode and grid, the anode current increases again, but dips sharply each time the accelerating voltage reaches a multiple of 4.9 volts (Fig. 7.4*b*). In these cases, the electron collides several times with mercury atoms, surrendering 4.9 ev of energy at each collision.

Apparently the smallest amount of energy that the mercury atom can accept is 4.9 ev. If it is offered less than this amount, it refuses it. Moreover, if it is offered slightly more than 4.9 ev of energy, it takes the 4.9 ev and refuses the rest. For example, if the initial electron energy is 5.5 ev, the final electron energy is 0.6 ev.

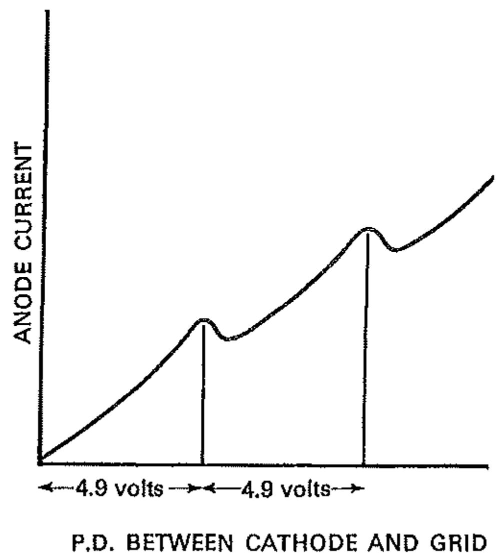


Fig. 7.4 (b). In the Franck-Hertz experiment, the anode current dips each time the accelerating voltage reaches a multiple of 4.9 volts.

The spectroscope (Fig. 7.4*a*) indicates the presence of radiation of wave length 2.5×10^3 A at the time the first dip in anode current occurs. We shall discuss the significance of this observation in Sections 7–4 and 7–5.

Further experiments show that 4.9 ev is not the only bundle of energy that the mercury atom will accept. If the initial electron energy is 6.7 ev or more, the atom will accept either 4.9 ev or 6.7 ev. If the initial energy is increased still further, it is found that there are still larger bundles of energy that the mercury atom will absorb.

Atoms other than those of mercury also absorb energy only in discrete

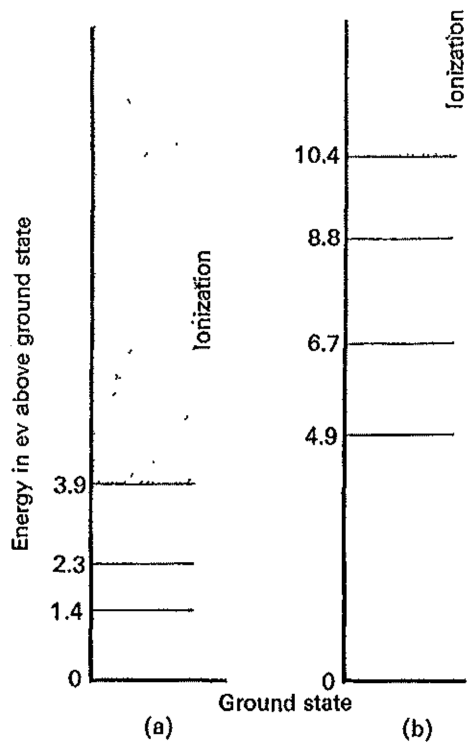


Fig. 7.5. Some of the energy levels for *(a)* cesium, *(b)* mercury.

amounts, but the amounts are not the same as for mercury; for each type of atom there are characteristic amounts of energy which it will accept. The internal energy of an atom can change, but the changes occur in definite steps. The resulting energy contents of the atom are called energy levels. For the minimum energy level, the atom is said to be in its ground state, and this energy level is usually taken as the zero of energy. The next energy level is reached when the atom accepts the smallest possible amount of energy. This added bundle of energy is called the first excitation energy and the atom is then in the first excited state.

In terms of Bohr's atomic model, the ground state of the hydrogen atom is associated with the smallest permitted orbital radius, the first excited state with the second permitted radius, etc. This orbital picture is now considered by many scientists to be too naive a picture of the hydrogen atom. It is perhaps better to describe the atom mathematically or graphically in terms of energy levels. Certainly the orbital model becomes complicated for atoms possessing many electrons. However, the idea of permitted orbits frequently assists us in our attempts to visualize the structure of the atom. Moreover, it helps us to understand one further observation from the Franck-Hertz experiment.

Referring again to mercury atoms, if the energy of the bombarding electrons is increased above 10.4 electron volts, the mercury atoms ionize, ejecting electrons and producing positive ions. In terms of the Bohr atomic model, sufficient energy has been supplied to remove an electron so far from the nucleus that the electron frees itself from the electrostatic attraction of the nucleus and no longer follows any orbit. The 10.4 electron volts is called the ionization energy for mercury. Each type of atom has its own ionization energy as well as its own energy levels. Figure 7.5 shows several of the energy levels for cesium and for mercury, and the ionization energy for each. An atom will accept any amount of energy greater than the ionization energy, for the electron liberated when an atom ionizes may have any amount of energy whatsoever.

Energy level diagrams such as those in Figure 7.5 are likely better pictures of atomic structure than diagrams showing a variety of orbits. And it is quite likely that still better pictures will be devised as our information concerning atoms grows. But Bohr's model and the energy level idea combine to explain very well the production of emission and absorption spectra.

7-4 WORKED EXAMPLE

Calculate the photon energy for radiation of wave length 25×10^2 A.

