Assignment 4.

1a) $\bar{p} = m\bar{v}$

$= 28 \times 1.67 \times 10^{-27}\ kg \cdot 300\ m/sec$

$= 1.4 \times 10^{-23}\ \frac{kg\,m}{sec}$

b) $\#\ states = \frac{V_p\, V}{h^3}$

$= \frac{\frac{4}{3}\pi (2.8 \times 10^{-23})^3 \cdot (3 \times 10 \times 12)}{(6.63 \times 10^{-34})^3}$

$= 1.1 \times 10^{35}$

c) $K.E. = \frac{1}{2} mv^2$

$= \frac{1}{2}\ 28 \times 1.67 \times 10^{-27}\ (300)^2$

$= 2.1 \times 10^{-21}\ J$

d) $K.E_{TOT} = N\ K.E._{one\ molecule}$

$= 8.7 \times 10^{27} \times 2.1 \times 10^{-21}$

$= 1.8 \times 10^{7}\ J$

$N = \frac{PV}{kT}$

$= \frac{10^5 \times 3 \times 10 \times 12}{1.38 \times 10^{-23} \times 300}$

$= 8.7 \times 10^{27}\ molecules$

e) $$1 = \frac{V_p V}{h^3}$$

$$= \frac{4}{3}\pi \frac{p_{min}^3 V}{h^3}$$

$$p_{min} = \left(\frac{3}{4\pi V}\right)^{1/3} h$$

$$= \left(\frac{3}{4\pi \times 3 \times 10 \times 12}\right)^{1/3} 6.63 \times 10^{-34}$$

$$= 5.8 \times 10^{-35} \ \frac{kg\,m}{sec}$$

f) $$\Delta E = \frac{p_{min}^2}{2m}$$

$$= \frac{(5.8 \times 10^{-35})^2}{2 \times 28 \times 1.67 \times 10^{-27}}$$

$$= 3.6 \times 10^{-44} \ J$$

$$\frac{\Delta E}{K.E_{molecule}} = \frac{3.6 \times 10^{-44}}{2.1 \times 10^{-21}}$$

$$= 1.7 \times 10^{-23}$$

2) # states $= 5^{10^{24}}$

3) # states $\Lambda \propto E^{\chi R}$ $\chi = \frac{1}{2}$ for nonrelativistic part.

If $E \to 2E$ then $\Lambda \to \Lambda\, 2^{2 \times 10^{24}}$.

4) # states $\Lambda_1 \propto E_1^{R_1/2} = E_1^{2}$

$\Lambda_2 \propto E_2^{R_2/2} = E_2^{5/2}$

$\Lambda_3 \propto E_3^{R_3/2} = E_3^{3}$

E_1	E_2	E_3	Λ_1	Λ_2	Λ_3	$\Lambda_1\Lambda_2\Lambda_3$
0	0	4	0	0	4^3	0
0	1	3	0	1	3^3	0
0	2	2	0	$2^{5/2}$	2^3	0
0	3	1	0	$3^{5/2}$	1	0
0	4	0	0	$4^{5/2}$	0	0
1	0	3	1	0	3^3	0
1	1	2	1	1	2^3	8
1	2	1	1	$2^{5/2}$	1	$2^{5/2}$
1	3	0	1	$3^{5/2}$	0	0
2	0	2	2^2	0	2^3	0
2	1	1	2^2	1	1	4
2	2	0	2^2	$2^{5/2}$	0	0
3	0	1	3^2	0	1	0
3	1	0	3^2	1	0	0
4	0	0	4^2	0	0	0

$\therefore \Lambda_{TOT} = 17.7$

b) Most probable state is $E_1 = E_2 = 1,\ E_3 = 2$

with probability $\frac{8}{17.7} = 45.3\%$

5) # states $\Lambda_1 \propto E_1^{R_1/2} = E_1^{12\times10^{24}}$

$\Lambda_2 \propto E_2^{R_2/2} = E_2^{10^{25}}$

E_1	E_2	Λ_1	Λ_2	$\Lambda_1\Lambda_2$
0	5	0	$5^{10^{25}} = 10^{7\times10^{24}}$	0
1	4	1	$4^{10^{25}} = 10^{6\times10^{24}}$	$10^{6\times10^{24}}$
2	3	$2^{12\times10^{24}} = 10^{3.6\times10^{24}}$	$3^{10^{25}} = 10^{4.8\times10^{24}}$	$10^{8.4\times10^{24}}$
3	2	$3^{12\times10^{24}} = 10^{5.7\times10^{24}}$	$4^{10^{25}} = 10^{3\times10^{24}}$	$10^{8.7\times10^{24}}$
4	1	$4^{12\times10^{24}} = 10^{7.2\times10^{24}}$	1	$10^{7.2\times10^{24}}$
5	0	$5^{12\times10^{24}} = 10^{8.4\times10^{24}}$	0	0

$\therefore \Lambda_{TOT} = 10^{8.7\times10^{24}}$

$\therefore$ most probable state is $E_1 = 3$, $E_2 = 2$.