Assignment 3

1. $\vec{v} = \frac{v_0}{l}(x, -y)$

For streamlines: $\frac{dy}{dx} = \frac{v_y}{v_x}$

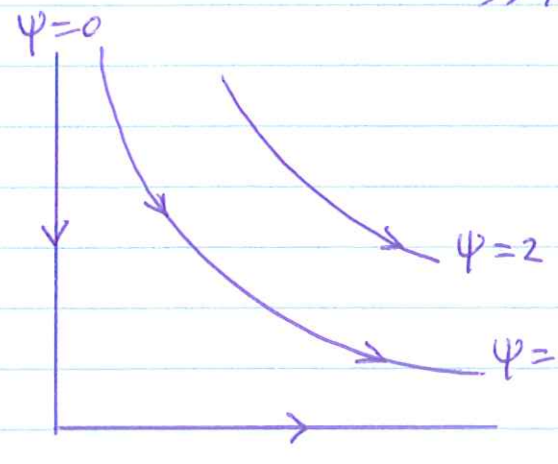
$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dx}{x} + \frac{dy}{y} = 0.$$

$$\therefore \ln x + \ln y = C.$$

$$\ln(xy) = C.$$

$$\therefore xy = \psi = \text{constant}$$



2. $\vec{v} = u_0 \sin \omega\left(t - \frac{y}{v_0}\right) \hat{x} + v_0 \hat{y}$

Streamlines: $\frac{dy}{dx} = \frac{v_y}{v_x}$

$$\frac{dy}{dx} = \frac{v_0}{u_0 \sin \omega\left(t - \frac{y}{v_0}\right)}$$

at $t=0$ $\quad \dfrac{dy}{dx} = \dfrac{-v_0}{u_0 \sin \frac{\omega y}{v_0}}$

$$-u_0 \sin \frac{\omega y}{v_0} = v_0\, dx$$

$$\frac{u_0 v_0}{\omega} \cos \frac{\omega y}{v_0} = v_0 x + K.$$

But $x = y = 0 \Rightarrow \quad \dfrac{u_0 v_0}{\omega} = K.$

$$\therefore \quad \frac{u_0 v_0}{\omega} \cos \frac{\omega y}{v_0} = v_0 x + \frac{u_0 v_0}{\omega}$$

$$x = \frac{u_0}{\omega}\left[\cos \frac{\omega y}{v_0} - 1\right]$$

at $t = \dfrac{\pi}{2\omega}$ $\quad \dfrac{dy}{dx} = \dfrac{v_0}{u_0 \cos \frac{\omega y}{v_0}}$

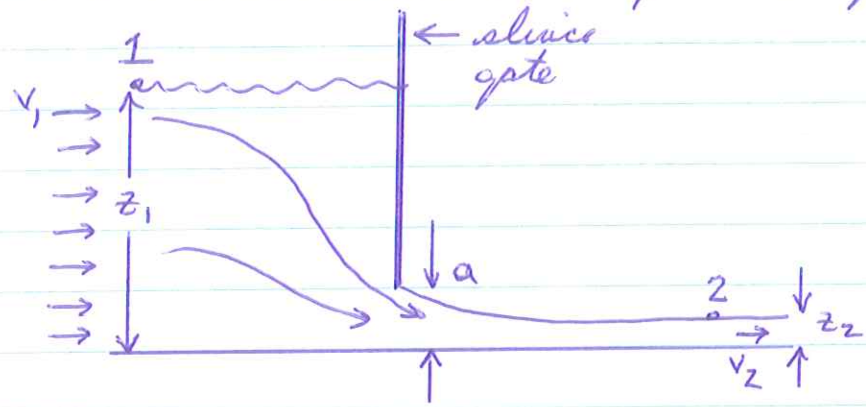
$$u_0 \cos \frac{\omega y}{v_0}\, dy = v_0\, dx$$

$$\frac{u_0 v_0}{\omega} \sin \frac{\omega y}{v_0} = v_0 x + K'$$

But $x = y = 0 \Rightarrow \quad K' = 0.$

$$\therefore \quad x = \frac{u_0}{\omega} \sin \frac{\omega y}{v_0}$$

3. Consider cross section of sluice gate.



Applying Bernoulli Eqn. between points 1 + 2 gives:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

But $P_1 = P_2$: $\quad \frac{V_1^2}{2} + g z_1 = \frac{V_2^2}{2} + g z_2 \quad (1)$

Continuity Eqn: Vol. flow in = Vol flow out

$$Q \equiv A_1 V_1 = A_2 V_2$$

$b z_1 V_1 = z_2 V_2 b \quad (2)$ where b is sluice gate width

Using (1) + (2) one can show volume flow rate

$$Q = z_2 b \sqrt{\frac{2g(z_1 - z_2)}{1 - (z_2/z_1)^2}}$$

Experimentally one finds that for $\frac{a}{z_1} < 0.2$,

$z_2 = C_c a$ where $C_c = 0.61$.

For particular sluice gate in problem,

flow rate per unit sluice gate length $\frac{Q}{b} = 0.61 \times 0.8\,\text{m} \left[\frac{2 \times 9.8\,(5 - 0.8 \times 0.61)}{1 - (0.8 \times 0.61/5)^2} \right]^{1/2}$

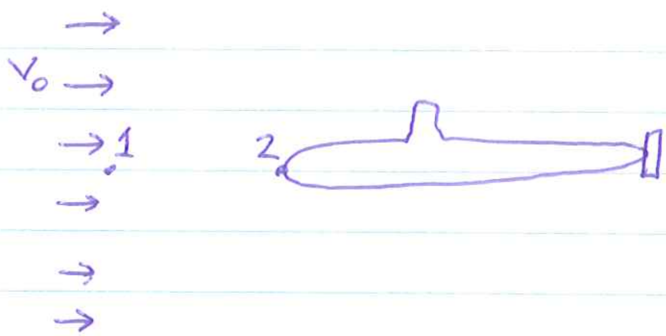
$= 4.61 \ \text{m}^2/\text{sec}.$

Note in limit $z_1 >> z_2 \Rightarrow \frac{Q}{b} = z_2 \sqrt{2 g z_1}$

$= 0.61 \times 0.8 \sqrt{2 \times 9.8 \times 5}$

$= 4.83 \ \text{m}^2/\text{sec}$

4. Consider reference frame where submarine is at rest + water moves as shown.



There is then steady flow and Bernoulli equation can be applied between points 1 + 2 at equal depth.

$$P_2 = P_1 + \frac{\rho v_1^2}{2}$$

$$= \rho g h_{depth} + \frac{\rho v_1^2}{2}$$

$$= 1.03 \times 10^3 \frac{kg}{m^3} \left[9.8 \frac{m}{s^2} \times 50m + \frac{(5 m/s)^2}{2} \right]$$

$$= 5.18 \times 10^5 \ Pa.$$

5. Water Speed $= \frac{1 \ l/sec.}{2 \times 30 \ mm^2} = 16.7 \ m/sec$

Torque $= |\vec{r} \times \vec{F}|$

$$= \left|\vec{r} \times \frac{d\vec{p}}{dt}\right|$$

$$= |\vec{r} \times \vec{v}| \frac{dm}{dt}$$

$$= 0.2 m \times 16.7 \ m/s \times 10^3 \frac{kg}{m^3} \cdot 10^{-3} \frac{m^3}{sec}$$

$$= 3.34 \ Nt \, m$$