# Assignment 7

1a) For one mole $RT = \left(P + \frac{a}{V^2}\right)(V - b)$

$$RT = \left(100\ atm + \frac{5.5\ l^2\ atm/mol^2}{(0.3\ l/mol)^2}\right)(0.3 - 0.03)\ l/mol$$

$$= 43.5\ \frac{atm\ l}{mol} \times 10^5\ \frac{Pa}{atm} \times \frac{1}{10^3}\ \frac{m^3}{l}$$

$$= 4.35 \times 10^3\ J/mol.$$

$$T = \frac{4.35 \times 10^3}{8.314}$$

$$\therefore T = 523\ K$$

b) Isobaric $\Rightarrow$ P is constant

$V_2 = 2V_1$

$$\therefore RT_2 = \left(100 + \frac{5.5}{(0.6)^2}\right)(0.6 - 0.03)\ \frac{atm\ l}{mol}$$

$$= 65.7\ \frac{atm\ l}{mol}$$

$$= 6.57 \times 10^3\ J/mol$$

$$T = 790\ K.$$

c) Isothermal $\Rightarrow$ T is constant

$$100\left(P_2 + \frac{5.5}{0.6^2}\right)(0.6 - 0.07) = 8.314 \times 523$$

$$P_2 = 66.8\ atm$$

2) $\Delta T = \frac{0.25 \, l/mol}{81 \frac{J}{mol\,K}} \left(1 - 2\times 10^{-3} \times 273\right) 1 \, atm$

$= 1.4 \times 10^{-3} \; \frac{atm\, l}{J/K}$

$\therefore \Delta T = 1.4 \times 10^{-1}$ K using 1 atm l = 100 J

3a) $W = \int_{V_i}^{V_f} P \, dV$

$= \int_{V_i}^{V_f} \frac{nRT}{V} \, dV$

$= nRT \ln\left(V_f / V_i\right)$

$W = P_i V_i \ln\left(V_f / V_i\right)$

b) $P_i V_i = P_f V_f \Longrightarrow P_f = \frac{P_i V_i}{V_f}$

c) i. $W = \int_{V_i}^{V_f} P \, dV$ where $P V^{\gamma} = K$ constant & $\gamma = \frac{7}{5}$.

$= K \int_{V_i}^{V_f} V^{-\gamma} \, dV$

$= \frac{K}{-\gamma+1} \left( V_f^{-\gamma+1} - V_i^{-\gamma+1} \right)$

$\therefore W = \left( P_f V_f - P_i V_i \right) \frac{1}{-\gamma+1}$

ii) $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$P_f = P_i \left( \frac{V_i}{V_f} \right)^{\gamma}$

4a) Helmholtz Free Energy $F \equiv E - TS$

$$dF = dE - TdS - SdT$$

$$= TdS - PdV + \mu dN - TdS - SdT$$ using 1st Law of T.D.

$$dF = -SdT - PdV + \mu dN$$

b) $\left(\frac{\partial F}{\partial T}\right)_{V,N} = -S \qquad \left(\frac{\partial F}{\partial V}\right)_{T,N} = -P$

$$\frac{\partial^2 F}{\partial T \partial V} = \frac{\partial^2 F}{\partial V \partial T} \Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

c) $\Delta Q = T \Delta S$

$$= T\left[\left(\frac{\partial S}{\partial T}\right)_V \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \Delta V\right]$$

$$= T\left(\frac{\partial P}{\partial T}\right)_V \Delta V$$ for isothermal process & using Maxwell relation

d) $P V^{10} T^{-5} = K.$

$$\left(\frac{\partial P}{\partial T}\right)_V V^{10} T^{-5} + P V^{10} (-5) T^{-6} = 0.$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{5P}{T}.$$

$$\therefore \Delta Q = T \frac{5P}{T} \Delta V$$

$$= 5 P \Delta V$$

$$= 5 \times 10^5 \, Pa \times (-.01 \times 0.5 \, l) \times 10^{-3} \frac{m^3}{l}$$

$$\Delta Q = -2.5 \, J$$

Note $\Delta Q < 0 \Rightarrow$ heat is produced.

5a) Heat Produced $= 57 \frac{eV}{\text{molecule}} \times \frac{0.7 \, kg}{114 \frac{amu}{\text{molecule}} \times 1.67 \times 10^{-27} \frac{kg}{amu}}$

$$= 2.1 \times 10^{26} \, eV$$

$$= 3.4 \times 10^{7} \, J$$

b) Engine Efficiency $\eta = \frac{6 \, kW \times 20 \, min}{3.4 \times 10^7 \, J} \times 60 \frac{sec}{min}$

$$= \frac{7.2 \times 10^6 \, J}{3.4 \times 10^7 \, J}$$

$$\eta = 21\%$$