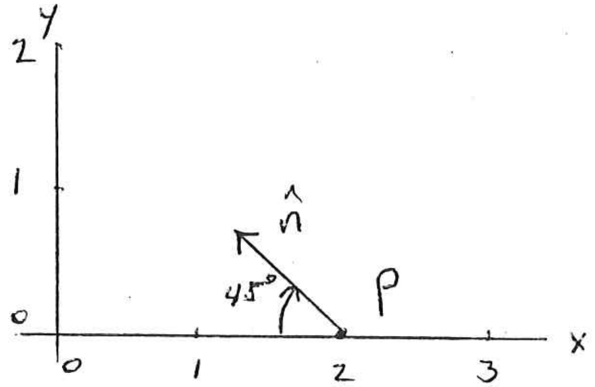
Quiz 1

Name: ______________________________

Total = 20 marks

1. (4 marks) A charge of +2 esu is located at the origin. Find the location of a charge of -1 esu if the electric field measured at point P (2, 0) where distances are in units of centimeters is in the direction $\hat{n}$ and has a magnitude of 3 esu/cm^2.



Let position of -1 esu be (x, y).

Electric field at P is $2\left(\frac{1}{2^2}, 0\right) - \frac{(2-x, -y)}{[(2-x)^2+y^2]^{3/2}} = 3\left(\frac{-1}{\sqrt{2}}, \frac{+1}{\sqrt{2}}\right)$

x eqn. $\frac{1}{2} - \frac{(2-x)}{[(2-x)^2+y^2]^{3/2}} = \frac{-3}{\sqrt{2}} \Rightarrow \frac{(2-x)}{[(2-x)^2+y^2]^{3/2}} = 2.62$ (1)

y eqn. $\frac{y}{[(2-x)^2+y^2]^{3/2}} = \frac{3}{\sqrt{2}} \Rightarrow \frac{y}{[(2-x)^2+y^2]^{3/2}} = 2.12$ (2)

(1) ÷ (2) $\Rightarrow \frac{2-x}{y} = 1.24$.

Subst. for x into (2) $\Rightarrow \frac{y}{[(1.24y)^2+y^2]^{3/2}} = 2.12.$

$\Rightarrow y = 0.34.$

$\Rightarrow x = 1.58.$

2. (6 marks) A proton with initial speed $v_o\,\hat{x}$ enters a region of length L having uniform electric field $\vec{E} = E_o\,(\hat{x} + \hat{y})$

a) Find an expression for the deflection in the $\hat{y}$ direction after the proton has traversed the region with the electric field.

b) Evaluate the deflection if v_o is 1% of the speed of light and $E_o = 10$ esu/cm^2 and L = 10 cm.

$$m\frac{d\vec{v}}{dt} = q\vec{E}$$

$$\therefore \quad m\frac{dv_x}{dt} = qE_o \quad \& \quad m\frac{dv_y}{dt} = qE_o$$

$$v_x = v_o + \frac{qE_o}{m}t \qquad v_y = \frac{qE_o}{m}t$$

$$x = v_o t + \frac{qE_o}{2m}t^2 \qquad y = \frac{qE_o}{2m}t^2$$

Time to traverse distance L is: $L = v_o t + \frac{qE_o}{2m}t^2$

$$0 = \frac{qE_o}{2m}t^2 + v_o t - L$$

$$t = \frac{-v_o \pm v_o\left(1 + \frac{2LqE_o}{mv_o^2}\right)^{1/2}}{qE_o/m}$$

$$\simeq \frac{L}{v_o} \text{ for low } E_o.$$

$$\therefore \text{deflection } y = \frac{qE_o}{2m}\frac{L^2}{v_o^2}$$

$$= \frac{4.8\times10^{-10}\,\text{esu} \times 10\,\text{esu/cm}^2 \times (10\,\text{cm})^2}{2\times1.67\times10^{-24}\,\text{gm} \times (3\times10^{8}\,\text{cm/sec})^2}$$

$$= 1.6\text{ cm}$$

3. (2 marks) Find the total charge on a sphere of radius R having charge density $\rho = \rho_o\, r / R$ where r is the spherical radial coordinate.

