PHYS 2020 Assignment 7

1) Suppose a potential difference existed between two points on a conductor. A current would then flow between the 2 points until charges are rearranged such that the potential difference is zero.
∴ all points of a conductor are at the same potential.

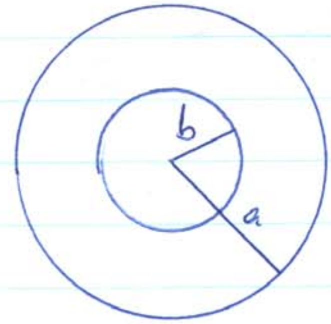
2a) charge of electron $= 1.6 \times 10^{-19}$ Coulomb
$= 4.8 \times 10^{-10}$ esu.
∴ 4.8×10^{-10} esu $= 1.6 \times 10^{-19}$ Coulomb.

$$1 \text{ esu} = \frac{1}{3 \times 10^{9}} \text{ Coulomb}$$

b) $$1 \text{ statvolt} \equiv \frac{\text{erg}}{\text{esu}}$$
$$= \frac{\text{erg}}{\text{esu}} \times \frac{1}{10^{7}} \frac{\text{J}}{\text{erg}} \times \frac{3 \times 10^{9}}{1} \frac{\text{esu}}{\text{Coulomb}}$$
$$= 300 \frac{\text{J}}{\text{Coul.}}$$
$$= 300 \text{ volts}$$

c) $$1 \text{ farad} \equiv \frac{\text{Coulomb}}{\text{volt}}$$
$$= \frac{\text{Coulomb}}{\text{volt}} \times \frac{3 \times 10^{9}}{1} \frac{\text{esu}}{\text{Coul}} \times \frac{300}{1} \frac{\text{volt}}{\text{statvolt}}$$
$$= 9 \times 10^{11} \frac{\text{esu}}{\text{statvolt}}$$
$$= 9 \times 10^{11} \text{ cm} \quad \text{since statvolt} = \frac{\text{esu}}{\text{cm}}.$$

3.10)



Let $\left\{ \begin{matrix} +Q \\ -Q \end{matrix} \right\}$ be charge on sphere of radius $\left\{ \begin{matrix} b \\ a \end{matrix} \right\}$.

From Gauss law $\vec{E} = 0$ except $\vec{E} = \frac{Q}{r^2}\hat{r}$ between spheres.

Potential difference between spheres is:

$$V = \left| -\int_b^a \vec{E} \cdot d\vec{r} \right|$$

$$= \left| -\int_b^a \frac{Q}{r^2}\, dr \right|$$

$$= \left| -Q \left[\frac{-1}{r} \right]_b^a \right|$$

$$= \left| Q \left(\frac{1}{a} - \frac{1}{b} \right) \right|$$

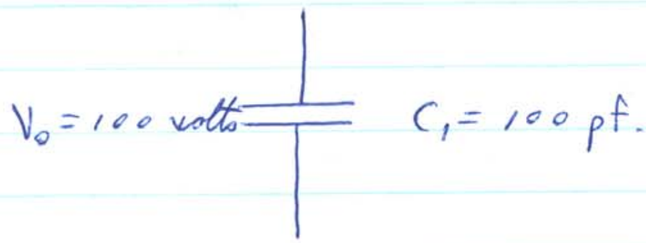
$$= Q \left(\frac{1}{b} - \frac{1}{a} \right)$$

$$V = Q\, \frac{a-b}{ab}$$

Capacitance $C = \frac{Q}{V} = \frac{ab}{a-b}$.

If $s \equiv a-b \ll b \Rightarrow a \simeq b$ and $C = \frac{a^2}{s} = \frac{A}{4\pi s}$
where $A = 4\pi a^2$ is sphere surface area.

