Assignment 2

1a) In one step displacement along z axis is

$s_z = s_0 \cos\theta$ where $s_0 = 1\,cm$.

Average displacement $\bar{s}_z = \frac{1}{4\pi}\int s_z \, d\Omega = 0$.

Standard deviation $s_{rms} = \left(\overline{s_z^2} - \bar{s}_z^2\right)^{1/2}$

$$= \frac{s_0}{\sqrt{3}}$$

$$= 0.58\ cm.$$

b) After t sec, and N collisions per sec,

Average displacement $\bar{S}_z = 0$.

Standard deviation $\sigma = \sqrt{Nt}\ s_{rms}$

Probability Distribution $P(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma}\right)^2}$

Probability photon is inside sun is

$$0.68 = \frac{1}{\sqrt{2\pi}}\int_{-R}^{R} \frac{e^{-\frac{1}{2}\left(\frac{z-0}{\sigma}\right)^2}}{\sigma}\, dz.$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-R/\sigma}^{R/\sigma} e^{-z'^2/2}\, dz' \quad \text{where } z' = \frac{z}{\sigma}$$

$$\therefore \frac{R}{\sigma} = 1$$

$$R^2 = Nt \, s_{rms}^2$$

$$t = \frac{3R^2}{N s_0^2}$$

$$= \frac{3 \, (7 \times 10^{10} \, cm)^2}{10^8 \, \frac{coll}{sec} \cdot (1 \, cm)^2}$$

$$= 1.47 \times 10^{14} \, sec$$

$$\therefore t = 4.7 \times 10^6 \text{ years}$$

2a) $\lim\limits_{\substack{N\to\infty \\ p\to 0 \\ Np=\mu}} \dfrac{N!}{k!\,(N-k)!}\, p^k (1-p)^{N-k}$

$= \dfrac{1}{k!} \lim\limits_{\substack{N\to\infty \\ p\to 0}} \dfrac{N!}{(N-k)!} \left(\dfrac{\mu}{N}\right)^k \left(1-\dfrac{\mu}{N}\right)^{N-k}$

$= \dfrac{\mu^k}{k!} \lim\limits_{N\to\infty} \dfrac{N(N-1)\ldots(N-k+1)}{N^k} \left(1-\dfrac{\mu}{N}\right)$

$= \dfrac{\mu^k}{k!} \lim\limits_{N\to\infty} \left(1-\dfrac{\mu}{N}\right)^N \underbrace{\left(1-\dfrac{\mu}{N}\right)^{-k}}_{=1 \text{ as } N\to\infty}$

$= \dfrac{\mu^k}{k!} e^{-\mu}$

b) $\sum\limits_{k=0}^{\infty} \dfrac{\mu^k}{k!} e^{-\mu} = e^{-\mu} \sum\limits_{k=0}^{\infty} \dfrac{\mu^k}{k!}$