SOLUTION

$\lambda = 25 \times 10^2 \text{ A} = 25 \times 10^{-8}$ m

$c = 3.0 \times 10^8$ m/sec

$$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{25 \times 10^{-8}} \text{ cps} = 12 \times 10^{14} \text{ cps}$$

$$E = h\nu$$

$$= 6.62 \times 10^{-34} \times 12 \times 10^{14} \text{ joules}$$

$$= \frac{6.62 \times 10^{-34} \times 12 \times 10^{14}}{1.60 \times 10^{-19}} \text{ ev}$$

$$= 4.9 \text{ ev}$$

7-5 ATOMIC EMISSION SPECTRA

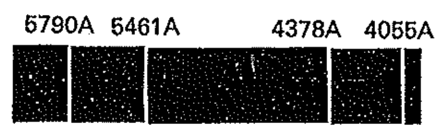
When an atom is excited (for example, in a discharge tube), its energy level increases from the ground state to one of the higher permitted states. But the atom remains in the excited state for only a short time (of the order of 10^{-8} sec) and then the energy drops to the ground state. The energy lost is radiated as a photon of radiation. Since the downward energy shifts are limited to certain permitted steps, the photon energy can have only several permitted values. That is, the radiation involves only a limited number of frequencies, and a line spectrum results.

Let us be more specific. A line of wave length 2537 A occurs in the ultraviolet portion of the spectrum of mercury. The worked example (Sect. 7-4) shows that a photon of about this wave length has an energy of about 4.9 ev. But this is the smallest amount of energy that a mercury atom can absorb—the energy that excites it from the ground state to the first excited state. This same amount of energy is reradiated as a photon when the atom returns to its ground state. Here we have an excellent correspondence between experimental result and theory. You can (and should) predict the wave lengths of several other lines in the spectrum of mercury, using the energy levels shown in Figure 7.5. Can you find any of these lines in Figure 7.6? Do you find any lines in this figure which do not correspond to the energy levels in Figure 7.5?

In general, the following relationship holds for emission spectra:

$$E = h\nu = E_i - E_f$$

where E or $h\nu$ is the energy of each of the radiated photons, E_i is the initial energy of the excited atom, and E_f is the final energy of the atom, frequently the ground state.



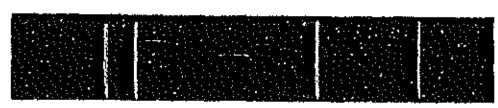


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Fig. 7.6. *(a)* A drawing of the visible portion of the spectrum of mercury. *(b)* A photograph of the visible portion of the mercury spectrum. You should calculate the energy of a photon of each of the wave lengths shown.

7–6 ATOMIC ABSORPTION SPECTRA

Incandescent sodium vapour removes from the continuous spectrum of white light passing through the vapour those frequencies normally occurring in the emission spectrum of sodium. (See Figure 7.7.) Here again the explanation in terms of energy levels is relatively simple. The sodium atoms absorb from the white light those photons whose energy is the difference between two of the energy levels for sodium. It is true that the atoms whose energy levels are thus raised later reradiate the energy. However, they reradiate it in all directions. As a result, the intensity of the energy of this wave length, radiated in the direction of the spectroscope, is so small as to pass unnoticed.

In general, the following relationship holds for absorption spectra:

$$E_f = E_i + h\nu$$

where E_i is the energy level of the atom before the photon is absorbed, E_f is the energy level of the atom after the photon is absorbed, and $h\nu$ is the energy of the photon.

7–7 THE SPECTRUM OF ATOMIC HYDROGEN

The discussion of energy levels and their connection with spectra should lead you to suspect that the lines in an emission spectrum are not scattered haphazardly throughout the spectrum. In fact, they form definite series. These series were studied by Balmer, Rydberg and others in the early part of this century. Formulae were developed connecting the frequencies or wave lengths of the lines in any series, mainly for elements of low atomic weight, and particularly for hydrogen. Then in 1913 Bohr showed the relationships between his planetary model of the atom and these series of spectral lines. Bohr developed the following formula from theoretical considerations and previously known universal constants. For hydrogen

$$E = -\frac{13.6}{n^2}\text{ ev}$$

where E is the energy of an electron in the n^{th} permitted orbit for hydrogen, and it is assumed that the electron energy is

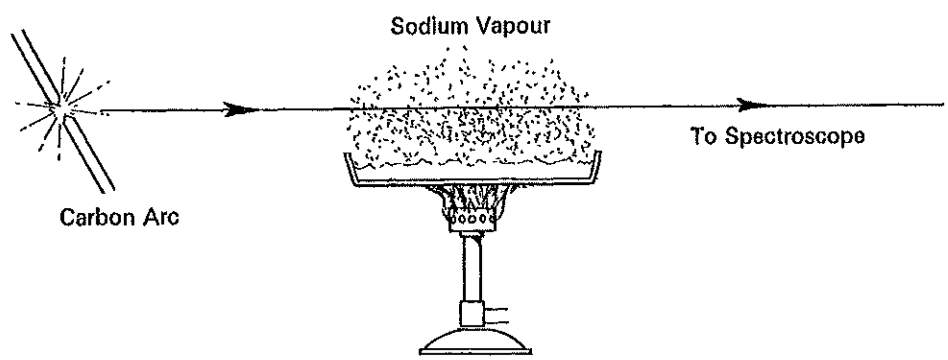


Fig. 7.7. A dark line absorption spectrum is produced when light from a carbon arc passes through incandescent sodium vapour.

