Assignment 12

1b) $T = 4000\ K \quad \nu_{max} = 2.3 \times 10^{14}\ Hz$

$T = 6000\ K \quad \nu_{max} = 3.6 \times 10^{14}\ Hz$

c) $T = 4000\ K \quad \lambda_{max} = \dfrac{3 \times 10^{8}\ m/sec}{2.3 \times 10^{14}\ Hz}$

$= 1.3 \times 10^{-6}\ m$

$= 1.3\ \mu m$

$T = 6000\ K \quad \lambda_{max} = \dfrac{3 \times 10^{8}}{3.6 \times 10^{14}}$

$= 8.3 \times 10^{-7}\ m$

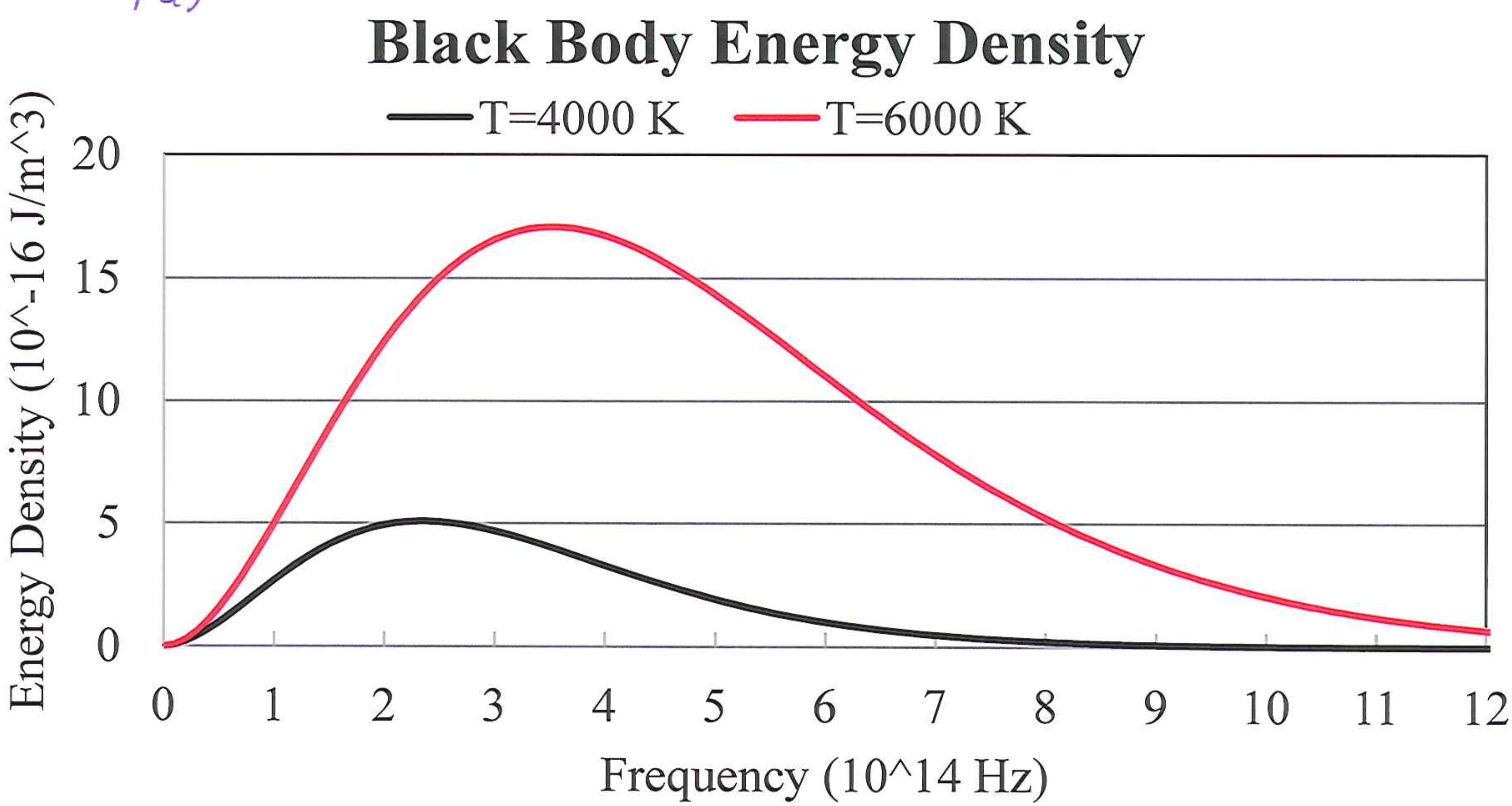
$= 830\ nm$

d) Total Energy Density $U = a T^4 \qquad a = 7.5 \times 10^{-16} \dfrac{J}{m^3 K^4}$

$T = 4000\ K \Rightarrow U = 1.9 \times 10^{-1}\ J/m^3$

$T = 6000\ K \Rightarrow U = 9.7 \times 10^{-1}\ J/m^3$

1a)


Black Body Energy Density
T=4000 K
T=6000 K
Energy Density (10^-16 J/m^3)
20
15
10
5
0
0
1
2
3
4
5
6
7
8
9
10
11
12
Frequency (10^14 Hz)

2a) Energy Radiated by Sun per second is

