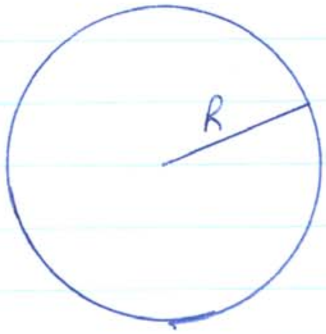
PHYS 2020 Assignment 5

1)



Let Q be charge on Earth

From Gauss Law

$$\vec{E} = \frac{Q}{r^2}\,\hat{r} \qquad r > R.$$

$$\therefore E(\text{earth's surface}) = \frac{Q}{R^2}$$

$$= \frac{10^9\ \text{esu}}{(6{,}400 \times 10^5\ \text{cm})^2}$$

$$= 2.44 \times 10^{-9}\ \text{esu/cm}^2$$

$$\varphi(r) - \varphi(\infty) = -\int_\infty^r \vec{E}\cdot d\vec{r}$$

$$= -\int_\infty^r \frac{Q}{r^2}\,dr$$

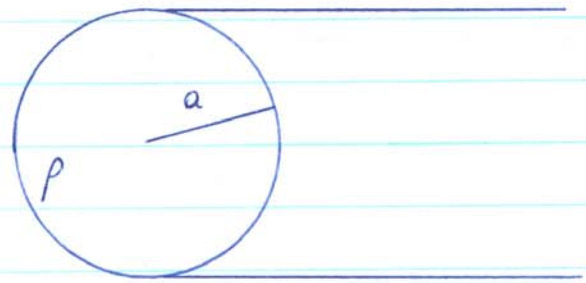
$$= Q\,\frac{1}{r}\Big|_\infty^r$$

$$\varphi(r) = \frac{Q}{r}$$

$$\therefore \varphi(\text{earth's surface}) = \frac{10^9\ \text{esu}}{6.4 \times 10^8\ \text{cm}}$$

$$= 1.56\ \text{statvolts}$$

2.8)



The electric field can be found from Gauss Law considering a cylinder of radius r and length l. By cylindrical symmetry $\vec{E} = E(r)\,\hat{r}$.

$r < a$
$$\int_{\text{cylinder surface}} \vec{E} \cdot d\vec{a} = 4\pi \int_{\text{cyl. volume}} \rho \, dV.$$

$$E(r) \cdot 2\pi r l = 4\pi \int_0^r \rho \, 2\pi r l \, dr$$

$$= 4\pi \; 2\pi \rho l \int_0^r r \, dr$$

$$= 4\pi \; 2\pi \rho l \frac{r^2}{2}$$

$$\therefore \vec{E}(r) = 2\pi \rho \vec{r}$$

$r > a$
$$\int \vec{E} \cdot d\vec{a} = 4\pi \int \rho \, dV.$$

$$E(r) \, 2\pi r l = 4\pi \int_0^a \rho \, 2\pi r l \, dr$$

$$= 4\pi \; 2\pi l \rho \int_0^a r \, dr$$

$$= 4\pi \; 2\pi l \rho \frac{a^2}{2}.$$

$$\vec{E}(r) = \frac{2\pi \rho a^2}{r} \; \hat{r}$$

b) $$\varphi(r) - \varphi(0) = -\int_0^r \vec{E}\cdot d\vec{r}$$

$r < a$ $$\varphi(r) = -\int_0^r 2\pi\rho\, r\, dr$$

$$= -2\pi\rho\,\frac{r^2}{2}$$

$$\varphi(r) = -\pi\rho r^2$$

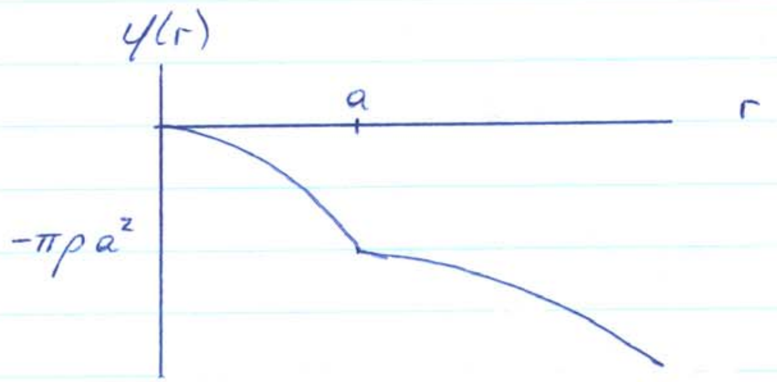
$r > a$ $$\varphi(r) - \varphi(a) = -\int_a^r \vec{E}\cdot d\vec{r}$$