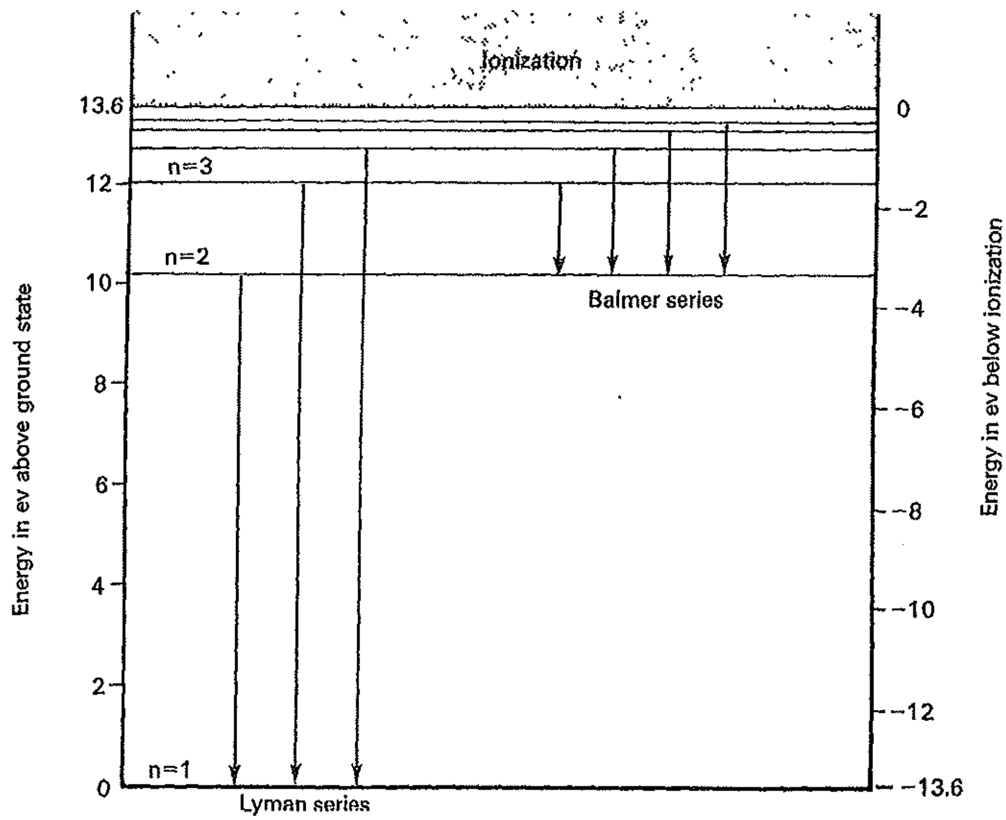


Fig. 7.8. The energy levels of hydrogen. Changes from higher energy levels to the ground state produce the Lyman series of spectral lines. Changes from higher energy levels to the second energy level produce the Balmer series, the four visible lines of which were shown in Figure 7.2.

zero at an infinite distance from the nucleus. (Recall that we made this assumption in Section 2-16. The negative sign in the above formula means that work has to be done on the electron to move it further from the nucleus. Therefore its energy increases, and if it is increasing toward zero, it must be negative.)

We will show later in this chapter how this formula may be developed. At the present time, let us consider some predictions which we can make from it.

(*a*) If $n = 1$, $E = -13.6$ ev. That is, the energy level of the ground state is -13.6 ev.

(*b*) If $n = 2$,

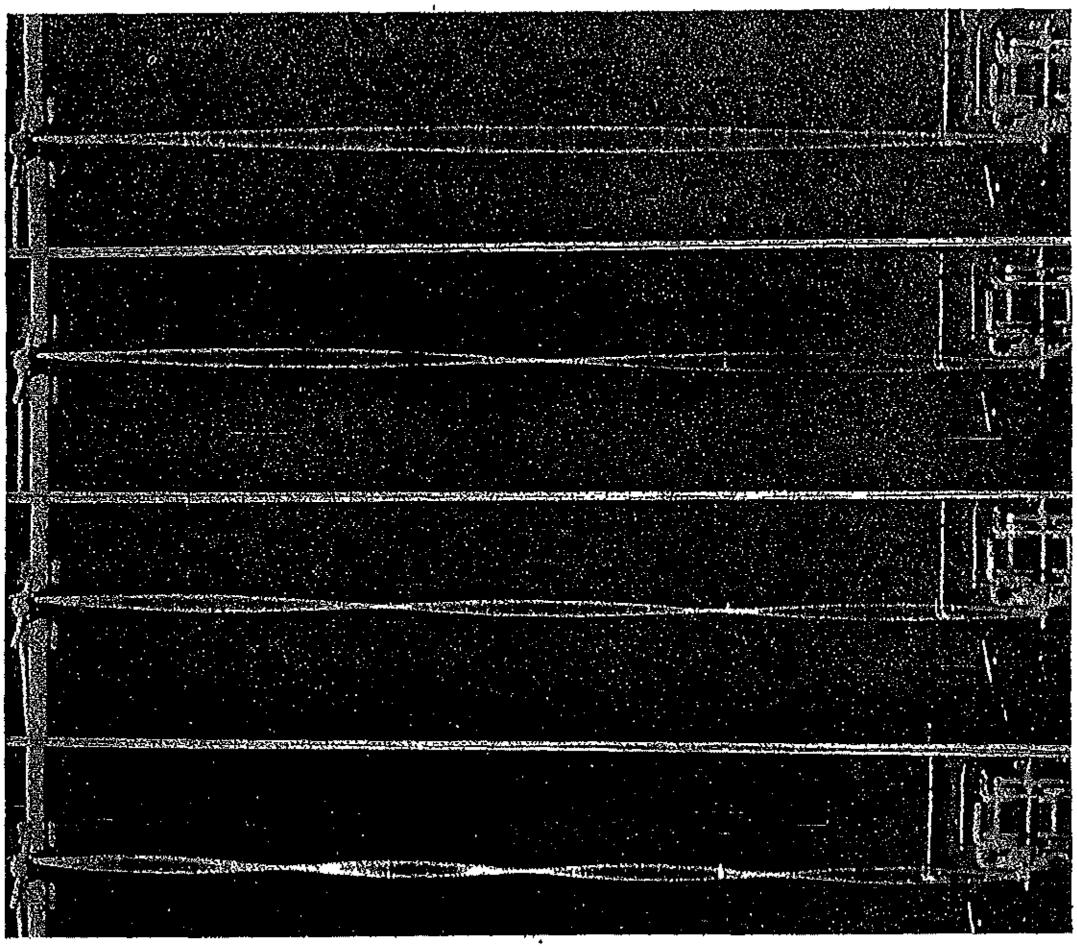
$$E = \frac{-13.6}{4}\text{ ev} = -3.4\text{ ev}$$

