Assignment 5

1. Consider two small interacting systems A_1 and A_2 for which $\Omega_1 = 2$ and $\Omega_2 = 4$.
   a) What is Ω_0?
   b) What are the entropies S_1 and S_2 in terms of Boltzmann's constant?
   c) What is S_0 in terms of Boltzmann's constant?

2. Repeat problem 1 for $\Omega_1 = 2 \times 10^{10^{24}}$ and $\Omega_2 = 4 \times 10^{10^{24}}$

3. Two small nonrelativistic systems A_1 and A_2 are in thermal equilibrium. The number of states accessible to each increases with its energy according to $\Omega_1 = C_1 E_1^{10}$ and $\Omega_2 = C_2 E_2^{8}$ where C_1 and C_2 are constants. The total energy of the combined system is fixed at $E_0 = E_1 + E_2 = 10^{-18}$ J.
   a) How many degrees of freedom have systems A_1 and A_2?
   b) What is the value of E_1 and E_2 when these systems are in equilibrium?
   c) What is the entropy of the combined system in equilibrium?
   d) What is the temperature of this system?

4. If the entropy of a nonrelativistic system changes by 1 J/K when the internal energy is doubled, how many degrees of freedom does the system have?

5. If a nonrelativistic system has 10^{24} degrees of freedom, by how much does the entropy increase when the internal energy is increased by 10%?

6. If you add 20 J of heat to a chunk of ice at -20 C (assume the chunk is big enough so that the temperature doesn't change):
   a) What is the change in entropy of the ice?
   b) By what factor does the number of states available to the ice increase?

7. a) How many joules of heat energy would you have to add to the Atlantic Ocean to double the number of states accessible to it? Assume that T = 4 °C, which is the water's average temperature.
   b) Would the answer be the same if you were dealing with a cup of water at 4 °C instead?