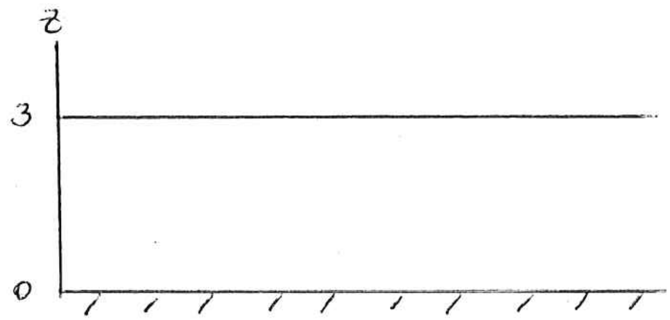
Quiz 2

Name: ______________________________

Total = 20 marks

1. (5 marks) Find the electric field everywhere due to 1) an infinite conducting plane located in the xy plane having surface charge density -4 esu/cm^2 and 2) an insulating plane with negligible thickness at height z = 3 cm having charge density +2 esu/cm^2.



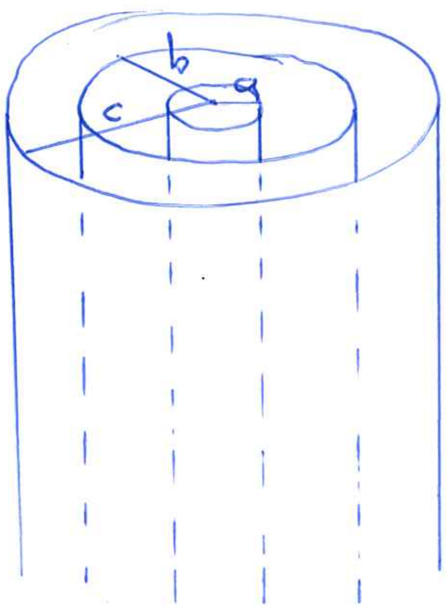
$\vec{E} = \vec{E}_{Ins. Plane} + \vec{E}_{Cond. Plane.}$ for $z > 0$.

$\vec{E}_{Ins\ Plane} = \begin{cases} \hat{z}\ 2\pi(2) = 4\pi \frac{esu}{cm^2}\ \hat{z} & \text{for } z > 3 \\ -4\pi\ \hat{z} & \text{for } z < 3 \end{cases}$

$\vec{E}_{Cond.\ Plane} = \hat{z}\, 4\pi(-4) = -16\pi \frac{esu}{cm^2}\ \hat{z}$ for $z > 0$.

$\therefore\ \vec{E} = \begin{cases} -\hat{z}\ 12\pi & z > 3 \\ -\hat{z}\ 20\pi & 3 > z > 0 \\ 0 & z < 0 \end{cases}$

3. (5 marks) An infinitely long insulating cylinder of radius a has charge density $\rho_o\, r / a$ where r is the cylindrical radial coordinate. It is surrounded by a grounded conducting cylindrical shell having inner radii b and outer radii c. Find the electric field everywhere.



Consider cylinder of radius r & length l.

$$\int \vec{E} \cdot d\vec{a} = 4\pi \int \rho \, dV.$$

By symmetry $\vec{E} = \hat{r}\, E(r)$

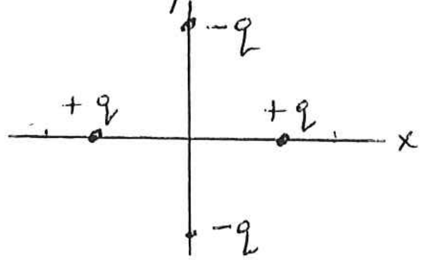
$$2\pi r l\, E(r) = 4\pi l \int_0^r \frac{\rho_o r}{a}\, 2\pi r\, dr$$

$$\therefore E(r) = \frac{4\pi}{3} \frac{\rho_o r^2}{a} \text{ for } r < a.$$

For $b > r > a$ $\quad E(r) = \frac{4\pi}{3} \rho_o \frac{a^2}{r}$

For $r > b$ $\quad E(r) = 0.$

4. (5 marks) An electric quadrupole is defined by the 4 point charges shown below in the xy plane.



The distance of each charge from the origin is L/2. The electrostatic potential at points far from the charges i.e. r >> L is given by the following expression.

$$\Phi(r,\theta,\phi) = \frac{3}{4} q L^2 \frac{\sin^2\theta \cos 2\phi}{r^3}$$

where r, θ and ϕ are spherical coordinates.

a) Find the electric field using:

$$\nabla\Phi = \hat{r}\frac{\partial\Phi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$E_r = -\frac{\partial\Phi}{\partial r} = \frac{9}{4} q L^2 \frac{\sin^2\theta\cos 2\phi}{r^4}$$

$$E_\theta = -\frac{1}{r}\frac{\partial\Phi}{\partial\theta} = -\frac{3}{2} q L^2 \frac{\sin\theta\cos\theta\cos 2\phi}{r^4}$$

$$E_\phi = \frac{-1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi} = \frac{3}{2} q L^2 \frac{\sin\theta\sin 2\phi}{r^4}$$

b) Find the electric field in Cartesian coordinates long the x axis.

On x axis: $\theta = \frac{\pi}{2}$, $\phi = 0$. and $r = x$, $\hat{r} = \hat{x}$

$\therefore E_r = \frac{9}{4} \frac{qL^2}{r^4}$

$E_\theta = 0$

$E_\phi = 0$.

$\therefore \vec{E} = \left(\frac{9}{4} \frac{qL^2}{x^4}, 0, 0 \right)$