3.11) <u>Part 1</u>

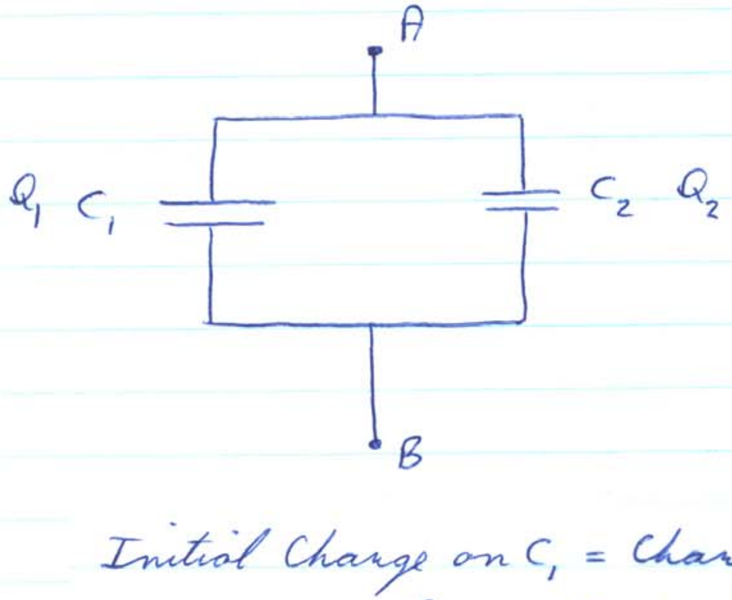


Charge on C_1 is $Q = C_1 V_0$

$= 100 \times 10^{-12}$ farad $\times$ 100 volts

$= 10^{-8}$ Coul.

<u>Part 2</u>



$V_{AB} = 30$ volts.

Initial Charge on C_1 = Charge on C_1 + Charge on C_2

$$Q = Q_1 + Q_2$$

$$C_1 V_0 = C_1 V_{AB} + C_2 V_{AB}$$

$$C_2 V_{AB} = C_1 V_0 - C_1 V_{AB}$$

$$C_2 = C_1 \left(\frac{V_0}{V_{AB}} - 1 \right)$$

$$\therefore C_2 = 100\,pf\left(\frac{100}{30} - 1\right)$$

$$= 233\ pf.$$

Initial energy $= \frac{1}{2} C_1 V_0^2$

$$= \frac{1}{2}\, 10^{-10}\ \text{farad} \times (100\ \text{volt})^2$$

$$= 5 \times 10^{-7}\ \text{joules}$$

Final energy $= \frac{1}{2} C_1 V_{AB}^2 + \frac{1}{2} C_2 V_{AB}^2$

$$= \frac{1}{2}\left(10^{-10}\ \text{farad} + 2.33 \times 10^{-10}\ \text{farad}\right) \times (30\ \text{volt})^2$$

$$= 1.5 \times 10^{-7}\ \text{joules}$$

$\therefore$ 3.5×10^{-7} joules has been used to do work or has been lost as heat.

3.12) Capacitance of parallel plate capacitor

$$C = \frac{A}{4\pi s}$$

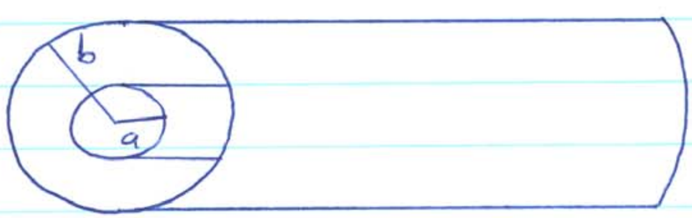
$$= \frac{\pi\ (15\ cm)^2}{4\pi\ (.004\ cm)}$$

$$= 14{,}063\ cm$$

$$= 15.6 \times 10^{-9}\ farad$$

$$C = 15.6\ nf.$$

3.15)



First the capacitance is found.

Let $\begin{Bmatrix} \lambda \\ -\lambda \end{Bmatrix}$ be charge per unit length on $\begin{Bmatrix} \text{inner} \\ \text{outer} \end{Bmatrix}$ cond.

