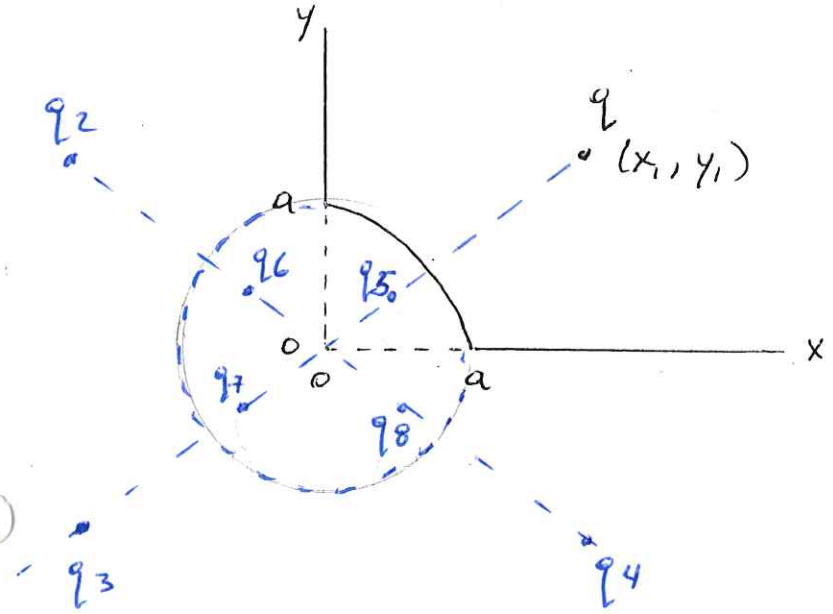
Quiz 3

Name: _____

Total = 20 marks

1. (5 marks) Consider a conducting surface shown below which consists of two semi infinite lanes with a spherical bubble at the corner of radius a. A point charge q is located at position (x1, y1) as shown. Find the potential everywhere.



$d_1 = \sqrt{x_1^2 + y_1^2}$

Consider image charges

$q_2 = -q$ at $\vec{r}_2 = (x_1, -y_1, 0)$

$q_3 = +q$ " $\vec{r}_3 = (-x_1, -y_1, 0)$

$q_4 = -q$ " $\vec{r}_4 = (-x_1, y_1, 0)$

$q_5 = -\frac{a}{d_1} q$ distance from O is a^2/d_1

$q_6 = \frac{a}{d_1} q$ "

$q_7 = -\frac{a}{d_1} q$ "

$q_8 = \frac{a}{d_1} q$ "

$\therefore \Phi(\vec{r}) = \sum_{i=1}^{8} \frac{q_i}{|\vec{r} - \vec{r}_i|}$

2. (5 marks) Consider a conducting sphere of radius R placed in a uniform electric field $\vec{E}_0$.

a) Find the potential at all points exterior to the conducting sphere.

Solution of Laplace's equation for azimuthally symmetric problems is:

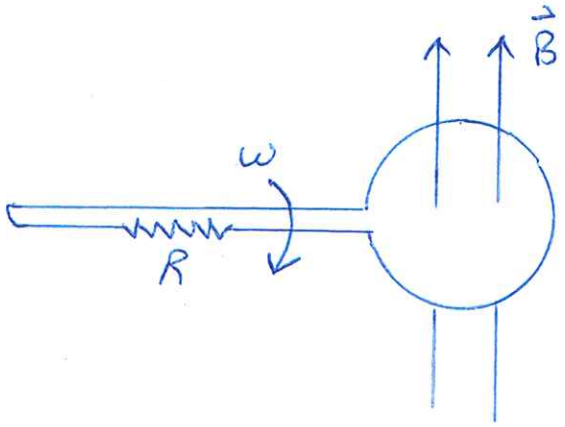
$$\Phi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + B_l r^{-(l+1)} \right) P_l(\cos\theta)$$

See class notes.

b) Find the electric field.

c) Find the surface charge density on the sphere.

3. (5 marks) A conducting loop of wire shown below spins at a rate of 60 Hz in a uniform 10^4 gauss magnetic field. The loop has an area of 10^3 cm^2. Find the current in a 1 kΩ resistor.



Voltage across R is $V = \frac{1}{c}\frac{\partial}{\partial t}\Phi$ ← Mag. Flux through Loop

$= \frac{1}{c} B A \omega$

$\therefore V = \frac{10^4 \text{ gauss} \times 10^3 \text{ cm}^2 \times 2\pi \times 60 \text{ Hz}}{3 \times 10^{10} \text{ cm/sec}}$

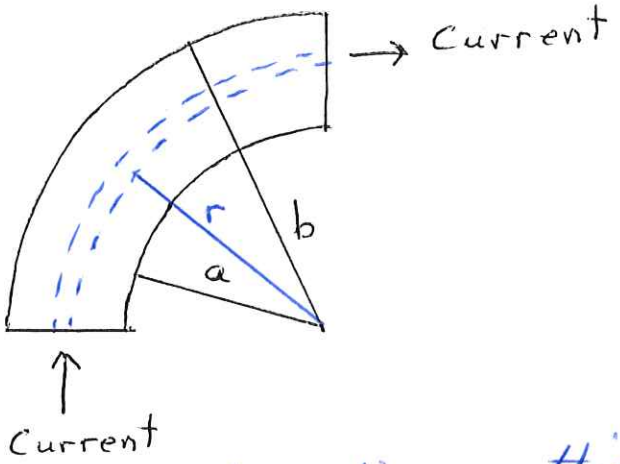
$= 1.3 \times 10^{-1}$ statvolt

$= 37.7$ volt

$\therefore$ current through R is $I = \frac{V}{R}$

$= 3.77 \times 10^{-2}$ amps.

4. (5 marks) A resistor is made of a rectangular slab of material of resistivity ρ that is bent as shown below. It has width w perpendicular to the page. Find at expression for its resistance.



Consider arc of radius r & thickness dr.

Resistance of arc $dR = \rho \frac{2\pi r}{4} \frac{1}{w\, dr}$

Adding parallel arcs of resistance gives:

$$\frac{1}{R_{TOT}} = \int_a^b \frac{1}{dR}$$

$$= \int_a^b \frac{4}{2\pi r \rho} w\, dr$$

$$\frac{1}{R_{TOT}} = \frac{2}{\pi} \frac{w}{\rho} \ln(b/a)$$

$$\therefore R_{TOT} = \frac{\pi \rho}{2w \ln(b/a)}$$