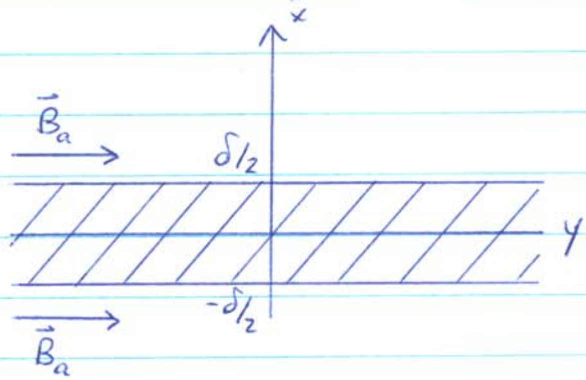
pg. 356 <u>Assignment 7</u>

1a)




$$\lambda^2 \nabla^2 B = B$$

$$\frac{d^2 B}{d x^2} = \frac{B}{\lambda^2}$$

$$B = C e^{-x/\lambda} + D e^{x/\lambda}.$$

Boundary conditions are $B\left(x = \pm \frac{\delta}{2}\right) = B_a$.

$$\Rightarrow B = B_a \frac{\cosh x/\lambda}{\cosh \delta/2\lambda}.$$

b) Magnetization $4\pi M = (B(x) - B_a)$

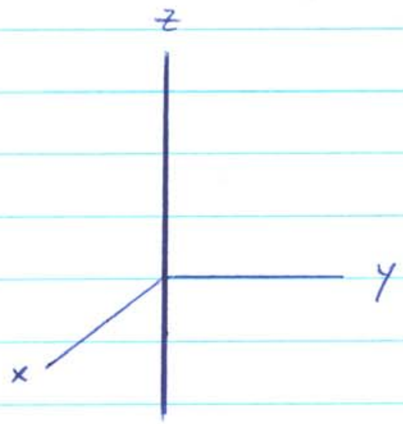
$$= B_a \left[ \frac{\cosh x/\lambda}{\cosh \delta/2\lambda} - 1 \right]$$

For $x \ll \lambda$ $\cosh x/\lambda = 1 + \frac{1}{2}\left(\frac{x}{\lambda}\right)^2$

$$\therefore 4\pi M = B_a \left[ \frac{1 + \frac{1}{2}\left(\frac{x}{\lambda}\right)^2}{1 + \frac{1}{2}\left(\frac{\delta}{2\lambda}\right)^2} - 1 \right]$$

$$\simeq -\frac{B_a}{8\lambda^2}\left(\delta^2 - 4x^2\right).$$

4a)




$\vec{B} = B(\rho)\ \hat{z}$

radial

in cylindrical coords.

$$B - \lambda^2 \nabla^2 B = 0.$$

In cylind. coords. this becomes:

$$B - \frac{\lambda^2}{\rho}\frac{d}{d\rho}\left(\rho \frac{dB}{d\rho}\right) = 0.$$

$$\frac{d^2 B}{d\rho^2} + \frac{1}{\rho}\frac{dB}{d\rho} - \frac{B}{\lambda^2} = 0.$$

Defining $x = \rho/\lambda$, we get:

$$\frac{d^2 B}{dx^2} + \frac{1}{x}\frac{dB}{dx} - B = 0.$$

From pg. 107 of Jackson, solution is

$$B = A\, I_0(x) + C\, K_0(x)$$

well defined at origin which we don't want.

$$\therefore B = C\, K_0(x).$$

Flux of $\vec{B}$ through $xy$ plane is:

$$\int_0^\infty 2\pi\rho \, d\rho \; B(\rho) = \Phi_0$$

$$\int_0^\infty 2\pi\rho \, d\rho \; c \, K_0(\rho/\lambda) = \Phi_0$$

$$2\pi c \, \lambda^2 \underbrace{\int_0^\infty x \, K_0(x) \, dx}_{=1 \text{ (prop. of } K_0)} = \Phi_0$$

$$\therefore \; c = \frac{\Phi_0}{2\pi\lambda^2}.$$

$$\therefore \; \vec{B}(\rho) = \frac{\Phi_0}{2\pi\lambda^2} K_0(\rho/\lambda)$$

b) $x \ll 1 \quad K_0(x) = -\ln\left(\frac{x}{2}\right) - .5772$

$x \gg 1 \quad K_0(x) = \sqrt{\frac{\pi}{2x}} \; e^{-x}$

$$\Rightarrow \rho \ll \lambda \quad B \simeq -\frac{\Phi_0}{2\pi\lambda^2}\left[\ln\left(\frac{\rho}{2\lambda}\right) + .5772\right]$$

$$\rho \gg \lambda \quad B \simeq \frac{\Phi_0}{2\pi\lambda^2}\left(\frac{\pi\lambda}{\rho}\right)^{1/2} \exp(-\rho/\lambda)$$

5a) $$\vec{J} = -\frac{c}{4\pi\lambda_L^2}\vec{A}$$

$$\frac{d\vec{J}}{dt} = \frac{-c}{4\pi\lambda_L^2}\frac{d\vec{A}}{dt}$$

$$= \frac{c^2}{4\pi\lambda_L^2}\vec{E}$$

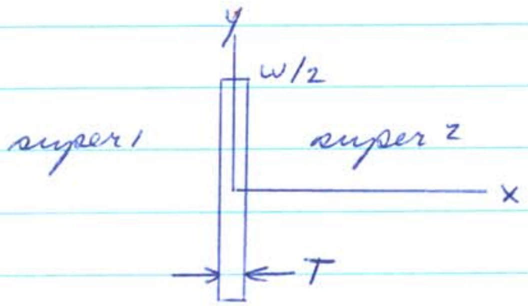
b) $$m\frac{d\vec{v}}{dt} = q\vec{E}$$

Now $\vec{J} = nq\vec{v} \Rightarrow \frac{d\vec{J}}{dt} = nq\frac{d\vec{v}}{dt}$

$$\frac{c^2\vec{E}}{4\pi\lambda_L^2} = \frac{nq^2}{m}\vec{E}$$

$$\therefore \lambda_L^2 = \frac{mc^2}{4\pi nq^2}.$$

6)




$\vec{B} \parallel \hat{z}$

Phase Diff. between 2 superconductors is

$$\delta = \theta_2 - \theta_1$$
$$= \frac{2e}{\hbar c}\Phi + \frac{\pi}{2}$$
$$= \frac{2e}{\hbar c} BTy + \frac{\pi}{2}$$

Current $J = J_0 \sin\delta$

$$= J_0 \sin\left(\frac{2e}{\hbar c} BTy + \frac{\pi}{2}\right)$$
$$= J_0 \cos \frac{2eBTy}{\hbar c}$$

$$J_{TOT} = \frac{1}{w}\int_{-w/2}^{w/2} J_0 \cos \frac{2eBTy}{\hbar c}\, dy$$

$$\therefore J_{TOT} = J_0 \frac{\sin eBwT/\hbar c}{\frac{weBT}{\hbar c}}$$ total current across junction