$$P_{sun} = \sigma\, T_{sun}^4 \cdot 4\pi R_{sun}^2$$

$$= 5.67 \times 10^{-8} \frac{J}{m^2 s K^4} (5800\,K)^2\, 4\pi (7 \times 10^8\, m)^2$$

$$= 4 \times 10^{26}\ J/sec.\ (\text{or Watt})$$

b) Sun's energy per second reaching Earth is

$$P_{s\ at\ Earth} = P_{sun} \frac{\pi R_E^2}{4\pi r_{ES}^2}$$

$$= 4 \times 10^{26}\ Watt\ \frac{\pi (6.4 \times 10^6\, m)^2}{4\pi (1.5 \times 10^{11}\, m)^2}$$

$$= 1.8 \times 10^{17}\ Watt.$$

c) Energy Radiated by Earth per second is

$$P_{Earth} = \sigma\, T_E^4 \cdot 4\pi R_E^2$$

at $= \underline{m} \Rightarrow$ $P_{s\ at\ Earth} = P_{Earth}$

$$1.8 \times 10^{17} = \sigma\, T_E^4\ 4\pi R_E^2$$

$$T_E = \left(\frac{1.8 \times 10^{17}}{\sigma\ 4\pi\ R_E^2}\right)^{1/4} = 280\,K.$$

d) Atmosphere acts as blanket to increase T_E.

