Assignment 3

1a) First Law of T.D. with $\Delta E = 0$ gives

Work done on gas = Heat Added $= 10^5$ Joules

b) Gas expands and does work $P\Delta V$.

$$\Delta V = \frac{10^5 \text{ Joules.}}{1 \times 10^5 \text{ Pascal}}$$

$$\therefore \Delta V = 1 \text{ m}^3$$

2. $$\Delta G_{\text{Path1}} = \int_{(1,1)}^{(1,3)} dG + \int_{(1,3)}^{(4,3)} dG$$

$$= \int_{(1,1)}^{(1,3)} x^2\,dy + \int_{(1,3)}^{(4,3)} 3xy\,dx.$$

$$= x^2 y \Big|_{(1,1)}^{(1,3)} + \frac{3x^2y}{2}\Big|_{(1,3)}^{(4,3)}$$

$$= 1.3 - 1.1 + \frac{3.4^2 3}{2} - \frac{3.1^2 3}{2}$$

$$= 2 + 72 - \frac{9}{2}$$

$$\Delta G_{\text{Path1}} = 69\tfrac{1}{2}$$

$$\Delta G_{Path\,3} = \int_{(1,1)}^{(4,1)} dG + \int_{(4,1)}^{(4,3)} dG$$

$$= \int_{(1,1)}^{(4,1)} 3xy\,dx + \int_{(4,1)}^{(4,3)} x^2\,dy$$

$$= \left.\frac{3x^2y}{2}\right|_{(1,1)}^{(4,1)} + \left.x^2y\right|_{(4,1)}^{(4,3)}$$

$$= \frac{3.4^2.1}{2} - \frac{3.1^2.1}{2} + 4^2.3 - 4^2.1$$

$$= 24 - \frac{3}{2} + 48 - 16.$$

$$\Delta G_{Path\,3} = 54\tfrac{1}{2}$$

$$\therefore \Delta G_{Path\,1} \neq \Delta G_{Path\,3}$$

The reason is dG is not an exact differential.
$\therefore$ result of integral is path dependent.

3a) $g = -y \sin x$ $\qquad h = \cos x$.

$\frac{\partial g}{\partial y} = -\sin x$ $\qquad \frac{\partial h}{\partial x} = -\sin x$.

$\therefore$ differential is exact.

b) $\frac{\partial g}{\partial y} = x^3 e^x$ $\qquad \frac{\partial h}{\partial x} = 3x^2 e^x + x^3 e^x$

$\frac{\partial g}{\partial y} \neq \frac{\partial h}{\partial x} \Rightarrow$ inexact differential.

c) $\frac{\partial g}{\partial y} = (1+x)e^x$ $\qquad \frac{\partial h}{\partial x} = e^x + xe^x$

$\therefore$ differential is exact.

d) $\frac{\partial g}{\partial y} = -8x^3 y^{-3}$ $\qquad \frac{\partial h}{\partial x} = -8x^3 y^{-3}$

$\therefore$ differential is exact.

4 a) # quantum states for electron $= \frac{\Delta x \; \Delta p_x}{h}$

$= \frac{10^{-9} m \cdot 2 \times 10^{7} \frac{m}{s} \times 9.67 \times 10^{-31} kg}{6.63 \times 10^{-34} \, J sec}$

$= 27.$

b) # quantum states for human $= \frac{\Delta V \; \Delta V_p}{h^3}$

$= \frac{10^3 m^3 \cdot (70 kg \cdot 1 m/sec)^3}{(6.63 \times 10^{-34} J sec)^3}$

$= 1.2 \times 10^{118}$

5) From notes, number of states is

$$n = \frac{4\pi V}{h^3} \int p^2 \, dp \quad \times 2$$

two types of photon polarization

For photon $E = pc$.

$$\therefore \; n = \frac{8\pi V}{h^3 c^3} \int E^2 \, dE.$$

$$\therefore \; \frac{dn}{dE} = \frac{8\pi V}{h^3 c^3} E^2$$