Assignment 5

1a) $\Lambda_0 = \Lambda_1 \cdot \Lambda_2 = 8.$

b) $S_1 = k_B \ln \Lambda_1 = k_B \ln 2$

$S_2 = k_B \ln \Lambda_2 = k_B \ln 4$

c) $S_0 = S_1 + S_2$

$= k_B \ln 2 + k_B \ln 4.$

$= k_B \ln 8.$

$= k_B \ln \Lambda_0$

2a) $\Lambda_0 = \Lambda_1 \Lambda_2 = 8 \times 10^{2 \times 10^{24}}$

b) $S_1 = k_B \ln \Lambda_1$

$= k_B \left[\ln 2 + 10^{24} \ln 10 \right]$

$S_2 = k_B \ln \Lambda_2$

$= k_B \left[\ln 4 + 10^{24} \ln 10 \right]$

c) $S_0 = S_1 + S_2$

$= k_B \left[\ln 8 + 2 \times 10^{24} \ln 10 \right]$

3a) A_1 has 20 degrees of freedom.
A_2 " 16 " " "

b) $\Lambda_0 = \Lambda_1 \cdot \Lambda_2$

$$= C_1 E_1^{10} \; C_2 E_2^{8}$$

$$\approx C_1 C_2 \; E_1^{10} \; (E_0 - E_1)^8$$

at =m $\quad 0 = \frac{d\Lambda_0}{dE_1}$

$$= C_1 C_2 \; 10 E_1^9 (E_0 - E_1)^8 - C_1 C_2 E_1^{10} \, 8 (E_0 - E_1)^7$$

$$= 10(E_0 - E_1) - 8E_1$$
$$= 10E_0 - 18E_1$$

$$\therefore E_1 = \frac{5}{9} E_0 \quad \& \quad E_2 = \frac{4}{9} E_0$$

c) $S_0 = S_1 + S_2$

$$= k_B \ln \Lambda_1 + k_B \ln \Lambda_2$$

$$= k_B \left[\ln C_1 + 10 \ln E_1 + \ln C_2 + 8 \ln E_2 \right]$$

d) $\frac{1}{T_1} = \frac{\partial S_1}{\partial E_1}$ $\qquad$ $\frac{1}{T_2} = \frac{\partial S_2}{\partial E_2}$ $\qquad$ $\therefore T_1 = T_2$ as expected.

$= \frac{10 k_B}{E_1}$ $\qquad$ $= \frac{8 k_B}{E_2}$

$= \frac{18 k_B}{E_0}$ $\qquad$ $= \frac{18 k_B}{E_0}$

4) $\Lambda = c E^{R/2}$

$S = k_B \ln \Lambda$

$= k_B \frac{R}{2} \ln E + k_B \ln c$

$\Delta S = S_2 - S_1$

$= \frac{k_B R}{2} \ln 2E - \frac{k_B R}{2} \ln E$

$1 \frac{J}{K} = \frac{k_B R}{2} \ln 2$

$\therefore$ # degrees of freedom $R = \frac{2}{\ln 2} \frac{1}{k_B}$.

5) $\Delta S = k_B \ln E_2^{R/2} - k_B \ln E_1^{R/2}$

$= \frac{k_B R}{2} \ln \left(\frac{E_2}{E_1}\right)$

$= \frac{k_B 10^{24}}{2} \ln (1.10)$

$= 4.8 \times 10^{22} k_B$

6a) $\Delta S = \dfrac{20\ J}{253\ K}$

$= 7.9 \times 10^{-2}\ J/K$

b) $\Delta S = k_B \ln \Omega_f - k_B \ln \Omega_i$

$= k_B \ln \left(\dfrac{\Omega_f}{\Omega_i}\right)$

$\dfrac{\Omega_f}{\Omega_i} = e^{\Delta S / k_B}$

$= e^{7.9 \times 10^{-2} / 1.38 \times 10^{-23}}$

$= e^{5.7 \times 10^{21}}$

$= 10^{2.5 \times 10^{21}}$

7a) $\Delta S = k_B \ln \left(\dfrac{\Omega_f}{\Omega_i}\right)$

$= k_B \ln 2$

$\Delta Q = T \Delta S$

$= 279 K \cdot 1.38 \times 10^{-23} \dfrac{J}{K} \ln 2$

$= 2.67 \times 10^{-21}\ J$

b) Same answer.