That is, the second energy level is 10.2 ev above the ground state. Similarly other energy levels may be calculated.

(*c*) If n becomes very large, E becomes very small. In the limit, when the electron is at an infinite distance from the nucleus, that is, when the hydrogen atom is ionized, the electron energy is zero. That is 13.6 ev above the ground state. Therefore the ionization energy of hydrogen should be 13.6 ev.

We have plotted some of these energy levels, and the ionization energy, in Figure 7.8. In this figure the zero of energy is the ground state, and the other energy levels are shown in electron volts above the ground state. Experiments of the Franck-Hertz type verify these predictions with amazing accuracy, and Bohr's atomic model is a tremendous success in this respect.

If we calculate the wave lengths of the radiation produced in energy level changes



Physics Department, University of Western Ontario

Fig. 7.9. Standing wave patterns produced by the interference of incident and reflected waves on a string.

from the second, third, fourth, etc. energy levels to the ground state (see Fig. 7.8) we obtain a set of wave lengths which belong to a series of lines in the spectrum of atomic hydrogen. This series is called the Lyman series, and had been identified, though it had not been explained, long before the energy level idea was developed. Another series of lines, the Balmer series (see Fig. 7.8 again), can be accounted for by energy level changes from the third, fourth, fifth, etc. energy levels to the second energy level. Again Bohr's model was a success, and after these successes the model was used constantly for many purposes. For example, because of the proven connection between energy levels and line spectra, considerable information about the structure of an atom can be obtained from the corresponding atomic spectrum.

7–8 AN EXPLANATION OF ENERGY LEVELS

Up to this point we have simply stated or observed experimentally that the orbital radii or energy levels can increase only by definite steps. Eventually we are bound to ask why. A reasonable explanation can be given with the help of deBroglie's concept of matter waves.

When standing waves are produced on a string (Fig. 7.9), the number of possible patterns is limited. The wave length of the incident and reflected waves must be such that the half wave length fits into the length of the string an integral number of times. That is, $L = n(\frac{1}{2}\lambda)$, where L is the length of the string, λ is the wave length, and $n = 1, 2, 3$, etc. Thus λ cannot be greater than $2L$, that is, the wave length has a maximum which it cannot exceed. Therefore the frequency has a minimum below which standing waves are impossible and above which standing waves are possible for a limited number of permitted values. We might be tempted to say that the string can vibrate in a "ground state" and in certain higher "excited states". For these states, the incident and reflected waves reinforce one another, resonance occurs, and the energy remains constant except for frictional losses.

Now consider the electron in a hydrogen atom. If the electron is moving at constant speed about the nucleus in a circular orbit of constant radius, its momentum must be constant. Therefore it must have a constant deBroglie wave length, given by the relationship $\lambda = \frac{h}{p}$. If we imagine this matter wave travelling along the circumference of the orbit, we realize that it cannot reinforce itself unless the circumference is an integral multiple of the deBroglie wave length (Fig. 7.10). Under these circumstances, and only under these circumstances, can the deBroglie wave maintain its energy; thus the permitted orbits and energy states are accounted for. Moreover, there is a maximum permitted wave length and a minimum permitted frequency. Thus there is a minimum electron energy and the ground state is accounted for.

The above explanation is oversimplified. The wave is actually three dimensional and is best described mathematically. Moreover, the electron does not necessarily follow a circular orbit, (or any orbit, for that matter). The visual model of the atom becomes more and more blurred as time goes on, and the mathematical description becomes more and more precise. And yet, assuming a

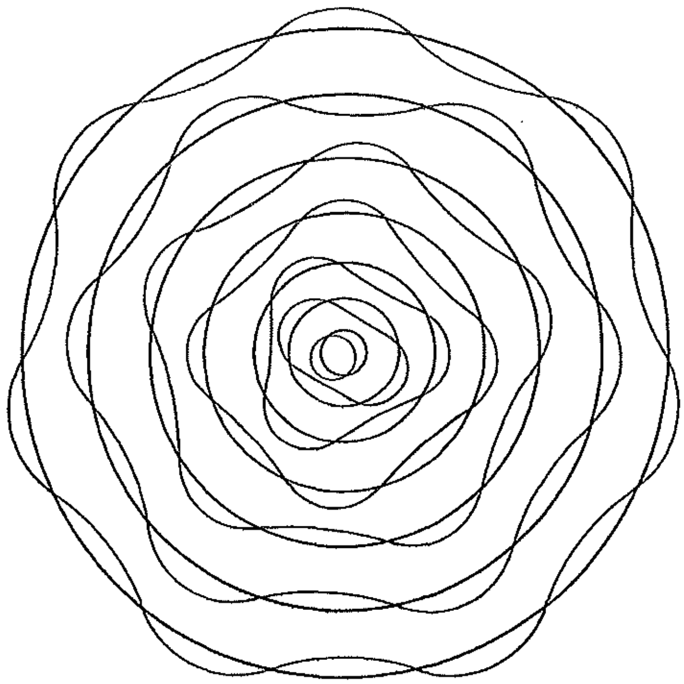


Fig. 7.10. deBroglie standing wave patterns for an electron on each of the first 7 Bohr orbits for hydrogen.

