Assignment 6 Solutions

1. solid angle of open net $= \frac{2}{3} \times \frac{3 \text{ meters} \times 2 \text{ meters}}{(30 \text{ meters})^2}$

$= 4.4 \times 10^{-3}$ steradians

2. Total Scatter Rate = # nuclei in target × incoming beam intensity I × σ_{TOT} (πa^2)

$$I = \frac{10 \times 10^3 \text{ counts/sec}}{10^{10} \text{ nuclei} \times \pi (10^{-13} \text{cm})^2}$$

$$= 3.2 \times 10^{19} \text{ particles/cm}^2\text{/sec}$$

3a. $$\sigma_{TOT} = \int_0^{\pi}\int_0^{2\pi} b^2 \sin^2\phi \, \sin\theta \, d\phi \, d\theta$$

$$= b^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} \sin^2\phi \, d\phi$$

$$= b^2 \left(-\cos\theta\right)_0^{\pi} \left(\frac{1-\cos 2\phi}{2}\right)_0^{2\pi}$$

$$= b^2 \cdot 2 \cdot \frac{2\pi}{2}$$

$$\therefore \sigma_{TOT} = 2\pi b^2$$

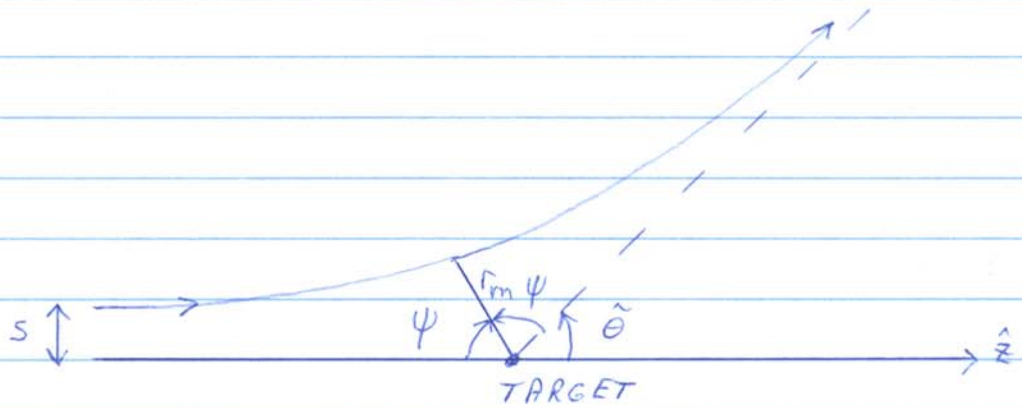
b. # scattered particles/sec = σ_{TOT} · incident flux.

$$\sigma_{TOT} = 10^8 / 10^{10}$$
$$= 10^{-2} \text{ cm}^2$$
$$\therefore b = 10^{-1}/\sqrt{2\pi} \text{ cm.}$$

c. $$\sigma_{TOT} = \frac{10^8}{10^{16}} = 10^{-8} \Rightarrow b = \frac{10^{-4}}{\sqrt{2\pi}} \text{ cm}$$

Assignment 6 Solution (Cont'd)

9-48)



$$F(r) = \frac{k}{r^3} \implies U(r) = \frac{k}{2r^2}$$

Particle path is found by solving the equation

$$\int d\theta = \int \frac{s/r^2 \, dr}{\left[1 - \frac{U}{E} - \frac{s^2}{r^2}\right]^{1/2}}$$

$$= \int \frac{s/r^2 \, dr}{\left[1 - \frac{k}{2Er^2} - \frac{s^2}{r^2}\right]^{1/2}}$$

$$= -\int \frac{du}{\left[1 - \left(1 + \frac{k}{2Es^2}\right)u^2\right]^{1/2}} \quad \text{where } u \equiv \frac{s}{r}$$

$$\theta - \theta_0 = -\frac{1}{A} \text{Arcsin } A u \quad \text{where } A \equiv \left(1 + \frac{k}{2Es^2}\right)^{1/2}$$

$\sin A(\theta_0 - \theta) = Au.$

$$u = \frac{1}{A} \sin A(\theta_0 - \theta)$$

Now $u(\theta = \pi) = 0 \implies 0 = \frac{1}{A} \sin A(\theta_0 - \pi)$

$$\therefore \theta_0 = \pi$$

$$\therefore \boxed{u = \frac{1}{A} \sin A(\pi - \theta)} \quad (1)$$

At closest approach $u = u_{max}$ and $\frac{du}{d\theta} = 0$.

Differentiating (1) $\implies 0 = -\cos A(\pi - \theta)$

$$\therefore A(\pi - \theta) = \frac{\pi}{2}$$

$$\therefore \theta(u_{max}) = \pi\left(1 - \frac{1}{2A}\right)$$

Hence $\psi \equiv \pi - \theta(u_{max})$

$$= \pi - \pi\left(1 - \frac{1}{2A}\right)$$

$$= \frac{\pi}{2A}$$

Scattering Angle $\hat{\theta} = \pi - 2\psi$

$$= \pi - \frac{\pi}{A}$$

$$= \pi - \pi\left(1 + \frac{k}{2Es^2}\right)^{-1/2}$$

$$\left(1 + \frac{k}{2Es^2}\right)^{-1/2} = \frac{\pi - \hat{\theta}}{\pi}$$

This can be solved to give:

$$s = \sqrt{\frac{k}{2E}} \frac{\pi - \hat{\theta}}{\sqrt{\hat{\theta}(2\pi - \hat{\theta})}}$$

Differential Cross Section is:

$$\sigma(\hat{\theta}) = \frac{s}{\sin\hat{\theta}} \left| \frac{ds}{d\hat{\theta}} \right|$$

$$= \sqrt{\frac{k}{2E}} \frac{(\pi - \hat{\theta})}{\sqrt{\hat{\theta}(2\pi - \hat{\theta})}} \frac{1}{\sin\hat{\theta}}$$

$$\cdot \sqrt{\frac{k}{2E}} \left| \frac{-1}{\sqrt{\hat{\theta}(2\pi - \hat{\theta})}} + (\pi - \hat{\theta})\left(-\frac{1}{2}\right)\left[\hat{\theta}(2\pi - \hat{\theta})\right]^{-3/2}\left((2\pi - \hat{\theta}) - \hat{\theta}\right) \right|$$

$$= \frac{k}{2E} \frac{(\pi - \hat{\theta})}{\hat{\theta}^2 (2\pi - \hat{\theta})^2} \left| \frac{-\hat{\theta}(2\pi - \hat{\theta}) - \frac{1}{2}(\pi - \hat{\theta})(2\pi - 2\hat{\theta})}{\sin\hat{\theta}} \right|$$

$$= \frac{k}{2E} \frac{(\pi - \hat{\theta})}{\hat{\theta}^2 (2\pi - \hat{\theta})^2} \left| \frac{-2\pi\hat{\theta} + \hat{\theta}^2 - (\pi - \hat{\theta})^2}{\sin\hat{\theta}} \right|$$

$$\therefore \sigma(E) = \frac{k}{2E} \frac{\pi^2 (\pi - \hat{\theta})}{\hat{\theta}^2 (2\pi - \hat{\theta})^2 \sin\hat{\theta}}$$