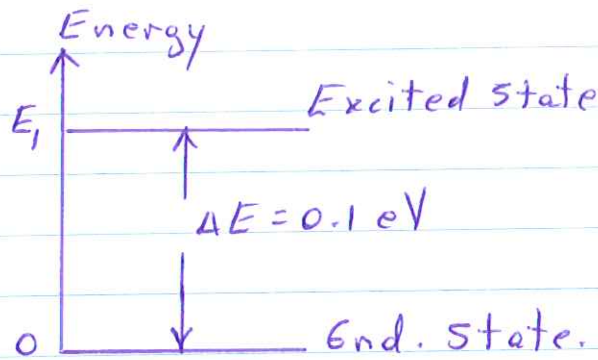
Assignment 8

1)



$$P(E_1) = e^{-\Delta E / k_B T}$$

$$0.01 = e^{-\Delta E / k_B T}$$

$$-\ln 100 = -\frac{\Delta E}{k_B T}$$

$$T = \frac{\Delta E}{k_B\, 2 \ln 10}$$

$$= \frac{0.1\,eV \times 1.6 \times 10^{-19}\, J/eV}{1.38 \times 10^{-23}\, J/K\ \ 2 \ln 10}$$

$$\therefore T = 252\ K.$$

2) Air Density $\rho = \frac{Pm}{k_B T}$ m = air molecule mass

$$= \frac{m}{k_B T} e^{-z/H} \quad \text{where } H \equiv \frac{k_B T}{gm}$$

When $\rho = \frac{\rho(z=0)}{2} \Rightarrow \frac{1}{2} = e^{-z/H}$

$$-\ln 2 = \frac{-z}{H}$$

$$z = H \ln 2$$

$$= \frac{1.38 \times 10^{-23} \times 275}{9.8 \times 5 \times 10^{-26}}$$

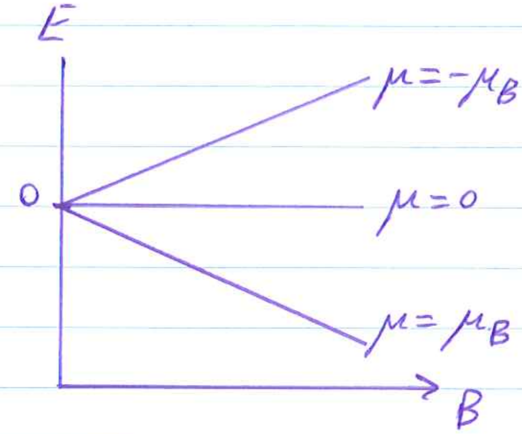
$$= 7745\,m$$

$$\therefore z \simeq 7.7\,km.$$

Hence airplanes must be pressurized for passengers to breathe.

3a) Energy $E = -\vec{\mu} \cdot \vec{B}$

$= \pm \mu_B B, 0$



$$P(+\mu_B) = \frac{e^{\mu_B B / k_B T}}{e^{\mu_B B/k_B T} + 1 + e^{-\mu_B B/k_B T}}$$

$$P(+\mu_B) = \frac{e^{x}}{e^{x} + 1 + e^{-x}} \quad \text{where } x \equiv \frac{\mu_B B}{k_B T}$$

Similarly $$P(0\mu_B) = \frac{1}{e^{x} + 1 + e^{-x}}$$

$$P(-\mu_B) = \frac{e^{-x}}{e^{x} + 1 + e^{-x}}$$

b) $$\bar{\mu} = \mu_B P(\mu_B) + 0\mu_B P(0\mu_B) + (-\mu_B) P(-\mu_B)$$

$$= \mu_B \frac{e^{x}}{e^{x}+1+e^{-x}} + 0 - \mu_B \frac{e^{-x}}{e^{x}+1+e^{-x}}$$

For $x \ll 1$, $e^{x} \simeq 1 + x$.

$$\therefore \bar{\mu} = \mu_B \frac{(1+x)}{3} - \mu_B \frac{(1-x)}{3}$$

$$= \frac{2}{3} \mu_B x$$

$$\bar{\mu} = \frac{2}{3} \frac{\mu_B^2 B}{k_B T}$$

c) Magnetic Moment of entire liquid

$$M = N_A \bar{\mu}$$

$$= \frac{2}{3} \frac{N_A \mu_B^2 B}{k_B T}$$

or $\frac{M}{B} = \frac{C}{T}$ where Curie constant $C = \frac{2}{3} \frac{N_A \mu_B^2}{k_B}$.