# Assignment 4

1. $u = x^2 + y^2 + z^2$
   $v = xy + yz + z$

Continuity Eqn. 

$$0 = \nabla \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2x + x + z + \frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial z} = -x - z$$

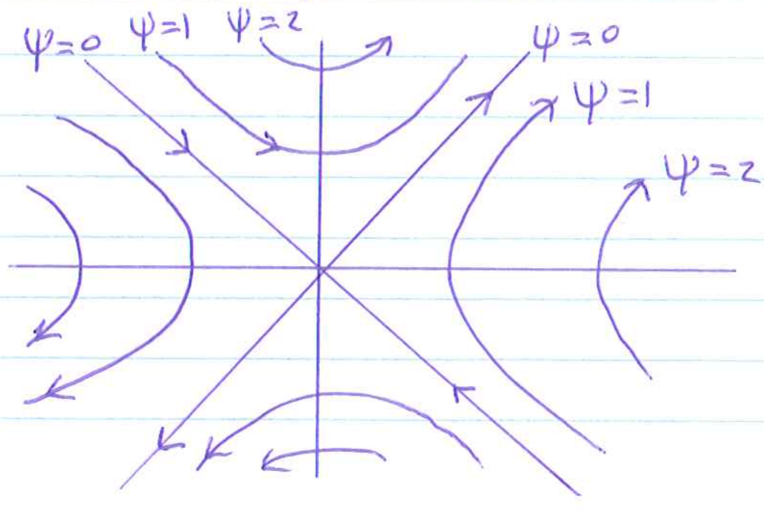
$$\therefore w = -xz - \frac{z^2}{2} + f(x, y)$$

2. $u = \frac{\partial \psi}{\partial y} = 2y \Rightarrow \psi = y^2 + f(x)$
   $v = -\frac{\partial \psi}{\partial x} = 4x \Rightarrow \psi = -2x^2 + g(y)$
   $\Rightarrow \psi = y^2 - 2x^2 + K$

K is a constant. It doesn't affect velocity + set to 0.

$\psi = 0 \Rightarrow 0 = y^2 - 2x^2 \Rightarrow y = \pm\sqrt{2}\,x$ straight lines

$\psi \neq 0 \Rightarrow \frac{y^2}{\psi} - \frac{x^2}{\psi/2} = 1$ Hyperbola



3a) $\psi = 2r^2 \sin 2\theta$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 4r\cos 2\theta$$

$$v_\theta = -\frac{\partial \psi}{\partial r} = -4r\sin 2\theta$$

But: $v_r = \frac{\partial \phi}{\partial r} \Rightarrow \phi = 2r^2\cos 2\theta + f(\theta)$

$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \Rightarrow \phi = 2r^2 \cos 2\theta + g(r)$

$\Rightarrow f(\theta) = g(r) = \text{constant}$

A constant doesn't affect velocity + is set to 0.

$\therefore \phi = 2r^2\cos 2\theta.$

b) Applying Bernoulli equation for streamline connecting 1 + 2 gives:

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2}$$

Now $v^2 = v_r^2 + v_\theta^2 = 16r^2 \Rightarrow v_1^2 = 16\ m^2/s^2$

$v_2^2 = 4\ m^2/s^2$

$$\therefore P_2 = P_1 + \frac{\rho}{2}(v_1^2 - v_2^2)$$

$$= 3\times 10^4\ Pa + \frac{10^3}{2}\frac{kg}{m^3}(16-4)\frac{m^2}{s^2}$$

$$= 36\ kPa$$

4) See notes.