# Phys 4050 Assignment 7

1. Magnetic field penetration in a plate. The equation describing the magnetic field penetration of a superconductor may be written as

$$\lambda^2 \nabla^2 B = B$$

where $\lambda$ is the penetration depth.

a) Show that $B(x)$ inside a superconducting plate perpendicular to the x axis and of thickness $\delta$ is given by

$$B(x) = B_a \frac{\cosh(x/\lambda)}{\cosh(\delta/2\lambda)}$$

where $B_a$ is the field outside the plate and parallel to it. Here $x = 0$ is at the center of the plate.

b) The effective magnetization $M(x)$ in the plate is defined by $B(x) - B_a = 4\pi M(x)$. For $\delta \gg \lambda$ show that:

$$4\pi M(x) = -\frac{B_a}{8\lambda^2}(\delta^2 - 4x^2)$$

2. Structure of a vortex.

a) Find a solution to the London equation that has cylindrical symmetry and applies outside a line core. In cylindrical polar coordinates, we want a solution of

$$B - \lambda^2 \nabla^2 B = 0$$

that is singular at the origin and for which the total flux is the flux quantum:

$$2\pi \int_0^\infty d\rho \, \rho B(\rho) = \Phi_0$$

The equation is in fact valid only outside the normal core of radius $\xi$.

b) Show that the solution has the limits

$$B(\rho) \simeq \frac{\Phi_0}{2\pi\lambda^2} \ln(\lambda/\rho) \qquad \xi \ll \rho \ll \lambda$$

$$B(\rho) \simeq \frac{\Phi_0}{2\pi\lambda^2} \left(\frac{\pi\lambda}{2\rho}\right)^{1/2} \exp(-\rho/\lambda) \qquad \rho \gg \lambda$$

3. London penetration depth.

a) Take the time derivative of the London equation to show that

$$\frac{\partial \vec{J}}{\partial t} = \frac{c^2}{4\pi \lambda_L^2} \vec{E}$$

b) If $m\, d\vec{v}/dt = q\vec{E}$, as for free carriers of charge q, mass m and density n, show that

$$\lambda_L^2 = \frac{mc^2}{4\pi n q^2}$$

4. Diffraction effect of Josephson junction. Consider a junction of rectangular cross section with a magnetic field B applied in the plane of the junction, normal to an edge of width w. Let the thickness of the junction be T. Assume for convenience that the phase difference of the two superconductors is $\pi/2$ when $B = 0$. Show that the dc current in the presence of the magnetic field is:

$$J = J_0 \frac{\sin(wTBe/\hbar c)}{wTBe/\hbar c}$$