$= e^{-\mu} e^{\mu}$

$\therefore \sum\limits_{k=0}^{\infty} P(k) = 1$

c) $\bar{k} = \sum\limits_{k=0}^{\infty} k\,P(k)$

$= \sum\limits_{k=0}^{\infty} k \dfrac{\mu^k}{k!} e^{-\mu}$

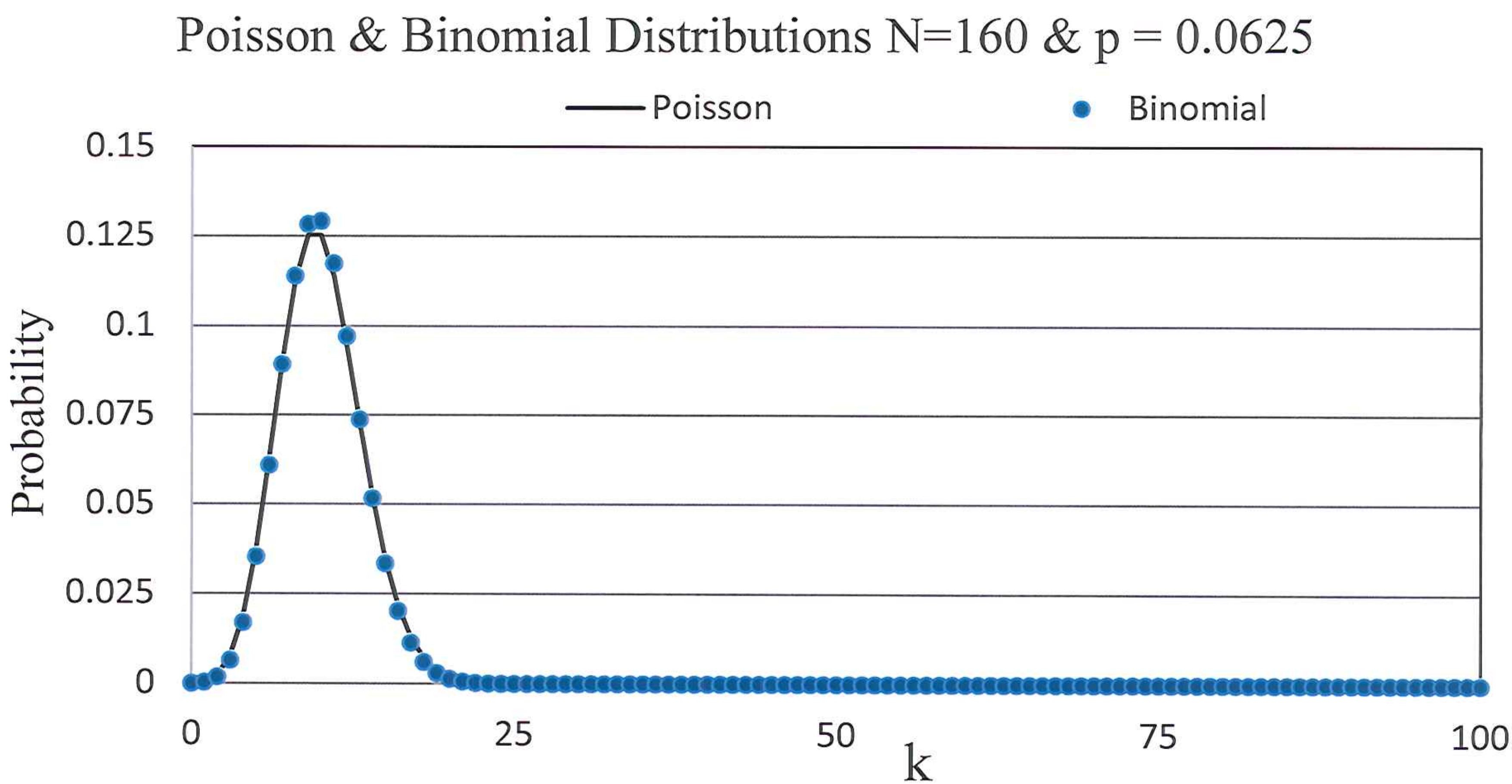
$= e^{-\mu} \sum\limits_{k=1}^{\infty} \dfrac{\mu^k}{(k-1)!}$

