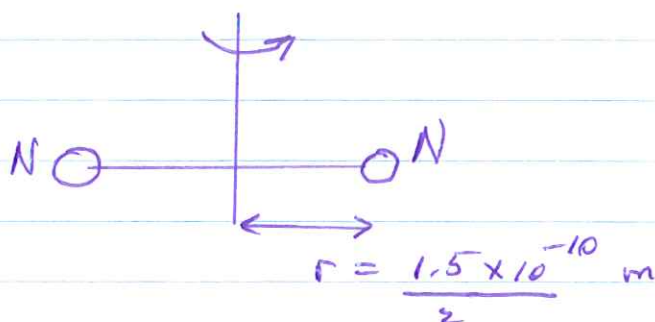


Assignment 9

1a)



$$I = 2 m_N r^2$$

$$= 2 \times 14 \times 1.67 \times 10^{-27} \text{ kg} \left(0.75 \times 10^{-10} \text{ m} \right)^2$$

$$= 2.6 \times 10^{-46} \text{ kg m}^2$$

b) Energy of first excited rotational state is:

$$\Delta E = \frac{L^2}{2I}$$

$$= \frac{(\sqrt{2} h)^2}{2I}$$

$$= \frac{(1.06 \times 10^{-34})^2}{2.6 \times 10^{-46}}$$

$$= 4.3 \times 10^{-23} \text{ J}$$

$$\text{Excitation Probability } P(\Delta E) = e^{-\Delta E/k_B T}$$

$$e^{-1} = e^{-\Delta E/k_B T}$$

$$\therefore T = \frac{\Delta E}{k_B} = \frac{4.3 \times 10^{-23}}{1.38 \times 10^{-23}} = 3 \text{ K}$$

2a) $\vec{v}_{ave} = 0$ i.e. there is no preferred direction of motion

$$b) \quad \bar{v} = \left(\frac{8kT}{\pi m} \right)^{1/2}$$
$$= \left(\frac{8 \times 1.38 \times 10^{-23} \times 300}{\pi \times 28 \times 1.67 \times 10^{-27}} \right)^{1/2}$$

$$= 524 \text{ m/sec.}$$

$$c) \quad v_{RMS} = \sqrt{\frac{3kT}{m}}$$

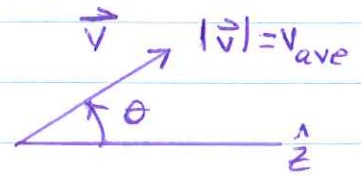
$$= 569 \text{ m/sec}$$

$$d) \quad v_{\text{Most Probable}} = \sqrt{\frac{2kT}{m}}$$
$$= 465 \text{ m/sec.}$$

3a) Consider gas of density n where particles each have speed v_{ave} but velocity is in random direction. Half of particles corresponding to density $\frac{n}{2}$ move in z direction. These particles

have average velocity z component given by:

$$\begin{aligned}\bar{v}_z &= \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi/2} v_z \sin\theta d\theta \\ &= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi/2} v_{ave} \cos\theta \sin\theta d\theta\end{aligned}$$



$$\therefore \bar{v}_z = \frac{v_{ave}}{2}$$

$$\text{Particle Flux in } z \text{ direction} = \frac{n}{2} \cdot \bar{v}_z$$

$$= \frac{n v_{ave}}{4}$$

$$= \frac{2.7 \times 10^{25} \times 524}{4}$$

$$= 3.5 \times 10^{27} \text{ part/m}^2/\text{sec.}$$

$$\text{b) Leak Rate} = 3.5 \times 10^{27} \pi \left(\frac{0.1 \times 10^{-3}}{2} \right)^2$$

$$= 1.1 \times 10^{20} \text{ part/sec.}$$

4a) Mean free path $\lambda = \frac{k_B T}{\sqrt{2} \pi d^2 P}$

$P = \frac{N k_B T}{V} \Rightarrow \lambda = \frac{V}{\sqrt{2} \pi d^2 N}$

$\therefore \lambda$ is independent of temperature.

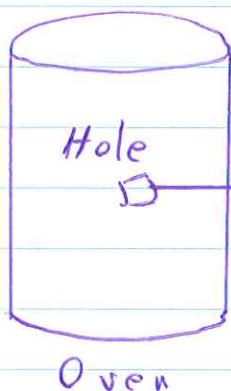
b) Mean time between collisions $\tau = \frac{\lambda}{\bar{v}_{Rel}}$

$\bar{v}_{Rel} \propto T^{1/2} \Rightarrow \tau \propto T^{-1/2}$

\therefore collision frequency $\tau^{-1} \propto T^{1/2}$

If T doubles, τ^{-1} increases by factor of $\sqrt{2}$.

5a)



For small hole, one may assume leak rate is small & doesn't significantly perturb velocity distribution in oven.

Particle flux $J_z \propto v_z P(v_z)$

Probability atom in oven has velocity v_z

$\therefore J_z \propto v_z e^{-mv_z^2/2k_B T}$

b) Velocity Probability Distribution in beam is

$$P(v_z) = N J_z$$

where N is normalization constant defined by

$$\int_0^{\infty} P(v_z) dv_z = 1$$

$$N \int_0^{\infty} v_z e^{-mv_z^2/2k_B T} dv_z = 1$$

$$N \frac{1}{2} \frac{2k_B T}{m} = 1$$

$$N = \frac{m}{k_B T}$$

$$\therefore P(v)_{\text{Beam}} = m\beta v e^{-m\beta v^2/2} \quad \text{where } \beta \equiv \frac{1}{k_B T}$$