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Assignment 5

3a) Kronig Penney Model.

(21b) $$\frac{P}{Ka} \sin Ka + \cos Ka = \cos ka$$

$$k = 0 \Rightarrow \frac{P}{Ka} \sin Ka + \cos Ka = 1 \qquad (1)$$

Since $P \ll 1 \Rightarrow \cos Ka = 1$

$$Ka = 2n\pi \quad n \in \text{integer}$$

$\therefore$ lowest energy corresponds to $K = 0$.

First Order Correction

Let $K = 0 + \Delta K$

$$(1) \Rightarrow \frac{P}{\Delta Ka} \sin \Delta Ka + \cos \Delta Ka = 1$$

$$\frac{P}{\Delta Ka}\left[\Delta Ka - \frac{(\Delta Ka)^3}{3!}\right] + 1 - \frac{(\Delta Ka)^2}{2!} = 1$$

$$P\left(1 - \frac{(\Delta Ka)^2}{6}\right) - \frac{(\Delta Ka)^2}{2} = 0.$$

$$P - \frac{(\Delta Ka)^2}{2} = 0.$$

$$\Delta K = \frac{\sqrt{2P}}{a}.$$

$\therefore$ lowest energy at $k = 0$ is $\frac{\hbar^2 \Delta K^2}{2m} = \frac{\hbar^2 P}{ma^2}$

b) at $k = \frac{\pi}{a}$ (2,b) $\Rightarrow$ $\frac{P}{Ka} \sin Ka + \cos Ka = -1$ (2)

Lowest order soln. yields $\cos Ka = -1$

$Ka = n\pi$ $n =$ odd integer

$\therefore$ Lowest unperturbed energy at $k = \frac{\pi}{a}$ corresponds to $K = \frac{\pi}{a}$.

First Order Correction

Let $K = \frac{\pi}{a} + \Delta K$

$$(2) \Rightarrow \frac{P}{\left(\frac{\pi}{a} + \Delta K\right)a} \sin\left(\frac{\pi}{a} + \Delta K\right)a + \cos\left(\frac{\pi}{a} + \Delta K\right)a = -1.$$

$$\frac{P}{\pi} \frac{1}{\left(1 + \frac{\Delta Ka}{\pi}\right)} \sin(\pi + \Delta Ka) + \cos(\pi + \Delta Ka) = -1$$

$$\frac{P}{\pi}\left(1 - \frac{\Delta Ka}{\pi}\right)(-\sin \Delta Ka) - \cos \Delta Ka = -1$$

$$\frac{P}{\pi}\left(1 - \frac{\Delta Ka}{\pi}\right)(-\Delta Ka) - \left(1 - \frac{(\Delta Ka)^2}{2!}\right) = -1$$

$$-\frac{P\Delta Ka}{\pi} + \frac{(\Delta Ka)^2}{2!} = 0.$$

$$\Delta Ka\left(-\frac{P}{\pi} + \frac{\Delta Ka}{2}\right) = 0.$$

$$\therefore \Delta Ka = 0,\ \frac{2P}{\pi}$$

Hence at $k = \frac{\pi}{a}$ there are two allowed values of K, to order ρ^1.

$$\Rightarrow \text{bandgap } \Delta E = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{a} + \frac{2\rho}{\pi a}\right)^2 - \left(\frac{\pi}{a}\right)^2\right]$$

$$\simeq \frac{\hbar^2}{\cancel{2}m}\left[\cancel{2} \cdot \frac{\cancel{\pi}}{a}\,\frac{2\rho}{\cancel{\pi} a}\right] \quad \text{to order } \rho^1$$

$$= \frac{2\hbar^2 \rho}{m a^2}$$

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2a) $E_d = 10^{-3}\ \mathrm{eV}$

$N = 10^{13}$ donors/cm^3

$m_e = .01\, m$

Probability donor electron is in conduction band is $\exp(-E_d/kT)$.

$\therefore$ density of donor conduction electrons at 4°K is

$$N_{cond} = N \exp(-E_d/kT)$$

$$= 10^{13} \exp\left\{-\frac{10^{-3}\ \mathrm{eV} \times 1.6 \times 10^{-12}\ \mathrm{erg/eV}}{1.38 \times 10^{-16}\ \mathrm{erg/K} \times 4\mathrm{K}}\right\}$$

$$= 5.5 \times 10^{11}\ \mathrm{cm}^{-3}$$

b) Since $kT << E_g$, n is given by 2a.

Hall coefficient $R_H = \dfrac{-1}{nec}$

$$= -\left(5.5\times10^{11}\,\text{cm}^{-3}\times4.8\times10^{-10}\,\text{esu}\times3\times10^{10}\,\frac{\text{cm}}{\text{sec}}\right)^{-1}$$

$$= -1.26\times10^{-13}\ \frac{\text{cm}^2\,\text{sec}}{\text{esu}}.$$

3) Using result for #9 pg. 155 we have:

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

For 2 types of carriers, holes + electrons we must add their respective currents to obtain total current:

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \left\{\frac{\sigma_e}{1+\omega_e^2\tau_e^2}\begin{pmatrix} 1 & -\omega_e\tau_e & 0 \\ \omega_e\tau_e & 1 & 0 \\ 0 & 0 & 1+\omega_e^2\tau_e^2 \end{pmatrix}\right.$$

$$\left. + \frac{\sigma_h}{1+\omega_h^2\tau_h^2}\begin{pmatrix} 1 & -\omega_h\tau_h & 0 \\ \omega_h\tau_h & 1 & 0 \\ 0 & 0 & 1+\omega_h^2\tau_h^2 \end{pmatrix}\right\}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Current flows in $\hat{x}$ direction.

$$\therefore\ j_x = \frac{\sigma_e}{1+\omega_e^2\tau_e^2}(E_x - \omega_e\tau_e E_y) + \frac{\sigma_h}{1+\omega_h^2\tau_h^2}(E_x - \omega_h\tau_h E_y)$$

$$0 = \frac{\sigma_e}{1+\omega_e^2\tau_e^2}(\omega_e\tau_e E_x + E_y) + \frac{\sigma_h}{1+\omega_h^2\tau_h^2}(\omega_h\tau_h E_x + E_y)$$

Neglecting terms of $O(B^2)$, the above eqns. become:

$$j_x = \sigma_e(E_x - \omega_e\tau_e E_y) + \sigma_h(E_x - \omega_h\tau_h E_y) \qquad (1)$$

$$0 = \sigma_e(\omega_e\tau_e E_x + E_y) + \sigma_h(\omega_h\tau_h E_x + E_y)$$

Last eqn. gives:

$$E_y = -E_x \frac{\sigma_e\omega_e\tau_e + \sigma_h\tau_h\omega_h}{\sigma_e + \sigma_h}$$

Subst. E_y into (1) + neglecting terms $O(B^2)$ we get:

$$j_x = E_x(\sigma_e + \sigma_h)$$

Hall Coefficient $R_H = \frac{E_y}{j_x B}$

$$R_H = \frac{-(\sigma_e\omega_e\tau_e + \sigma_h\omega_h\tau_h)}{(\sigma_e+\sigma_h)^2 B}$$

After tedious algebra we get:

$$R_H = \frac{1}{ec}\,\frac{n_h - b^2 n_e}{(n_h + b n_e)^2} \quad \text{where } b \equiv \frac{\mu_e}{\mu_h} \ + \ \mu_h \equiv \frac{e\tau_h}{m_h}.$$