Quiz 10

Name: ______________________________

Total = 20 marks

1. (10 marks) The scalar and vector potentials generated by an oscillatory electric dipole aligned along the z axis are given as follows in the radiation zone.

$$\Phi = -\frac{p_0 \omega}{c} \frac{\cos\theta}{r} \sin\omega\left(t - \frac{r}{c}\right)$$

$$\vec{A} = \frac{-p_0 \omega}{rc} \sin\omega\left(t - \frac{r}{c}\right)\left(\cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}\right)$$

a) What approximations were made regarding the dipole size, wavelength and distance to the observer?

$r \gg \lambda \gg s$ ← dipole size

r ↑ distance to observer

λ ↑ wavelength

b) Find the electric and magnetic fields.

$$\vec{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$= -p_0 \frac{\omega^2}{c^2} \frac{\sin\theta}{r} \cos\omega\left(t - \frac{r}{c}\right)\hat{\theta}$$

$$\vec{B} = \nabla \times \vec{A}$$

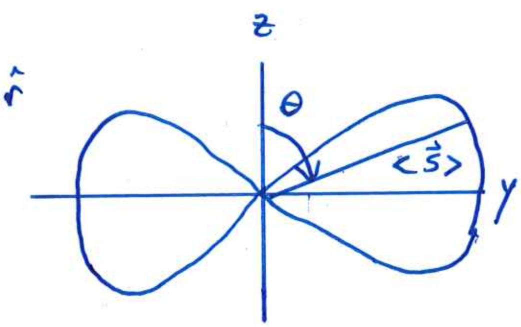
$$= -p_0 \frac{\omega^2}{c^2} \frac{\sin\theta}{r} \cos\omega\left(t - \frac{r}{c}\right)\hat{\phi}$$

c) Find the time averaged Poynting vector. Sketch the angular dependence where θ is the angle between the dipole and the direction to the observer.

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$$

$$= \frac{c}{4\pi} p_o^2 \frac{\omega^4}{c^4} \frac{\sin^2\theta}{r^2} \cos^2\omega\left(t - \frac{r}{c}\right) \hat{r}$$

$$\langle \vec{S} \rangle = \frac{1}{8\pi} p_o^2 \frac{\omega^4}{c^3} \frac{\sin^2\theta}{r^2} \hat{r}$$



d) A photomultiplier detects the radiation. Compare the signals of a photomultiplier detecting light emitted in the z direction compared to the x directions.

No radiation is emitted along $\hat{z}$ direction.

e) Derive an expression for the total radiated power.

$$P = \int_{\substack{\text{surface}\\ \text{of sphere}}} \langle \vec{S} \rangle \cdot d\vec{o}$$

$$= \frac{p_o^2}{8\pi} \frac{\omega^4}{c^3} \int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{p_o^2}{8\pi} \frac{\omega^4}{c^3} 2\pi \int_0^\pi \sin^3\theta \, d\theta$$

$$P = \frac{p_o^2}{3} \frac{\omega^4}{c^3}$$

2. (4 marks) Make an argument using equations as appropriate showing by what factor (which you should evaluate) the power resulting from electric dipole radiation is greater/smaller than magnetic dipole radiation.

$$P_{ED} = \frac{p_0^2 \omega^4}{3c^3} \qquad P_{MD} = \frac{m_0^2 \omega^4}{3c^3}$$

$$\therefore \frac{P_{MD}}{P_{ED}} = \frac{m_0^2}{p_0^2}$$

$$= \frac{(\pi a^2 q_0 \omega)^2}{q_0^2 a^2 c^2} \qquad \text{where } m_0 = \frac{\pi a^2 I_0}{c} \text{ \& } I_0 \sim q_0 \omega, \quad p_0 = q_0 a.$$

$$\sim \left(\frac{a\omega}{c}\right)^2$$

$$\sim \left(\frac{a}{\lambda}\right)^2$$

$\ll 1$ since $a \sim 1\,\text{Å}$ & $\lambda_{\text{visible}} \sim 500$ nm.

3. (6 marks) Bremmstrahlung: A proton travelling at 0.1 c hits a heavy nucleus.

a) Assuming it decelerates linearly in a distance of a nuclear radius (10^{-13} cm) find how much energy is radiated.

Power $P = \frac{2}{3} \frac{e^2 a^2}{c^3}$

Acceleration $a = -\frac{0.1c}{\Delta t}$

Braking Distance $s = \frac{1}{2} a (\Delta t)^2$

$\Rightarrow a = \frac{0.01c^2}{2s}$

$\therefore P = \frac{2}{3} \frac{(4.8 \times 10^{-10}\,esu)^2}{(3 \times 10^{10}\,cm/s)^3} \cdot \frac{10^{-4} (3 \times 10^{10}\,cm/s)^4}{4\,(10^{-13}\,cm)^2}$

$= 1.15 \times 10^{13}$ erg/s

Energy Radiated $E = P \Delta t$

$a = \frac{0.01 c^2}{2s} = 4.5 \times 10^{31}\,cm/s^2 \Rightarrow \Delta t = \sqrt{\frac{2s}{a}} = 6.7 \times 10^{-23}$ sec.

$\therefore E = 7.67 \times 10^{-10}$ erg.

b) What would be the wavelength of a photon that contained all this energy?

$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

$= \frac{6.63 \times 10^{-27}\,erg\,sec \times 3 \times 10^{10}\,cm/sec}{7.67 \times 10^{-10}\,erg.}$

$= 2.6 \times 10^{-7}$ cm

$\therefore \lambda = 26$ Å $\Rightarrow$ X-ray