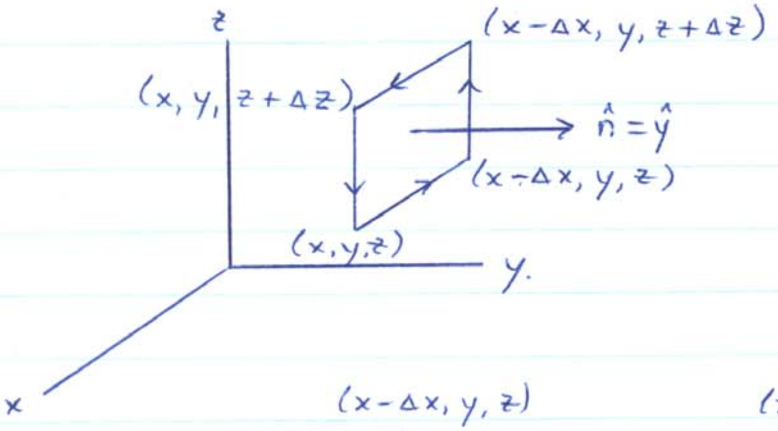
# PHYS 2020 Assignment 6

1) To find $(\nabla \times \vec{E})_y$ consider area $\Delta A = \Delta x \Delta z$ oriented as shown below.




$$\int_{\substack{\text{Contour}\\ \text{around } \Delta A}} \vec{E}\cdot d\vec{l} = \int_{(x,y,z)}^{(x-\Delta x,y,z)} E_x\,dx + \int_{(x-\Delta x,y,z)}^{(x-\Delta x,y,z+\Delta z)} E_z\,dz$$

$$+ \int_{(x-\Delta x,y,z+\Delta z)}^{(x,y,z+\Delta z)} E_x\,dx + \int_{(x,y,z+\Delta z)}^{(x,y,z)} E_z\,dz.$$

$$\simeq E_x\left(x - \tfrac{\Delta x}{2}, y, z\right)(-\Delta x) + E_z\left(x-\Delta x, y, z+\tfrac{\Delta z}{2}\right)\Delta z$$

$$+ E_x\left(x - \tfrac{\Delta x}{2}, y, z+\Delta z\right)\Delta x + E_z\left(x, y, z+\tfrac{\Delta z}{2}\right)(-\Delta z).$$

$$= \left\{-E_x\left(x-\tfrac{\Delta x}{2}, y, z\right) + E_x\left(x-\tfrac{\Delta x}{2}, y, z+\Delta z\right)\right\}\Delta x$$

$$+ \left\{E_z\left(x-\Delta x, y, z+\tfrac{\Delta z}{2}\right) - E_z\left(x, y, z+\tfrac{\Delta z}{2}\right)\right\}\Delta z.$$

$$= \frac{\partial E_x}{\partial z}\Delta z \Delta x - \frac{\partial E_z}{\partial x}\Delta x \Delta z.$$

In the limit as $\Delta A \to dA$ we get:

$$\oint_{\substack{\text{Contour}\\ \text{around } dA}} \vec{E}\cdot d\vec{l} = \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) dA.$$

$$\therefore \oint_{\substack{\text{contour} \\ \text{around unit area}}} \vec{E} \cdot d\vec{l} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}.$$

$$\therefore (\nabla \times \vec{E})_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}.$$

2) $\vec{E} = (6xy,\ 3x^2 - 3y^2,\ 0)$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & 3x^2 - 3y^2 & 0 \end{vmatrix}$$

$$= \left(-\frac{\partial}{\partial z}(3x^2 - 3y^2),\ \frac{\partial}{\partial z}(6xy),\ \frac{\partial}{\partial x}(3x^2 - 3y^2) - \frac{\partial}{\partial y}(6xy)\right)$$

$$= (0, 0, 0)$$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= 6y - 6y$$

$$= 0.$$

3a) $\vec{F} = (x + y,\ -x + y,\ -2z)$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + y & -x + y & -2z \end{vmatrix}$$

$$= (0,\ 0,\ -2)$$

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$= 1 + 1 - 2$$

$$= 0.$$

Similarly b + c are done.

4) $$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \, , \, -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} \, , \, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla \cdot (\nabla \times \vec{A}) = \frac{\partial}{\partial x}\left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y}\left( -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z}\left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y}$$

$$= 0$$