simple planetary model, assuming circular orbits, and using relatively simple mathematics, we can develop the relationship that we have already used so successfully for hydrogen, namely

$$E = \frac{-13.6}{n^2} \text{ ev}$$

The development of this formula, shown in the next section, is not quite as precise or mathematically rigid as Bohr's method, but it yields a satisfactory result. And perhaps it provides as good a summary and as fitting a conclusion for this series of books as it is possible to find, for it employs and ties together deBroglie's ideas, Bohr's ideas, Coulomb's law, the centripetal force formula, and our concepts of standing waves.

7-9 CALCULATING ENERGY LEVELS AND ORBITAL RADII

When an electron rotates about a nucleus, the centripetal force necessary to keep the electron in orbit is supplied by the electrostatic force of attraction between the electron and the nucleus. Thus

$$\frac{mv^2}{r} = \frac{kqQ}{r^2}$$

or

$$mv^2 = \frac{kqQ}{r} \qquad (1)$$

where m is the mass of the electron,
v is its linear speed,
r is its orbital radius,
q and Q are the charges on the electron and the nucleus respectively,
and k is the constant in Coulomb's law.

In order that the associated deBroglie waves may form a standing wave,

$$n\lambda = 2\pi r \qquad (n = 1, 2, 3, \ldots)$$

But $$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\therefore \frac{nh}{mv} = 2\pi r \qquad (2)$$

From (2) $$m = \frac{nh}{2\pi rv}$$

Substitute in (1)

$$\frac{nh}{2\pi rv} \cdot v^2 = \frac{kqQ}{r}$$

whence $$v = \frac{2\pi kqQ}{nh} \qquad (3)$$

This equation (3) permits us to calculate the possible speeds of an orbital electron. Suppose we do so for the hydrogen atom. Both q and Q in this case are 1 elementary charge, $k = 2.3 \times 10^{-28}$ newton-(metres)^2/(elementary charge)^2; and $h = 6.62 \times 10^{-34}$ joule-sec. Substituting these values in (3), we obtain

$$v = \frac{2.18 \times 10^6}{n} \text{ m/sec}$$

Thus the electron speed in the first orbit is 2.18×10^6 m/sec; and for all orbits the speed varies inversely as the number of the orbit.

Let us next compute the permitted orbital radii. From equation (2)

$$v = \frac{nh}{2\pi rm}$$

From equation (3)

$$v = \frac{2\pi kqQ}{nh}$$

$$\therefore \frac{nh}{2\pi rm} = \frac{2\pi kqQ}{nh}$$

$$\therefore r = \frac{n^2h^2}{4\pi^2kqQm} \qquad (4)$$

For the hydrogen atom we substitute the previously stated values for k, q, Q, and h, and 9.11×10^{-31} kg for the mass m of the electron. Then

$$r = 5.3 \times 10^{-11} \times n^2 \text{ metres}$$

We have already discussed the significance of this prediction in Sections 5–2 and 5–8.

Next we may compute electron energy levels. The energy of the electron is partly kinetic and partly potential; its total energy is the sum of the kinetic and potential energies. Therefore

$$E = \tfrac{1}{2} mv^2 - \frac{kqQ}{r} \qquad (5)$$

But, from (3), $$v = \frac{2\pi kqQ}{nh}$$

and from (4) $$r = \frac{n^2h^2}{4\pi^2kqQm}.$$

Substituting these values in (5) we obtain

$$E = -\frac{(2\pi kqQ)^2m}{2n^2h^2} \qquad (6)$$

Therefore for the hydrogen atom

$$E = \frac{-2.17 \times 10^{-18}}{n^2} \text{ joules}$$

But one joule $= 6.25 \times 10^{18}$ ev

Therefore $$E = -\frac{13.6}{n^2} \text{ ev}$$

This, of course, is the relationship which we discussed and used earlier in this chapter.

7–10 PROBLEMS

Where necessary assume that
(*a*) Planck's constant $= 6.62 \times 10^{-34}$ joule-sec
(*b*) the mass of the electron $= 9.11 \times 10^{-31}$ kg
(*c*) the constant in Coulomb's law

$$= 2.3 \times 10^{-28} \frac{\text{newton-(metre)}^2}{\text{(elementary charge)}^2}$$

1. How do the energy changes in an atom differ from the kinetic energy changes of a car speeding up or slowing down?

2. Refer to Figure 7.5. What are the possible energy changes when (*a*) cesium is bombarded with 3.0 ev electrons, (*b*) mercury is bombarded with 9.5 ev electrons?

3. What energy level changes within an atom will give rise to radiation of wave length (*a*) 1240 A, (*b*) 6200 A, (*c*) 62000 A?

4. What energy level changes within an atom will give rise to radiation of frequency (*a*) 2.0×10^{14} c/s, (*b*) 4.0×10^{15} c/s, (*c*) 1.0×10^{15} c/s?