Total Charge $Q = \int_0^R \int_0^{2\pi} \int_0^{\pi} \rho \; r^2 \sin\theta \, d\theta \, d\phi \, dr$

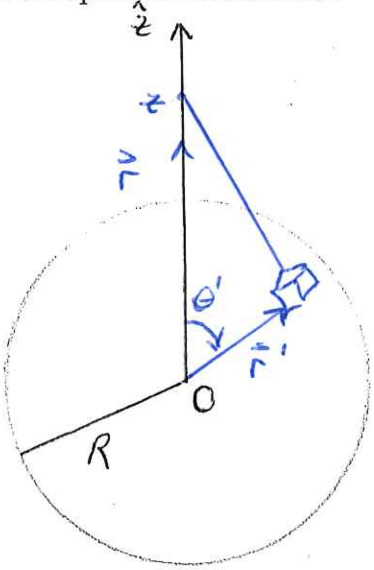
$$= \frac{4\pi \rho_0}{R} \int_0^R r^3 \, dr$$

$$= \frac{4\pi \rho_0}{R} \frac{R^4}{4}$$

$$= \pi \rho_0 R^3$$

4. (8 marks) This problem asks you to derive an expression for the electric field outside a uniformly charged sphere just using Coulomb's law i.e. NOT using Gauss Law. For computation ease you may wish to find the electric field on the z axis of a sphere of radius R centered about the origin.

Hint: To do one of the integrals you may wish to define $s^2 = z^2 + r^2 - 2rz \cos\theta$ where r and θ are spherical coordinates.



$\vec{r} = (0, 0, z)$

Consider infinitesimal charge at $\vec{r}' = r'(\sin\theta' \cos\phi', \sin\theta' \sin\phi', \cos\theta')$

$d^3r' = r'^2 \sin\theta' \, dr' \, d\theta' \, d\phi'$

$$\vec{E}(\vec{r}) = \iiint \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,\rho(\vec{r}')\,d^3r'$$

$$\vec{E}(0,0,z) = \hat{z}\rho \int_0^R \int_0^\pi \frac{z - r'\cos\theta'}{(z^2+r'^2-2r'z\cos\theta')^{3/2}}\, 2\pi\, r'^2 \sin\theta'\, d\theta'\, dr'$$

Now $s^2 = z^2 + r'^2 - 2r'z\cos\theta' \Rightarrow s\,ds = r'z\sin\theta'\,d\theta'$

$\theta = 0 \Rightarrow s^2 = z^2 + r'^2 - 2r'z \Rightarrow s = z - r'$

$\theta = \pi \Rightarrow \qquad s = z + r'$

Also $r'\cos\theta' = \dfrac{z^2 + r'^2 - s^2}{2z}$

$$\therefore \vec{E}(0,0,z) = \hat{z}\, 2\pi\rho \int_0^R \int_{z-r'}^{z+r'} \frac{\left[z - \frac{(z^2+r'^2-s^2)}{2z}\right]}{s^3}\, \frac{s\,ds}{r'z}\, r'^2\, dr'$$

$$= \frac{\hat{z}\, 2\pi\rho}{2z^2} \int_0^R \int_{z-r'}^{z+r'} \frac{z^2 - r'^2 + s^2}{s^2}\, ds\, r'\, dr'$$

$$= \hat{z}\, \frac{\pi\rho}{z^2} \int_0^R \left[s + (z^2-r'^2)\left(\frac{-1}{s}\right)\right]_{z-r'}^{z+r'} r'\, dr'$$

$$= \hat{z}\, \frac{\pi\rho}{z^2} \int_0^R \left\{2r' + (z^2-r'^2)\left[\frac{-1}{z+r'} + \frac{1}{z-r'}\right]\right\} r'\, dr'$$

$$= \hat{z}\, \frac{\pi\rho}{z^2} \int_0^R 4\, r'^2\, dr'$$

$$= \hat{z}\, \frac{4\pi R^3 \rho}{3\, z^2}$$

$$= \hat{z}\, \frac{Q}{z^2} \quad \text{where } Q \equiv \tfrac{4}{3}\pi R^3 \rho.$$