Assignment 6

1. Drag $D = D(d, v, \mu)$

$[d] = L$, $[v] = \frac{L}{T}$, $[\mu] = \frac{M}{LT}$, $[D] = \frac{ML}{T^2}$

$\left.\begin{array}{l}\#\text{ variables} = 4 \\ \#\text{ dimensions} = 3\end{array}\right\} \Rightarrow \#\ \text{Pi terms} = 4 - 3 = 1$

Let $\Pi_1 = D\, d^a\, v^b\, \mu^c$

$$M^0 L^0 T^0 = \frac{ML}{T^2}\, L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{LT}\right)^c$$

M: $0 = 1 + c \Rightarrow c = -1$

T: $0 = -2 - b - c \Rightarrow 0 = -2 - b + 1 \Rightarrow b = -1$

L: $0 = 1 + a + b - c \Rightarrow 0 = 1 + a - 1 + 1 \Rightarrow a = -1$

$$\therefore \frac{D}{\mu\, v\, d} = C = \text{constant}$$

$$D = C \mu v d.$$

Theoretical result is $D = 3\pi \mu v d$.

2. Define $\lambda \equiv \frac{w_m}{w_p} = \frac{1}{15}$ w_m = model width
w_p = prototype spillway

$\therefore w_m = \frac{20\,m}{15} = 1.33\ m.$

Equating Froude # for model + spillway gives:

$$\frac{v_m}{\sqrt{g_m l_m}} = \frac{v_p}{\sqrt{g_p l_p}}$$

Now $g_m = g \Rightarrow \frac{v_m}{v_p} = \sqrt{\frac{l_m}{l_p}} = \lambda^{1/2}$

Flow rate: $\frac{Q_m}{Q_p} = \frac{v_m A_m}{v_p A_p} = \lambda^{1/2}\lambda^2 = \lambda^{5/2}$

$\therefore Q_m = \left(\frac{1}{15}\right)^{5/2} 125\ \frac{m^3}{sec} = 0.14\ \frac{m^3}{sec}$

Now $\frac{v_p}{v_m} = \frac{l_p/t_p}{l_m/t_m} \Rightarrow \frac{t_m}{t_p} = \frac{v_p}{v_m}\frac{l_m}{l_p}$

$$= \lambda^{-1/2}\lambda$$
$$= \lambda^{1/2}$$

$\therefore t_p = 24$ hr. corresponds to $t_m = \frac{24\ hr}{\sqrt{15}} = 6.2\ hr$

3. In notes, it was shown that for laminar flow volume flow rate is:

$$Q = \frac{\Delta P \,\pi D^4}{128 \mu l}$$

ΔP = pressure drop over distance l
D = pipe diameter
μ = viscosity

Also from notes: $\frac{\Delta P}{l} = \frac{C \mu v}{D^2}$ $\quad C$ = constant

Volume flow rate $Q = A v$

$$= \pi \left(\frac{D}{2}\right)^2 v$$

$$= \frac{\pi D^2}{4} \frac{\Delta P}{l} \frac{D^2}{\mu C}$$

$$= \frac{\Delta P \,\pi D^4}{4C \mu l}$$

Comparing to previous result $\Rightarrow C = 32$

$$\therefore \frac{\Delta P}{\frac{1}{2}\rho v^2} = \frac{32 \mu v}{D^2} \frac{l}{\frac{1}{2}\rho v^2}$$

$$= 64 \frac{\mu}{\rho v D} \frac{l}{D}$$

$$\frac{\Delta P}{\frac{1}{2}\rho v^2} = \frac{64}{Re} \frac{l}{D}$$

4a) $w = k \ln z$

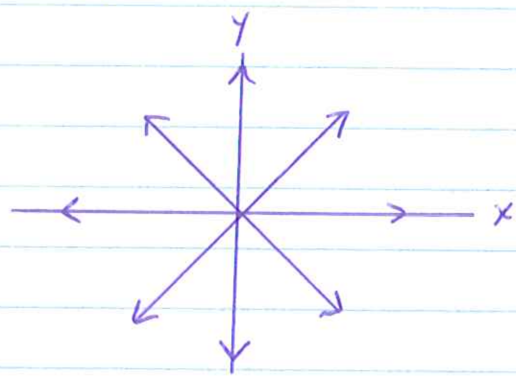
$= k \ln(re^{i\theta})$

$= k \ln r + ik\theta$

$\therefore \psi = k\theta$ & $\phi = k \ln r$.

b) $v_r = \frac{\partial \phi}{\partial r} = \frac{k}{r}$, $v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0$

c)



Plot is for source $k>0$. Sink $k<0$ has flow reversed.

5a) $w = U z^{1/2}$

$= U (re^{i\theta})^{1/2}$

$= Ur^{1/2}\left(\cos\frac{\theta}{2} + i \sin\frac{\theta}{2}\right)$

$\therefore \psi = Ur^{1/2}\sin\frac{\theta}{2}$ & $\phi = Ur^{1/2}\cos\frac{\theta}{2}$

b) $v_r = \frac{\partial \phi}{\partial r} = \frac{U}{2} r^{-1/2} \cos\frac{\theta}{2}$

$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{U}{2} r^{-1/2} \sin\frac{\theta}{2}$

c) Streamlines: $\psi = U r^{1/2} \sin\frac{\theta}{2} = \sqrt{K}$ constant

$$U^2 r \sin^2\frac{\theta}{2} = K.$$

$$U^2 r \left(\frac{1-\cos\theta}{2} \right) = K$$

$$r = \frac{C}{1-\cos\theta} \quad \text{where } C \equiv \frac{K}{2U^2}$$

$$r - r\cos\theta = C$$

$$\sqrt{x^2+y^2} - x = C$$

$$x^2 + y^2 = (C+x)^2$$

$$y^2 = 2Cx + C^2 \quad \text{- Parabola.}$$