5. What radiation wave length corresponds to an energy level change of (*a*) 1.0 ev, (*b*) 2.5 ev, (*c*) 5.0 ev?

6. What radiation frequency corresponds to an energy level change of (*a*) 2.0 ev, (*b*) 6.0 ev, (*c*) 8.0 ev?

7. If the energy levels of an atom are known, its line spectrum can be predicted. Is the converse necessarily true?

8. Consider the relationship $E = -\frac{13.6}{n^2}$. (*a*) What is the effect on E of changing n by a factor of 3? (*b*) Is there any limit to the number of energy levels? What is true of the energy levels for large values of n?

9. Use the relationship $E = -\frac{13.6}{n^2}$ ev to compute the first 6 energy levels for the hydrogen atom. Draw the energy level diagram, showing two scales, one with zero at ionization and one with zero at the ground state. Then calculate the wave length, in Angstroms, of the first three spectral lines produced by transitions from higher energy levels to (*a*) $n = 1$ (the Lyman series), (*b*) $n = 2$ (the Balmer series), (*c*) $n = 3$ (the Paschen series). In what portion or portions of the electromagnetic spectrum does each series fall?

10. Use your answers to problem 9 to list wave lengths that you would expect to be absorbed when white light is passed through incandescent hydrogen.

11. Calculate the radii, in metres, of the first 5 Bohr orbits for hydrogen.

12. The nucleus of the hydrogen atom is thought to be about 1.5×10^{-13} cm in diameter. If the nucleus were magnified to have a diameter of 0.10 mm (the size of a speck of dust), how far away would the ground state electron be?
13. Calculate the speed, in m/sec, of an electron in each of the first 5 Bohr orbits for hydrogen.
14. Use your answers to problem 11 to calculate the circumference of each of the first 2 Bohr orbits for hydrogen. Then use your answers to problem 13 to calculate the wave length in each of these orbits. Show that, in each case, the circumference is an integral multiple of the electron wave length.
15. Calculate the kinetic energy of an electron in each of the first two Bohr orbits for hydrogen.
16. Calculate the electric potential energy of an electron in each of the first two Bohr orbits for hydrogen.
17. (*a*) Calculate the total energy (kinetic plus potential) for an electron in each of the first two Bohr orbits for hydrogen. (*b*) Calculate the binding energy for an electron in each of these two orbits.
18. Refer to Figure 7.5(*b*). What effect would you predict when mercury vapour is bombarded by (*a*) a 7.0 ev electron, (*b*) an 11.4 ev electron, (*c*) a 7.0 ev photon, (*d*) an 11.4 ev photon.

7–11 SUMMARY

Associated with each type of atom there are definite and discrete energy levels. These energy levels may be explained in terms of deBroglie standing waves for the orbital electrons. The permitted orbits are such that their circumferences are integral multiples of the electron wave lengths.

Atomic emission spectra are produced when the electrons in an atom, after the atom has been excited, drop to a lower energy level and emit photons of radiation. In such a case,

$$E = h\nu = E_i - E_f$$

Atomic absorption spectra are produced when white light passes through an incandescent gas. Electrons in atoms of the gas are excited by the incident light, and absorb energy from the light. In these cases,

$$E_f = E_i + h\nu$$

For atomic hydrogen,

$$E = -\frac{13.6}{n^2}\ \text{ev}$$

QUESTIONS FOR REVIEW

1. Define each of the new terms introduced in the seven chapters of this book.

2. State each of the laws developed in Chapters 1 to 7. Indicate clearly the restricting or limiting conditions, if any, associated with each law.

3. Summarize the formulae developed in Chapters 1 to 7. For each formula, state (*a*) the meaning of each of the symbols used, (*b*) the restricting or limiting conditions, if any, associated with the formula, (*c*) the units associated with each symbol in the formula.

4. The formula $p = \frac{h}{\lambda}$ implies that the momentum of a photon is inversely proportional to the wave length. What proportions are implied by each of the following formulae?

   (*a*) $F = \frac{kqQ}{r^2}$ (*b*) $E = QV$ (*c*) $QV = Fs$

   (*d*) $QV = \frac{1}{2}mv^2$ (*e*) $E = \frac{kqQ}{r}$ (*f*) $V = \frac{kQ}{r}$

   (*g*) $Q = It$ (*h*) $Q = Ze$ (*i*) $E = h\nu$

   (*j*) $\lambda' = \frac{h}{p}$ (*k*) $n\lambda = 2d \sin \theta$ (*l*) $n\lambda = 2\pi r$

   (*m*) $v = \frac{2.18 \times 10^6}{n}$ (*n*) $r = 5.3 \times 10^{-11}\, n^2$ (*o*) $E = -\frac{(2\pi kqQ)^2 m}{2n^2h^2}$

   (*p*) $E = -\frac{13.6}{n^2}$

5. What units has k in each of the following formulae?

   (*a*) $F = \frac{kqQ}{r^2}$ (*b*) $E = \frac{kqQ}{r}$ (*c*) $V = \frac{kQ}{r}$

6. What units has h in each of the following formulae?

   (*a*) $E = h\nu$ (*b*) $p = \frac{h}{\lambda}$ (*c*) $\lambda = \frac{h}{p}$

7. (*a*) List examples of quantities encountered daily (coinage, for example) for which there is a smallest unit, or quantum. State the magnitude of the quantum in each case. (*b*) List examples of quantized quantities from the field of science.