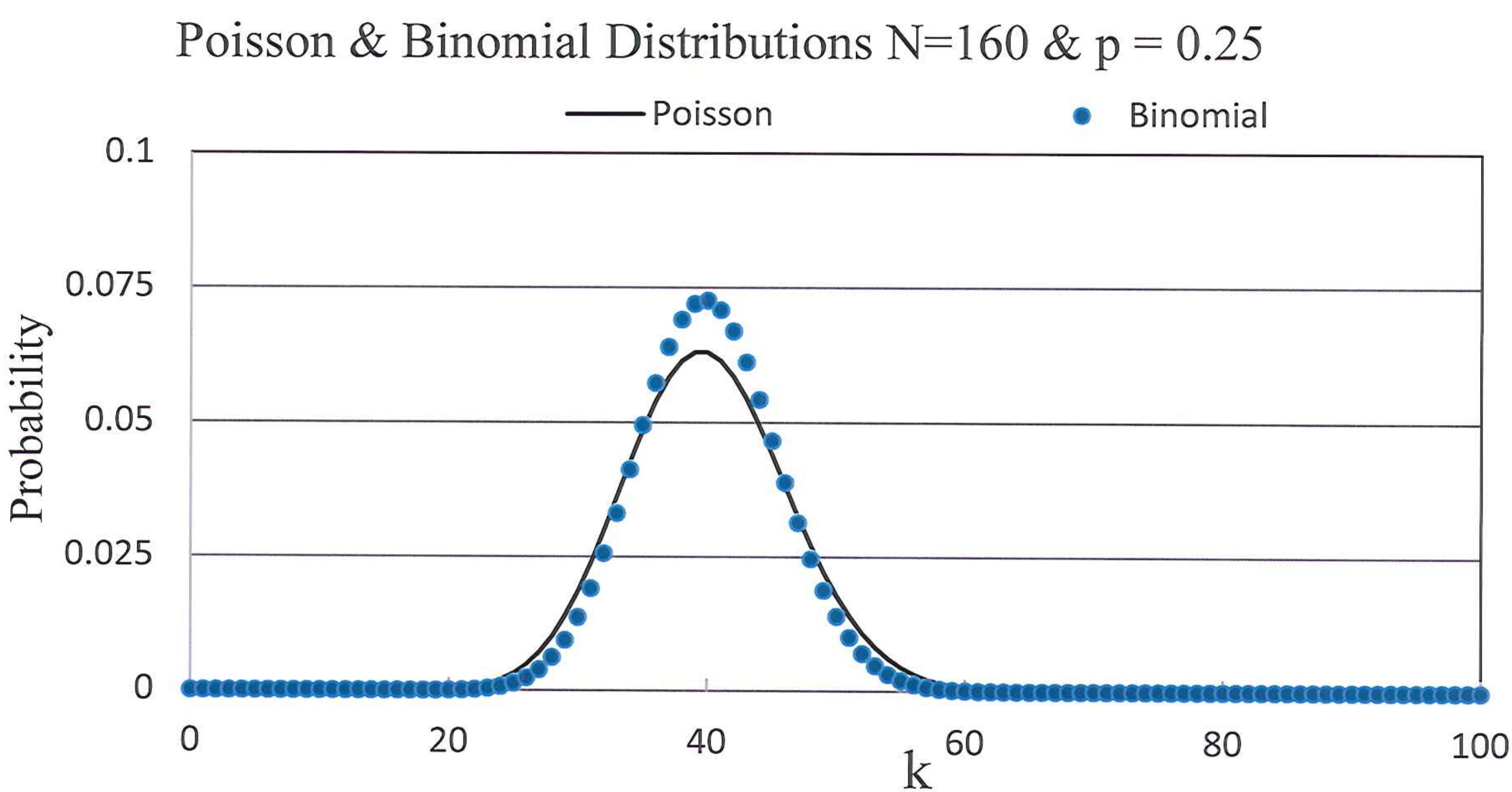
$= e^{-\mu} \sum\limits_{k'=0}^{\infty} \dfrac{\mu^{k'+1}}{k'!}$ where $k' = k-1$

$\rightarrow \bar{k} = \mu e^{-\mu} \sum\limits_{k'=0}^{\infty} \dfrac{\mu^{k'}}{k'!}$

$= \mu e^{-\mu} e^{\mu}$

$\therefore \bar{k} = \mu.$


Poisson & Binomial Distributions N=160 & p = 0.0625
Poisson
Binomial
Probability
0.15
0.125
0.1
0.075
0.05
0.025
0
0
25
50
75
100
k


Poisson & Binomial Distributions N=160 & p = 0.25
Poisson
Binomial
Probability
0.1
0.075
0.05
0.025
0
0
20
40
60
80
100
k

4a) # ^{137}Cs atoms = $\dfrac{10^{-6} \text{ gm.}}{133 \times 1.67 \times 10^{-24} \text{ gm.}}$

$= 4.5 \times 10^{15}$ atoms

b) Decay probability in 1 sec = $1 - e^{-\ln 2\, t / t_{1/2}}$

$\simeq \dfrac{\ln 2}{t_{1/2}}\, t.$

$= \dfrac{\ln 2 \;\; . \; 1 \text{ sec.}}{3 \times 10^{7} \frac{\text{sec}}{\text{yr}} \cdot \times 27 \text{ yrs.}}$