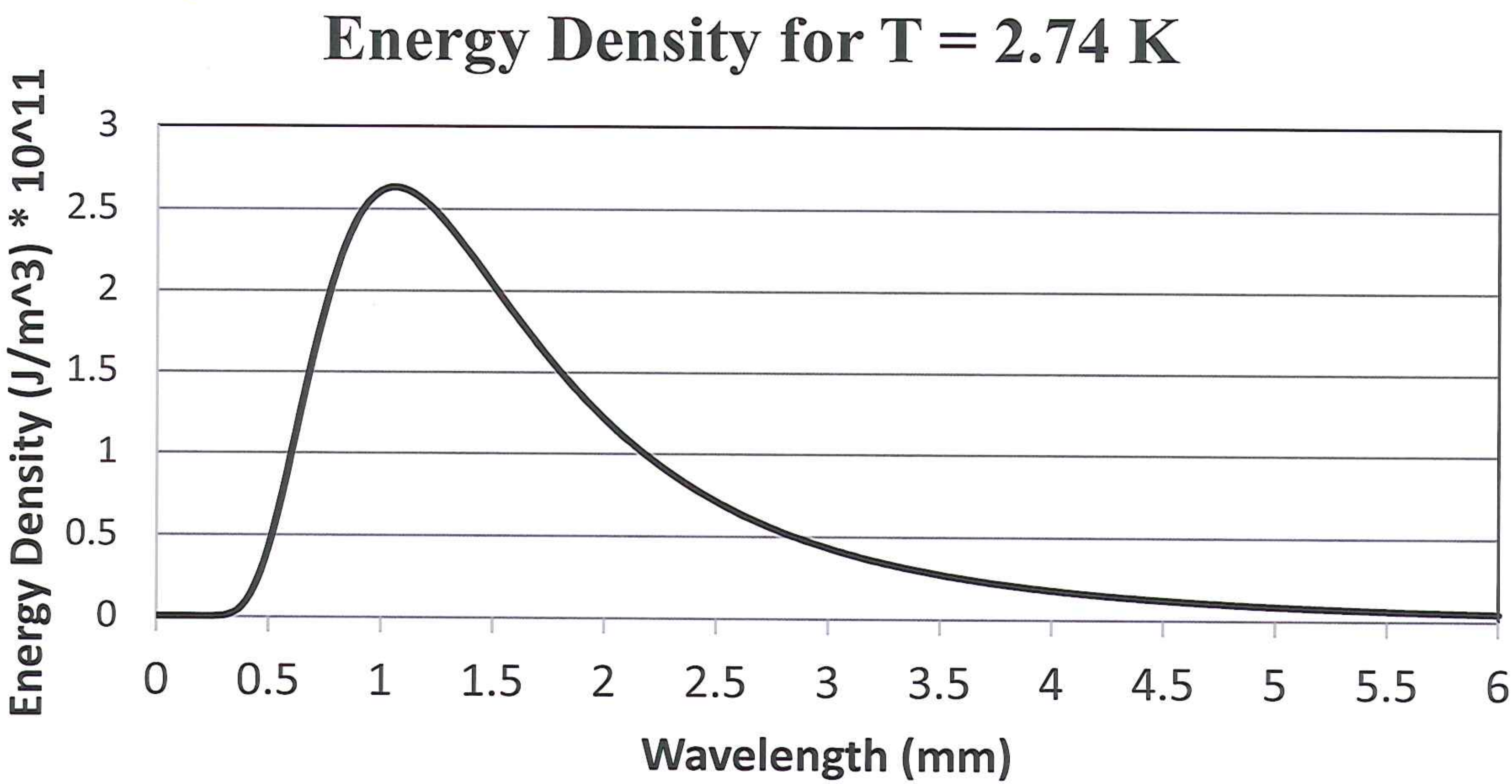
3a) $\left|\frac{dU}{d\lambda}\right| = \left|\frac{dU}{d\nu}\frac{d\nu}{d\lambda}\right|$

$$= \left|\frac{-c}{\lambda^2}\right| \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\beta h\nu}-1}$$

$$= \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\beta hc/\lambda}-1}$$

c) Max. value of $\frac{dU}{d\lambda}$ for $T = 2.74\,K$ occurs at $\lambda = 1.06$ mm.

3b)



Energy Density for T = 2.74 K
Energy Density (J/m^3) * 10^11
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Wavelength (mm)

4a) Photon Energy $h\nu > W$.

$$\therefore \frac{hc}{\lambda_{max}} = W$$

$$\lambda_{max} = \frac{6.63 \times 10^{-34} \text{ Jsec} \times 3 \times 10^{8} \text{ m/sec}}{1 \text{eV} \times 1.6 \times 10^{-19} \text{ J/eV}}$$

$$= 1240 \text{nm}$$

b) From attached plot of solar spectrum $I(\lambda)$ we find:

$$\text{Fraction of light with } \lambda < 1240 \text{m} = \frac{\int_0^{1240\text{nm}} I(\lambda)\, d\lambda}{\int_0^{\infty} I(\lambda)\, d\lambda} = 84\%$$

c) Average wavelength of photons between 0 + 1240nm is:

$$\bar{\lambda} = \frac{\int_0^{1240\text{nm}} \lambda\, I(\lambda)\, d\lambda}{\int_0^{1240\text{nm}} I(\lambda)\, d\lambda} = 663 \text{ nm } (1.9 \text{ eV})$$

$$\therefore \text{Ideal Solar Cell Efficiency} = 84\% \times \frac{1 \text{eV}}{1.9 \text{eV}}$$

$$= 44\%.$$

d) Solar Flux during day

$= \frac{1.8 \times 10^{17}\ W}{\pi R_E^2}$ (See question 2)

$= 1.4\ \frac{kW}{m^2}$

Average Flux over 24 hrs is 0.7 kW/m^2.

