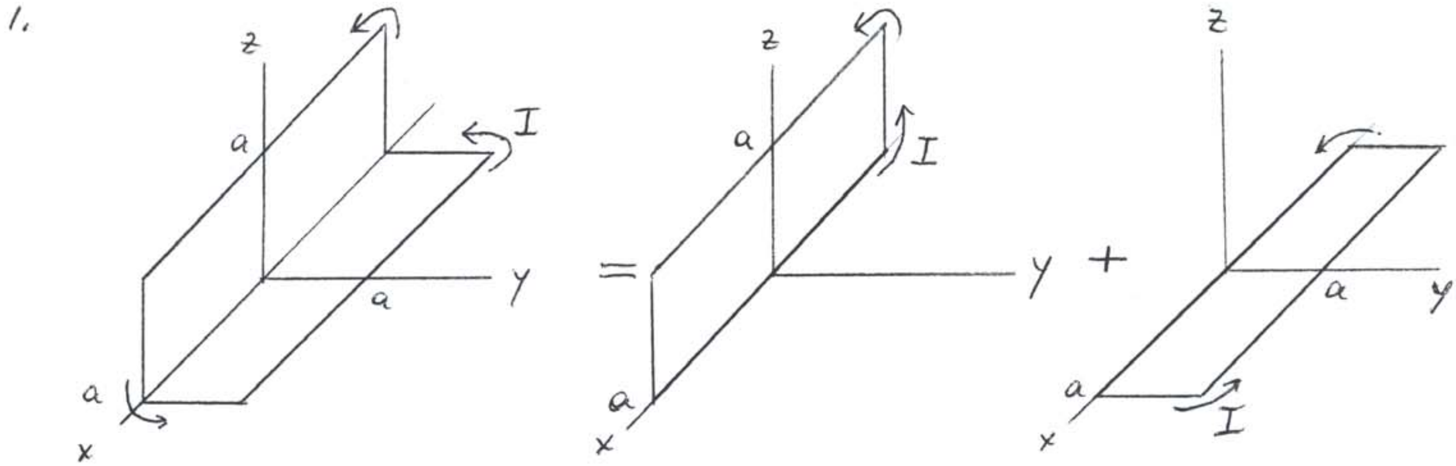
Assignment 8

1.



$$\vec{m} = \frac{I\,2a^2}{c}\,\hat{y} + \frac{I\,2a^2}{c}\,\hat{z}.$$

$$= \frac{2Ia^2}{c}(\hat{y} + \hat{z})$$

2a) $$\vec{A} = \frac{\vec{m} \times \vec{r}}{r^3}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$= \nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

$$= -(\vec{m}\cdot\nabla)\left(\frac{\vec{r}}{r^3}\right) + \vec{m}\underbrace{\left(\nabla\cdot\left(\frac{\vec{r}}{r^3}\right)\right)}_{=0 \text{ class exercise}}$$

$$= -(\vec{m}\cdot\nabla)\left(\frac{\vec{r}}{r^3}\right)$$

$$B_x = -(\vec{m}\cdot\nabla)\left(\frac{x}{r^3}\right)$$

$$= -\left[m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z}\right]\left(\frac{x}{r^3}\right)$$

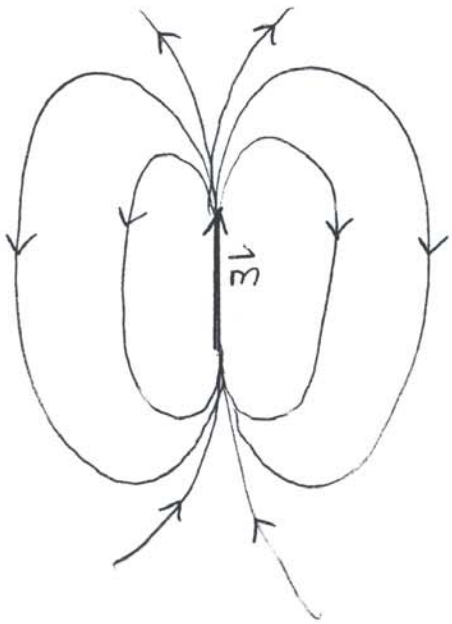
$$= -\left\{ m_x \frac{1}{r^3} + x\left[m_x \frac{\partial r^{-3}}{\partial x} + m_y \frac{\partial r^{-3}}{\partial y} + m_z \frac{\partial r^{-3}}{\partial z}\right]\right\}$$

$$= -\frac{m_x}{r^3} + 3x\left[\frac{m_x x}{r^5} + \frac{m_y y}{r^5} + \frac{m_z z}{r^5}\right]\Big\}$$

$$B_x = -\frac{m_x}{r^3} + 3x\,\frac{(\vec{m}\cdot\vec{r})}{r^5}$$

$$\therefore\ \vec{B} = -\frac{\vec{m}}{r^3} + \frac{3\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}$$

b).