8. If the energy of the photon incident on an atom is greater than the ionization energy, what effect takes place?

9. What would be the effect on the penetrating properties of the X-rays produced by an X-ray tube of (*a*) failure to remove as much air as possible from the tube, (*b*) too high a voltage applied to the tube? Justify your answers.

10. If light is emitted in bursts, as the photon idea suggests, why does the source not twinkle?
11. When a scintillation counter detects an alpha particle, a flash of light energy is emitted. What is the source of this light energy?
12. The results of measurements in photoelectric experiments depend on the preparation of the photoelectric surface. Explain why this is so.
13. How can the charge on the nucleus of an atom be measured?
14. How can the diameter of an atomic nucleus be measured?
15. Does it make sense to talk about the colour of an atom?
16. If the relationship $E = h\nu$ held for sound as it does for light, what likelihood would there be of measuring E?
17. A magnetic field is applied to a parallel beam of radiation from a radioactive substance. The effect of the magnetic field is to break the beam into two portions, one of which is deflected slightly from its straight line path. The other portion continues to travel in a straight line. What type of radiation is definitely present? How could the other type be identified?
18. Calculate the deBroglie wave length of an electron in the first Bohr orbit for hydrogen. Then calculate the energy of a photon of radiation of this wave length. Show that the energy of such a photon is many times the ionization energy for hydrogen.
19. Calculate the current for an electron moving in the first Bohr orbit for hydrogen.
20. In what ways would our universe be different if the value of Planck's constant were of the order of 10^{-20} joule-sec?
21. What is the function of a resistor in an electric circuit?
22. Summarize the evidence for the existence of electrons.
23. Can you explain why two like charges repel each other? Can you describe the force of repulsion?
24. List the experiments described in this book, in which it is assumed that electric charge is never created nor destroyed.
25. Show that electric field intensity may be measured either in newtons/coulomb or volts/metre.
26. In a cathode-ray oscilloscope, the vertically-mounted deflecting plates are called horizontal deflecting plates. Explain.
27. In what ways do X-rays differ from cathode rays?
28. Outline the contributions made by Thomson, Millikan, Rutherford, Bohr, and deBroglie to the understanding of atomic structure.
29. What contributions have chemists made to our knowledge of the atom?
30. What facts concerning the photoelectric effect cannot be explained in terms of waves?
31. Summarize (*a*) what you know to be true, and (*b*) what you think may be true of the structure of a hydrogen atom.

INDEX

ANSWERS

Chapter 2—Section 2-17, page 24

1. F changes by a factor of (*a*) 3 (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{1}{6}$
2. (*a*) 9.0×10^{-6} newt (*b*) 5.8×10^{-6} newt (*c*) 4.0×10^{-6} newt
3. (*a*) 1.5×10^{-5} newt (*b*) 1.2×10^{-4} newt (*c*) 1.0×10^{-5} newt
4. 9 5. 0.577 6. 3.6×10^{-4} newt
7. 1.8×10^{-2} newt 8. 34 newt 9. 3.3×10^{-6} coul
10. 4.5×10^{-6} newt 11. 2.0×10^{2} newt/coul 12. 1.3×10^{2} m
13. 3.0×10^{3} newt/coul 14. 2.0×10^{2} volts 15. 1.0×10^{-4} coul
16. 1.5 joules; 5.0 newt 17. 10^{3} volts/m 18. 8.0×10^{3} newt/coul
19. (*b*) 1.28×10^{-16} joules 20. 3.0×10^{3} volts 21. 5 volts
22. (*a*) 3.2×10^{-4} joules (*b*) 6.4×10^{-4} newt (*c*) 3.2×10^{-4} joules
(*d*) 8 m/sec 23. 1.6×10^{-13} joules; 1.4×10^{7} m/sec
24. (*a*) 7.3×10^{14} m/sec^2 (*b*) 6.6×10^{-16} newt (*c*) 83 volts
25. 5.3×10^{7} m/sec 26. 26 volts 27. 9.2×10^{6} m/sec
29. V changes by a factor of (*a*) 10 (*b*) 5
30. (*b*) (*i*) 2.7 joules (*ii*) 1.35 joules (*iii*) 0.90 joules
31. (*b*) (*i*) 4.8×10^{4} volts (*ii*) 3.6×10^{4} volts (*iii*) 2.4×10^{4} volts

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1. 6 3. 0.25 amp 4. 1.5 min
5. (*a*) 600 (*b*) 3.75×10^{21}
6. (*a*) 1 coul (*b*) 2.5 amp (*c*) 5 min
7. (*a*) 7.2×10^{4} joules (*b*) 120 watts
8. (*a*) (*i*) 0.50 amp (*ii*) 1.0 amp (*b*) 2.4×10^{2} watts
(*d*) 2.16×10^{5} joules 9. 0.51
10. 3.2×10^{-15} joules; 0.96 watts
11. (*a*) 4.8×10^{2} watts (*b*) 88 watts (*c*) 1.6×10^{2} coul

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3. (*a*) 1.7×10^{-3} m
5. (*a*) 1.8×10^{4} newt/coul (*b*) negatively (*c*) 8.0×10^{-19} coul
(*d*) 5 surplus 6. (*a*) 1.2×10^{4} volts 7. 2.9×10^{-15} kg
8. 3.75×10^{15} 9. 1.88×10^{12} 10. 2.5×10^{11}
11. (*a*) -4.00×10^{-6} coul (*b*) 2.00×10^{-4} coul
12. (*a*) 1.3×10^{19} (*b*) 2.5×10^{20} (*c*) 5×10^{16}
13. (*a*) 0.100 (*b*) 0.02 (*c*) 8.0×10^{-3}
14. 2.6×10^{-2} amp

