Assignment 4

1) # modes with wavevector less than k. is

$$N(k) = \left(\frac{L}{2\pi}\right)^3 \frac{4}{3}\pi k^3 \quad 2$$

2 electron spins

But $E = \frac{\hbar^2 k^2}{2m} \Rightarrow N(E) = \left(\frac{L}{2\pi}\right)^3 \frac{4}{3}\pi \left(\frac{2mE}{\hbar^2}\right)^{3/2} 2$

∴ energy density of electron states is:

$$D(E) = \frac{dN}{dE}$$

$$= \left(\frac{L}{2\pi}\right)^3 4\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$$

electrons $N \equiv \int_0^{\epsilon_F} D(E)\, dE$ This is definition of Fermi energy.

$$N = \int_0^{\epsilon_F} \left(\frac{L}{2\pi}\right)^3 4\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}\, dE$$

$$= \frac{V}{2\pi^2} \left(\frac{3m}{\hbar^2}\right)^{3/2} \frac{2}{3}\, \epsilon_F^{3/2}$$

energy at $T = 0$ $U_0 \equiv \int_0^{\epsilon_F} E\, D(E)\, dE.$

$$= \left(\frac{L}{2\pi}\right)^3 4\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{\epsilon_F} E^{3/2}\, dE.$$

∴ $U_0 = \frac{3}{5} N \epsilon_F$ using expression for N

4a) $M_s = 2 \times 10^{33}$ gm.

Assuming sun is all H; # electrons in sun = # protons.

$$\therefore \quad N = \frac{M_s}{m_{prot.}}$$

$$= \frac{2 \times 10^{33}}{1.67 \times 10^{-24}}$$

$$= 1.2 \times 10^{57} \text{ electrons}$$

White Dwarf: $r = 2 \times 10^{9}$ cm.

$$\epsilon_F = \left(\frac{3\pi^2 N}{V}\right)^{2/3} \frac{\hbar^2}{3m}$$

$$= \left(\frac{3\pi^2 \; 1.2 \times 10^{57}}{\frac{4\pi}{3} \; (2 \times 10^{9})^3 \text{cm}^3}\right)^{2/3} \frac{(1 \times 10^{-27} \text{ erg sec})^2}{3 \times 9.11 \times 10^{-28} \text{ gm.}}$$

$$= 9.8 \times 10^{-9} \text{ erg.}$$

$$\therefore \epsilon_F = 6100 \text{ eV.}$$

b) # electron states having wavevector less than k is

$$N(k) = \left(\frac{L}{2\pi}\right)^3 \frac{4}{3}\pi k^3 \cdot 2$$

Using $\epsilon = \hbar k c \implies N(\epsilon) = \frac{V}{3\pi^2} \frac{\epsilon^3}{(\hbar c)^3}$

$\therefore$ density of states $D(\epsilon) = \frac{V}{\pi^2} \frac{\epsilon^2}{(\hbar c)^3}$

$$N = \int_0^{\epsilon_F} D(\epsilon)\, d\epsilon$$ definition of Fermi level

$$= \frac{V}{3\pi^2} \frac{\epsilon_F^3}{(\hbar c)^3}$$

$$\therefore \epsilon_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3} \hbar c$$

c) If $r = 10^6$ cm
$N = 1.2 \times 10^{57}$ electrons

$$\Rightarrow \epsilon_F = \left(\frac{3\pi^2 \times 1.2 \times 10^{57}}{\frac{4\pi}{3} \times 10^{18}\,\text{cm}^3}\right)^{1/3} 1 \times 10^{-27}\,\text{erg sec} \times 3 \times 10^{10}\,\frac{\text{cm}}{\text{sec}}$$

$$= 6.5 \times 10^{-4}\,\text{erg}$$
$$= 4 \times 10^{8}\,\text{eV}$$

5) Fermi Energy For liquid He^3 is:

$$\epsilon_F = \left(\frac{3\pi^2 N}{V}\right)^{2/3} \frac{\hbar^2}{2m}$$ where $\frac{N}{V} = \frac{Nm}{mV} = \frac{\rho}{m}$ $\quad m_{He^3} = 3\, m_{prot}$