Magnetic field lines of mag. dipole are the same as electric field lines of electric dipole

3a) Current $I = e\,\frac{v}{2\pi r}$

Magnetic dipole moment $\mu = \frac{IA}{c}$

$$= \frac{ev}{2\pi r}\,\frac{\pi r^2}{c}$$

$$= \frac{e}{2mc}\,mvr$$

$$\therefore\ \mu = \frac{eL}{2mc}$$

b) Torque $= \dfrac{d\vec{L}}{dt}$

$$\frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B}.$$

$$\frac{d\vec{\mu}}{dt} = \gamma \vec{\mu} \times \vec{B} \qquad \text{using } \vec{\mu} = \gamma \vec{L}.$$

$$= \gamma(\mu_y B, -\mu_x B, 0) \qquad \text{since } \vec{B} = (0, 0, B).$$

$$\frac{d\mu_z}{dt} = 0 \qquad \Longrightarrow \mu_z = \mu_z(0) = \text{constant (zero in our case)}$$

$$\frac{d\mu_y}{dt} = -\gamma B \mu_x \qquad (1)$$

$$\frac{d\mu_x}{dt} = \gamma B \mu_y \qquad (2)$$

$$(1) + (2) \Longrightarrow \frac{d^2}{dt^2}\begin{Bmatrix}\mu_x \\ \mu_y\end{Bmatrix} = -(\gamma B)^2 \begin{Bmatrix}\mu_x \\ \mu_y\end{Bmatrix}$$

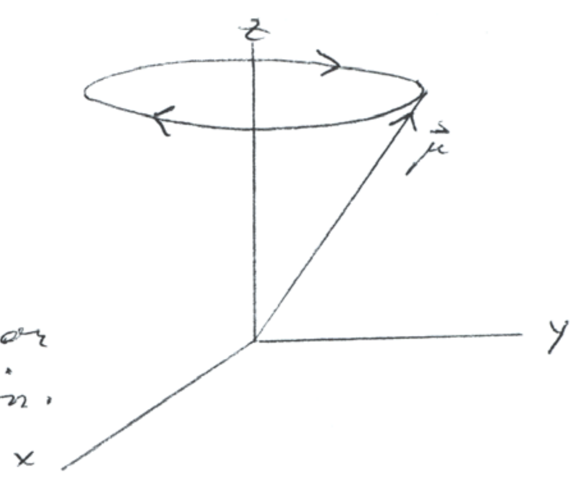
$$\mu_x = A \cos \gamma B t + B \sin \gamma B t$$
$$\mu_y = C \cos \gamma B t + D \sin \gamma B t$$

where $A, B, C + D$ are constants.

Using initial conds. $\mu_x(0) = \mu_0$, $\mu_y(0) = 0$ we get:

$$\mu_x = \mu_0 \cos \gamma B t$$

$$\mu_y = -\mu_0 \sin \gamma B t.$$



$\vec{\mu}$ precesses about $\hat{z}$ or magnetic field direction.

4a) $\vec{H}$ is found using $\oint \vec{H}\cdot d\vec{\ell} = \frac{4\pi}{c} I_{\text{enclosed by loop}}$.

By symmetry $\vec{H} = H(r)\,\hat{\phi}$.
Consider a loop of radius r centered about conductor axis.

$r < a \qquad H \cdot 2\pi r = \frac{4\pi}{c}\, I \,\frac{\pi r^2}{\pi a^2}$

$$\vec{H} = \frac{2 I r}{c a^2}\,\hat{\phi}$$

$b > r > a \qquad \vec{H} = \frac{2I}{rc}\,\hat{\phi}.$

$r > b \qquad \vec{H} = 0.$

Magnetization $\vec{M} = \chi_m \vec{H}$.

$r < a \qquad \vec{M} = 0$

$b > r > a \qquad \vec{M} = \frac{2I}{rc}\,\chi_m\,\hat{\phi}$

$r > b \qquad \vec{M} = 0$

$$\vec{B} = \vec{H} + 4\pi\vec{M}$$

$r < a \qquad \vec{B} = \frac{2Ir}{ca^2}\,\hat{\phi}$

$b > r > a \qquad \vec{B} = (1 + 4\pi\chi_m)\,\frac{2I}{rc}\,\hat{\phi}$

$r > b \qquad \vec{B} = 0.$

b) Volume bound current density

$$\vec{J}_b = c\,(\nabla \times \vec{M})$$

$$= c\,\frac{1}{r}\frac{\partial}{\partial r}(r\,M_\phi)\,\hat{z}$$

$$\vec{J}_b = 0$$

Surface bound current density

$$\vec{K}_b = c\,(\vec{M} \times \hat{n})$$ $\hat{n}$ = normal to surface

$$\vec{K}_b(r=a) = c\,(\vec{M} \times (-\hat{r}))$$

$$= c\,\frac{2I}{ac}\,\chi_m\,\hat{\phi} \times (-\hat{r})$$

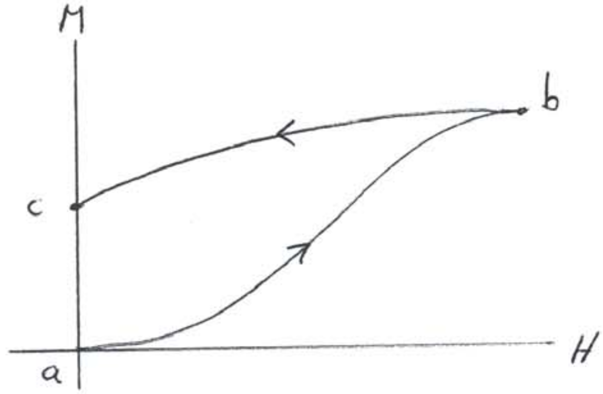
$$= \frac{2I\chi_m}{a}\,\hat{z}.$$

$$\vec{K}_b(r=b) = c\,(\vec{M} \times \hat{r})$$

$$= c\,\frac{2I}{bc}\,\chi_m\,\hat{\phi} \times \hat{r}.$$

$$= -\frac{2I\chi_m}{b}\,\hat{z}.$$

5.



a = begin with chunk of unmagnetized iron.

b = magnetization is maximum when H is max.

c = permanent magnet i.e. $\vec{M} \neq 0$ with $\vec{H} = 0$.