Average Power at $10^3\ m^2$ = 700 kW

Electrical Power generated = $0.15 \times \frac{1}{2} \times 700$ ← cloudiness factor

= 53 kW

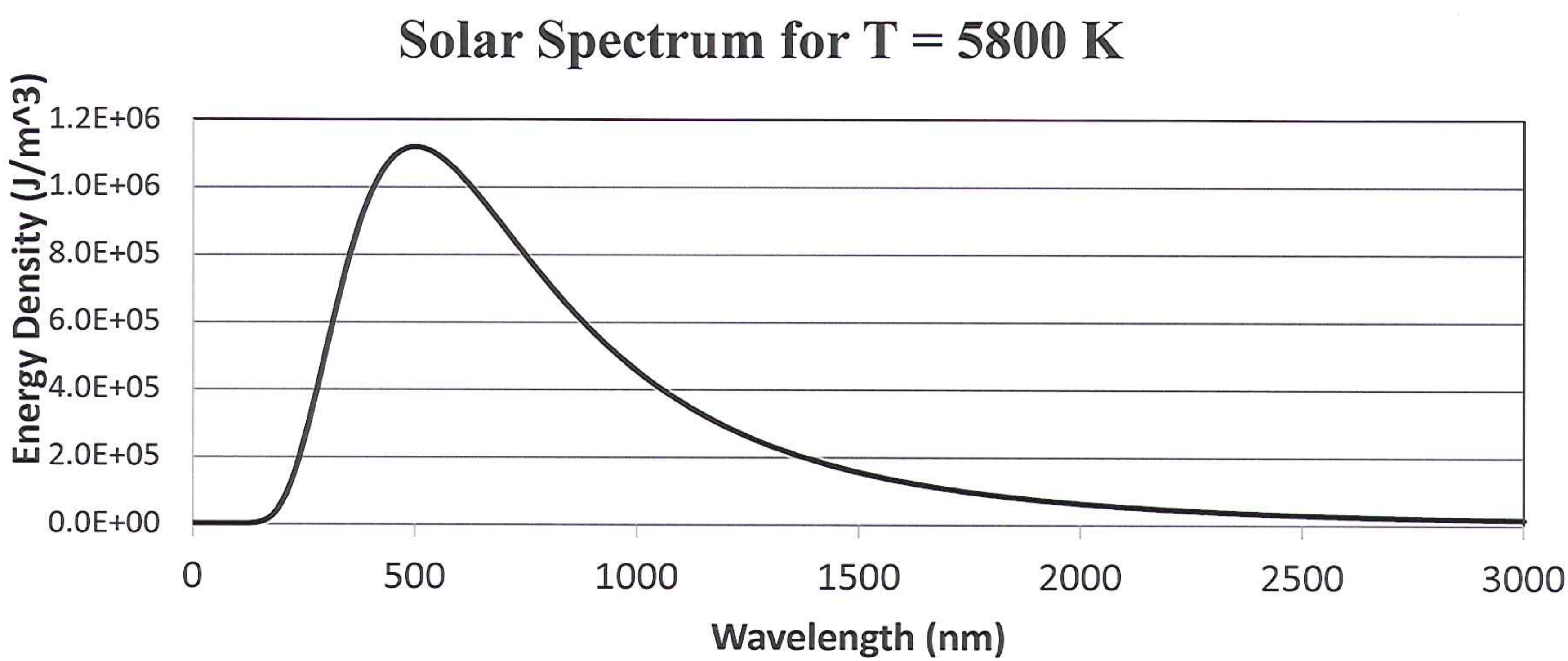
∴ electrical energy generated in 1 year

$= 53\,kW \times 24\ \frac{hr}{day} \times 365\ \frac{day}{yr}$

$= 4.6 \times 10^5$ kW hr/yr

e) at 10 ¢/kW ⟹ \$46,000 per year.

4c)


Solar Spectrum for T = 5800 K
Energy Density (J/m^3)
1.2E+06
1.0E+06
8.0E+05
6.0E+05
4.0E+05
2.0E+05
0.0E+00
0
500
1000
1500
2000
2500
3000
Wavelength (nm)