Assignment 11

1) $\bar{n}_{FD} = \frac{1}{e^{B(\epsilon-\mu)}+1}$ $\bar{n}_{BE} = \frac{1}{e^{B(\epsilon-\mu)}-1}$

$\beta = (k_B T)^{-1}$

$= (1.38 \times 10^{-23} \times 273)^{-1}$

$= (4.14 \times 10^{-21} J)^{-1}$

$= (.025\ eV)^{-1}$

$= 40\ eV^{-1}$

$\epsilon - \mu$	$\beta(\epsilon-\mu)$	$\bar{n}_{FD}$	$\bar{n}_{BE}$
0.002eV	0.08	0.48	12
0.02	0.8	0.31	0.82
0.2	8	3×10^{-4}	3×10^{-4}

4a) Lowest Electron State $2S_{1/2}$ $L=0,\ J=1/2$

Adding nuclear spin of proton $I=\frac{1}{2}$

i.e. $\vec{F}=\vec{J}+\vec{I} \Rightarrow F=0,1$

b) $F=0,1 \Rightarrow$ H is boson

c) Protons $I=1/2$ Electrons $S=1/2$

Half integral spin particles are fermions

3a) # quantum states $= \frac{V V_p}{h^3}$

states per volume $= \frac{V_p}{h^3}$

$= \frac{4\pi}{3}\frac{p^3}{h^3}$ where $\frac{p^2}{2m}=\frac{3}{2}k_BT$

$= \frac{4\pi\ (3k_BTm)^{3/2}}{3h^3}$

$= 1\times10^{31}\ m^{-3}$

particles per volume (density) $= \frac{6\times10^{23}}{22.4\times10^{-3}\ m^3}$

$= 2.7\times10^{25}\ m^{-3}$

$\therefore$ n << # states per volume $\Rightarrow$ classical statistics applies.

b) # quantum states per volume = density

$$\frac{4\pi}{3}\frac{(3k_BTm)^{3/2}}{h^3} = n$$

$$3k_BTm = \left(\frac{3nh^3}{4\pi}\right)^{2/3}$$

$$T = \frac{1}{3}\left(\frac{3}{4\pi}\right)^{2/3}\frac{n^{2/3}h^2}{k_B m}$$

$$= \frac{1}{3}\left(\frac{3}{4\pi}\right)^{2/3}\frac{(2.7\times10^{25})^{2/3}(6.63\times10^{-34})^2}{1.38\times10^{-23}\times 4\times1.67\times10^{-27}}$$

$\therefore T = 0.05$ K.

4a) Density of conduction electrons

$$= \frac{8.9 \text{ gm/cm}^3}{64 \text{ gm} \times 1.67\times10^{-24}}$$

$$= 8.3\times10^{22} \text{ cm}^{-3}$$

b) # quantum states per volume $= \frac{1}{h^3}\frac{4\pi}{3}p^3$

$$= \frac{4\pi}{3h^3}(2m_{elect}E)^{3/2}$$

at room temp $E \simeq \frac{1}{40}$ eV. =

$\therefore$ # quantum states per volume $= 8.9\times10^{18}$ cm^{-3} $<$ elect. density

$\therefore$ classical statistics doesn't hold.

5a) $$0 = \frac{dE_{TOT}}{dR}$$

$$= -\frac{N^{5/3}(9\pi\hbar^3/4)^{2/3}}{m_e R^3} + \frac{GM^2}{R^2}$$

$$R = \frac{N^{5/3}(9\pi\hbar^3/4)^{2/3}}{GM^2 m_e}$$

For relativistic case $0 \neq \frac{dE_{TOT}}{dR}$

b) $$p_F = \left(\frac{3\pi^2 N\hbar^3}{V}\right)^{1/3}$$

$$m_e c = \left(\frac{9\pi}{4} N\hbar^3\right)^{1/3}\frac{1}{R} \quad \text{where } V = \frac{4}{3}\pi R^3$$

$$R = \frac{N^{1/3}(9\pi\hbar^3/4)^{1/3}}{m_e c}$$

Using results from a + b gives:

$$\frac{N^{5/3}(9\pi\hbar^3/4)^{2/3}}{GM^2 m_e} = \frac{N^{1/3}(9\pi\hbar^3/4)^{1/3}}{m_e c}$$

$$M^2 = \frac{c\,N^{4/3}(9\pi\hbar^3/4)^{1/3}}{G}$$

electrons in star made mainly of He is:

$$N = \underbrace{\frac{M}{4m_p}}_{\text{He Mass}} \cdot \overset{\uparrow\ \text{electrons/He}}{2}$$

$$\therefore M^2 = \left(\frac{M}{2m_p}\right)^{4/3} \frac{c}{G} \left(\frac{9\pi \hbar^3}{4}\right)^{1/3}$$

$$M^{2/3} = \frac{c}{G} \frac{1}{(2m_p)^{4/3}} \left(\frac{9\pi \hbar^3}{4}\right)^{1/3}$$

$$M_{crit} = \sqrt{\frac{9\pi}{64}} \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{m_p^2}$$

$$= 2.5 \times 10^{30}\ \text{kg}$$

c) $\therefore M_{crit} \simeq M_{sun} \quad (2 \times 10^{30}\ \text{kg})$