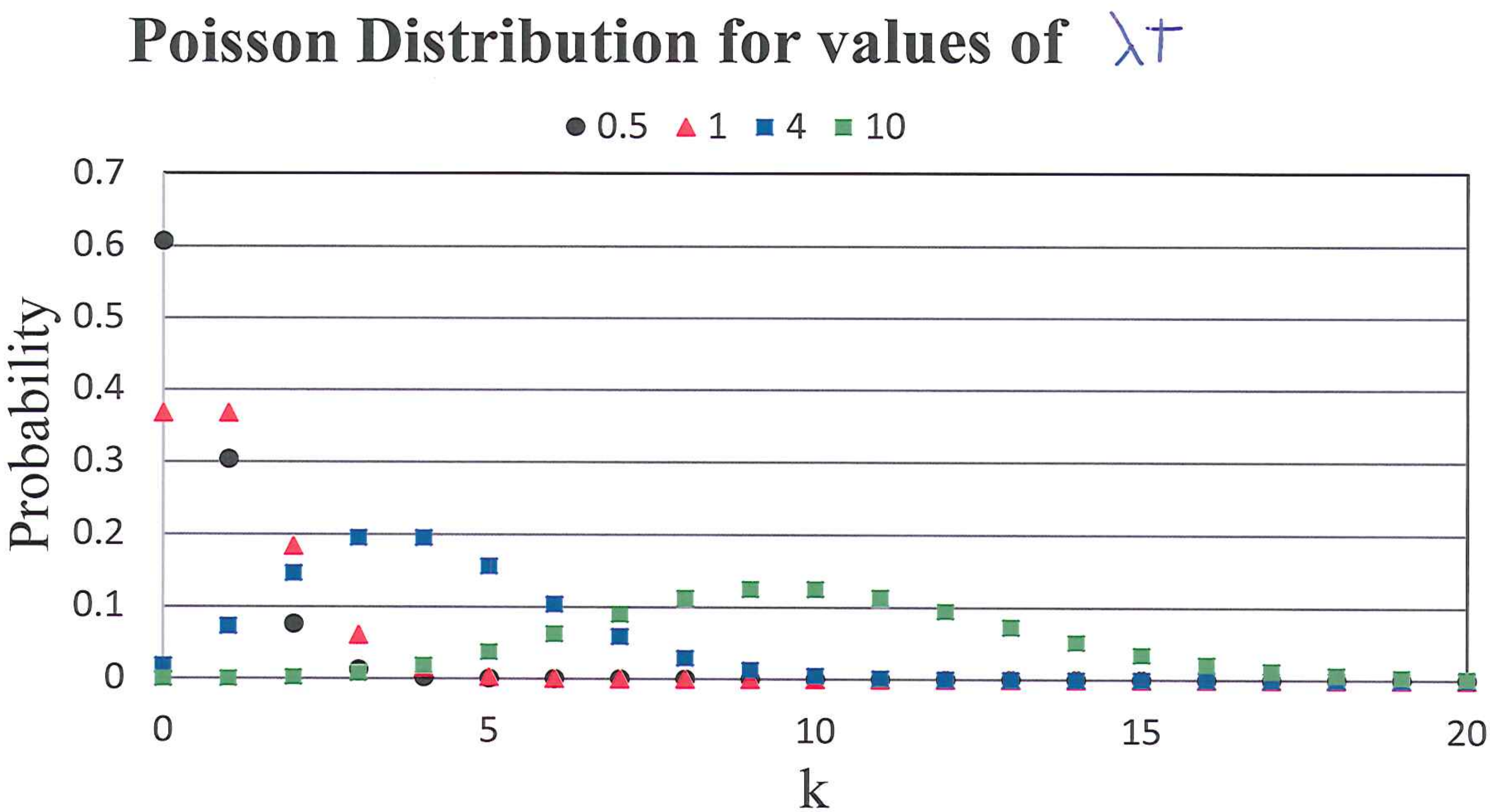
$= 8.6 \times 10^{-10}$

c) $\lim_{\substack{N \to \infty \\ p \to 0}}$ reasonable for $N = 4.5 \times 10^{15}$ + $p = 8.6 \times 10^{-10}$.

d) # Decays per second $\lambda = Np$

$= 4.5 \times 10^{15}$ atoms $\times 8.6 \times 10^{-10} \dfrac{\text{decay prob}}{\text{sec}}$

$= 3.9 \times 10^{6}$ atoms


Poisson Distribution for values of λt
0.5
1
4
10
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Probability
0
5
10
15
20
k