$$\epsilon_F = \left(\frac{3\pi^2 \times .081\,\text{gm/cm}^3}{3 \times 1.67 \times 10^{-24}\,\text{gm}}\right)^{2/3} \frac{(1 \times 10^{-27}\,\text{erg sec})^2}{2 \times 2 \times 1.67 \times 10^{-24}\,\text{gm}.}$$
$$= 0.7 \times 10^{-15}\,\text{erg}$$
$$= 4.2 \times 10^{-4}\,\text{eV}$$

$$T_F = \frac{\epsilon_F}{k} \simeq 5K.$$

6) $$m\frac{dv}{dt} + \frac{mv}{\tau} = -eE.$$

Let $E = E e^{-i\omega t}$

$v = v e^{-i\omega t}$

$$\therefore \; -im\omega v + \frac{mv}{\tau} = -eE.$$

$$mv\left(\frac{1}{\tau} - i\omega\right) = -eE.$$

$$v = -\frac{eE}{m}\frac{1}{\frac{1}{\tau} - i\omega}.$$

$$= -\frac{e\tau}{m}\frac{1}{1-i\omega\tau}E.$$

Current Density $J = -nev.$

$$= \frac{ne^2\tau}{m}\frac{1}{1-i\omega\tau}E.$$

$\therefore$ conductivity $$\sigma = \frac{ne^2\tau}{m}\frac{1+i\omega\tau}{1+\omega^2\tau^2}.$$

$$\therefore \sigma(\omega) = \sigma(0)\frac{1+i\omega\tau}{1+\omega^2\tau^2}$$

9) Equation of motion $\vec{B} = B\hat{z}$ is:

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right) v_x = -e\left(E_x + \frac{B}{c} v_y\right)$$

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right) v_y = -e\left(E_y - \frac{B}{c} v_x\right)$$

$$m\left(\frac{d}{dt} + \frac{1}{\tau}\right) v_z = -eE_z.$$

In the static case, all time derivatives are zero.

$$\therefore \quad \frac{m}{\tau} v_x = -e\left(E_x + \frac{B}{c} v_y\right) \qquad (1)$$

$$\frac{m}{\tau} v_y = -e\left(E_y - \frac{B}{c} v_x\right) \qquad (2)$$

$$\frac{m}{\tau} v_z = -eE_z. \qquad (3)$$

Subst. v_y from (2) into (1), we get:

$$\frac{m}{\tau} v_x = -eE_x - \frac{eB}{c} \frac{\tau}{m} (-e)\left(E_y - \frac{B}{c} v_x\right)$$

$$v_x = -\frac{e\tau}{m} E_x - \omega_c \frac{\tau^2}{m} (-e)\left(E_y - \frac{B}{c} v_x\right) \qquad \omega_c \equiv \frac{eB}{mc}$$

$$= \frac{-e\tau}{m} E_x + \frac{e\tau^2}{m} \omega_c E_y - \tau^2 \omega_c^2 v_x.$$

$$v_x(1 + \omega_c^2 \tau^2) = -\frac{e\tau}{m}\left(E_x - \omega_c \tau E_y\right)$$

$$v_x = \frac{-e\tau/m}{1 + \omega_c^2 \tau^2}\left(E_x - \omega_c \tau E_y\right)$$

$$J_x = -n e v_x$$

$$= \frac{n e^2 \tau / m}{1 + \omega_c^2 \tau^2} \left(E_x - \omega_c \tau E_y \right)$$

$$J_x = \frac{\sigma(0)}{1 + \omega_c^2 \tau^2} \left(E_x - \omega_c \tau E_y \right)$$

Similarly one solves for J_y + J_z.

$$\Rightarrow \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \frac{\sigma(0)}{1 + \omega_c^2 \tau^2} \begin{pmatrix} 1 & -\omega_c \tau & 0 \\ \omega_c \tau & 1 & 0 \\ 0 & 0 & 1 + \omega_c^2 \tau^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$