15. (*a*) 30 (*b*) 1.9×10^{20}
16. 1.0×10^{16}
17. (*a*) 3.8×10^{2} (*b*) 5.0×10^{19} (*c*) 1.25×10^{14}
18. (*a*) 3.2×10^{-19} (*b*) 8.0×10^{-12} (*c*) 9.6×10^{-21}
19. (*a*) 3.2×10^{-5} amp (*b*) 6.4×10^{-5} amp
20. 2.1×10^{-25} kg
21. (*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{2}{\sqrt{3}}$
22. 2.0×10^{5}
23. (*a*) 3.0×10^{2} ev (*b*) 2.4×10^{-27} kg
24. (*a*) 1.4×10^{-17} (*b*) 1.8×10^{2} ev
25. (*a*) (*i*) 45 ev (*ii*) 7.2×10^{-17} newt (*b*) 1.8×10^{2} volts
26. (*a*) $\frac{1}{\sqrt{2}}$ (*b*) 100
27. 2×10^{4} m/sec²

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1. 10^{-11}
3. (*a*) 1.92×10^{-18} (*b*) 4.64×10^{-18} (*c*) 2.08×10^{-18} (*d*) 1.26×10^{-17}
4. $2.4 \times 10^{-11}\%$
7. 0.40 newt
8. (*a*) 1.6×10^{-36} newt (*b*) 2.5×10^{35} (*c*) 2.5×10^{35}
9. 1.2×10^{-18} joules
10. 2.8×10^{-14} m
11. 3.7×10^{-13}
12. -3.3×10^{4} m/sec; 6.7×10^{4} m/sec
14. (*a*) 1.5×10^{4} m (*b*) 2.0 m (*c*) 5.0×10^{3} A (*d*) 1.0 A
15. (*a*) 7.5×10^{3} c/s (*b*) 5.0×10^{14} c/s (*c*) 1.5×10^{20} c/s
16. 10^{9}
17. 8.3×10^{7} c/s
20. (*a*) 4.0×10^{7} m/sec (*b*) 4.5×10^{3} volts
21. 2.0×10^{18} c/s
22. 1.5×10^{15}

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3. 4.14×10^{-15}
4. 3.3×10^{-19}
5. 1.5×10^{14} c/s; 2.0×10^{-6} m
6. $1\frac{1}{2}$ to 3
7. (*a*) 4.1×10^{-3} (*b*) 1.8 (*c*) 3.1 (*d*) 4.1×10^{3}
8. (*a*) 8.3×10^{3} (*b*) 4.1 (*c*) 2.1 (*d*) 1.2×10^{-3}
11. 0.31 A
12. 10^{15}
13. 2.5×10^{-19}
14. 2.0×10^{-19} joules
15. 2.1 ev/electron
16. (*a*) 3.9×10^{14} c/s; 8.0×10^{14} c/s; 1.1×10^{15} c/s
    (*b*) 1.5 ev; no emission; no emission
17. 5.0
18. (*b*) 1.24 A
19. 6.6×10^{-21} kg-m/sec
20. 1.3×10^{-27} kg-m/sec
21. 6.6×10^{-27} newt-sec
22. (*a*) 4.1×10^{-7} (*b*) 4.9×10^{-19} (*c*) 1.6×10^{-27}

24. (*a*) 2.7 A (*b*) 6.6×10^{-15} m (*c*) 10^{-38} m
25. (*a*) 1.5×10^{-8} m (*b*) (*i*) 1.2×10^{-9} m
(*ii*) 8.5×10^{-10} m (*iii*) 6.0×10^{-10} m
26. 8.3×10^{-34} m
27. (*a*) (*i*) 4.8×10^{2} ev (*ii*) 7.7×10^{-17} joules
(*b*) 1.3×10^{7} m/sec (*c*) 5.6×10^{-11} m
28. (*a*) 2.5×10^{-7} (*b*) 5.5×10^{-10}
29. (*a*) 2.7×10^{-27} kg-m/sec (*b*) 1.2×10^{-24} kg-m/sec

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3. (*a*) 10.0 ev (*b*) 2.0 ev (*c*) 0.20 ev
4. (*a*) 0.83 ev (*b*) 17 ev (*c*) 4.1 ev
5. (*a*) 1.2×10^{4} A (*b*) 5.0×10^{3} A (*c*) 2.5×10^{3} A
6. (*a*) 4.8×10^{14} c/s (*b*) 1.4×10^{15} c/s (*c*) 1.9×10^{15} c/s
8. (*a*) E changes by a factor of $\frac{1}{9}$
9. (*a*) 1.2×10^{3}; 1.0×10^{3}; 9.7×10^{2}
(*b*) 6.5×10^{3}; 4.9×10^{3}; 4.3×10^{3}
(*c*) 1.9×10^{4}; 1.3×10^{4}; 1.1×10^{4}
11. 5.3×10^{-11}; 2.1×10^{-10}; 4.8×10^{-10}; 8.5×10^{-10}; 1.3×10^{-9}
12. about 3 m
13. 2.18×10^{6}; 1.09×10^{6}; 7.3×10^{5}; 5.5×10^{5}; 4.4×10^{5}
15. 2.2×10^{-18} joules; 5.4×10^{-19} joules
16. -4.3×10^{-18} joules; -1.1×10^{-18} joules
17. (*a*) -2.2×10^{-18} joules; -5.5×10^{-19} joules
(*b*) 2.2×10^{-18} joules; 5.5×10^{-19} joules

ACKNOWLEDGEMENTS

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