$\therefore \vec{E}$ is only nonzero between cylinders. Using Gauss Law for a cylinder of radius ρ + length l we get:

$$\underbrace{\int \vec{E} \cdot d\vec{a}}_{\substack{\text{surface of cylinder} \\ \text{radius } a < \rho < b \\ \text{length } l}} = 4\pi \underbrace{\int \rho\, dV}_{\substack{\text{charge enclosed} \\ \text{by cylinder}}}$$

Neglecting end effects $\vec{E} = E(\rho)\,\hat{\rho}$.

$$E \cdot 2\pi \rho l = 4\pi \lambda l.$$

$$\vec{E} = \frac{2\lambda}{\rho}\,\hat{\rho}.$$

Potential difference between inner & outer cylinder

$$V = \left| -\int_a^b \vec{E} \cdot d\vec{\rho} \right|$$

$$= \left| -\int_a^b \frac{2\lambda}{\rho} \, d\rho \right|$$

$$= \left| -2\lambda \ln\rho \Big|_a^b \right|$$

$$V = 2\lambda \ln\left(\frac{b}{a}\right)$$

$\therefore$ Capacitance $C = \dfrac{Q}{V}$

$$= \frac{\lambda L}{2\lambda \ln(b/a)} \qquad L = \text{cylinder length}$$

$$= \frac{L}{2\ln(b/a)}$$

Energy stored by electric field
= Energy stored by capacitor

$$= \frac{1}{2} C V^2$$

$$= \frac{L}{4\ln(b/a)} V^2$$

$$= \frac{1}{4} \frac{30 \text{ cm}}{\ln(4/3)} \left(45 \text{ volts} \times \frac{1}{300} \frac{\text{statvolt}}{\text{volt}} \right)^2$$

$= .59$ cm statvolt^2

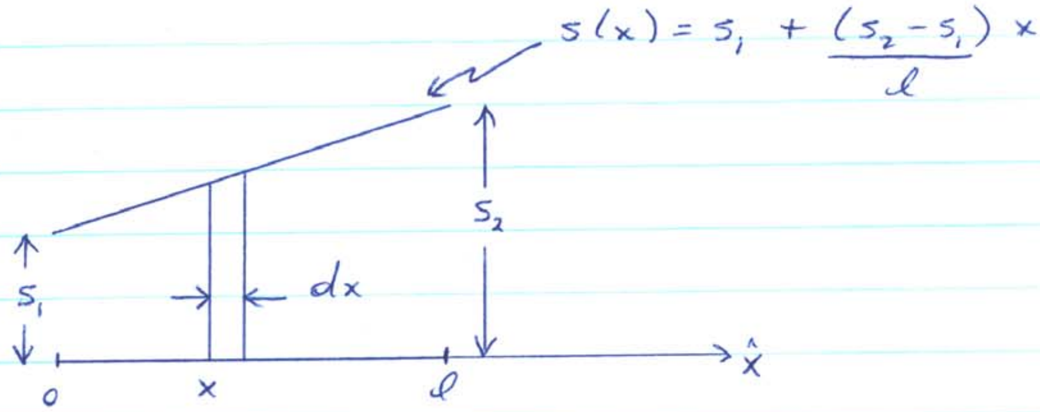
$= .59$ erg since cm (statvolt)^2 = cm $\left(\frac{\text{esu}}{\text{cm}}\right)^2$

= cm dyne = erg

7a). Capacitance of parallel plate capacitor $C = \frac{A}{4\pi s}$

b) Capacitance of C_1 + C_2 in parallel is $C = C_1 + C_2$.

c)



Consider capacitor at x of length dx + width w.
Area of capacitor " " is $w\,dx$.

Space between plates is $s(x)$.

$\therefore$ capacitance $dC = \frac{w\,dx}{4\pi\, s(x)}$

$\therefore$ Capacitance of angled wedge

C = sum of all capacitors in parallel of length dx

$$= \int_0^l \frac{w\,dx}{4\pi\, s(x)}$$

$$= \frac{w}{4\pi} \int_0^l \frac{dx}{s_1 + \frac{(s_2 - s_1)}{l} x}$$

$$\therefore C = \frac{w\,l}{4\pi\,(s_2 - s_1)} \ln\left(s_2/s_1\right)$$