$$= -\int_a^r \frac{2\pi\rho a^2}{r}\, dr$$

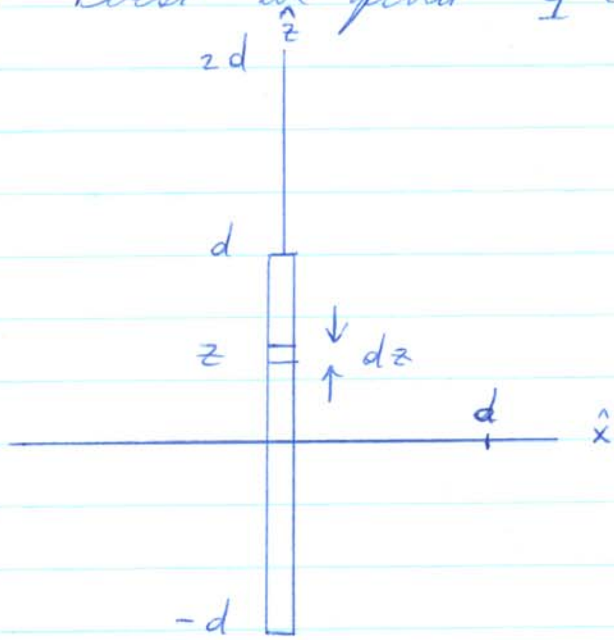
$$= -2\pi\rho a^2 \ln(r/a)$$

$$\varphi(r) = -\pi\rho a^2 - 2\pi\rho a^2 \ln(r/a)$$

$$= -\pi\rho a^2\left[1 + 2\ln(r/a)\right]$$



3) First we find $\Phi(0,0,2d)$.



Consider an infinitesimal piece of rod of length dz at height z.

Charge on dz is $\lambda\, dz$.

Distance of dz to $(0,0,2d)$ is $2d-z$.

Contribution of $\lambda\, dz$ to potential at $(0,0,2d)$ is

$$d\varphi(0,0,2d) = \frac{\lambda\, dz}{2d-z}$$

Potential at $(0,0,2d)$ due to entire rod is:

$$\varphi(0,0,2d) = \int_{-d}^{d} \frac{\lambda\, dz}{2d-z}$$

$$= -\lambda \ln(2d-z)\Big|_{-d}^{d}$$

$$\varphi(0,0,2d) = -\lambda(\ln d - \ln 3d)$$

$$= \lambda(\ln 3d - \ln d)$$

$$= \lambda \ln 3$$

<u>Next we find $\Phi(d,0,0)$.</u>

Once again consider an infinitesimal piece of rod of length dz at height z.

Charge on dz is $\lambda\, dz$.

Distance of dz to $(d,0,0)$ is $\sqrt{d^2+z^2}$.

Contribution of $\lambda\, dz$ to potential at $(d,0,0)$ is:

$$d\varphi(d,0,0) = \frac{\lambda\, dz}{\sqrt{d^2+z^2}}$$

$$\varphi(d,0,0) = \int_{-d}^{d} \frac{\lambda\, dz}{\sqrt{d^2+z^2}}$$

$$= 2\int_{0}^{d} \frac{\lambda\, dz}{\sqrt{d^2+z^2}}$$

Let $z = d\tan\theta \qquad dz = d\sec^2\theta\, d\theta$

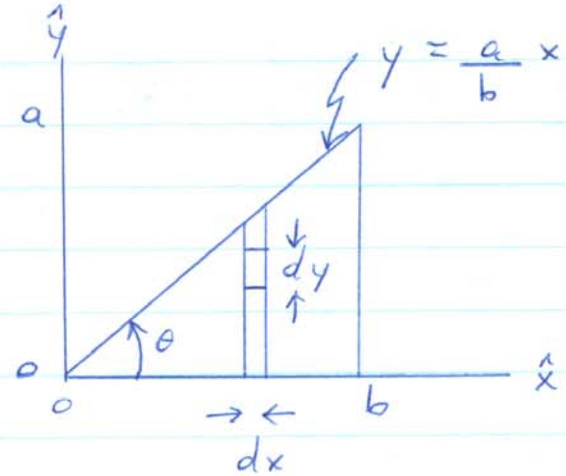
$$z=0 \Rightarrow \theta = 0 \qquad z = d \Rightarrow \theta = \frac{\pi}{4}.$$

$$\therefore \varphi(d,0,0) = 2\lambda \int_0^{\pi/4} \frac{d \sec^2\theta}{d \sec\theta}\, d\theta$$

$$= 2\lambda \int_0^{\pi/4} \sec\theta \, d\theta$$

$$= 2\lambda \ln|\sec\theta + \tan\theta| \Big|_0^{\pi/4}$$

$$= 2\lambda \ln|\sqrt{2} + 1|$$

2.12) 

Consider infinitesimal area $dx\,dy$ at position (x, y)

Charge on $dx\,dy$ is $\sigma\,dx\,dy$.

Distance of $dx\,dy$ to P (origin) is $\sqrt{x^2+y^2}$.

∴ contribution to potential at origin from $\sigma\,dx\,dy$ is

$$d\varphi(o) = \frac{\sigma\,dx\,dy}{\sqrt{x^2+y^2}}$$

$$\varphi(o) = \int_0^b \int_0^{y=ax/b} \frac{\sigma}{\sqrt{x^2+y^2}}\,dy\,dx$$

The triangle has been divided into a series of vertical strips starting at $x=0$ + ending at $x=b$. A strip at x has height $y = \frac{ax}{b}$.

Let $y = x\tan\theta'$

$$dy = x\sec^2\theta'\,d\theta'$$

$y = 0 \Rightarrow \theta' = 0$

$y = \frac{ax}{b} \Rightarrow \theta' = \theta$

$$\therefore \varphi(0) = \int_0^b \int_0^\theta \sigma \frac{x \sec^2\theta'}{x \sec\theta'} \, d\theta' \, dx$$

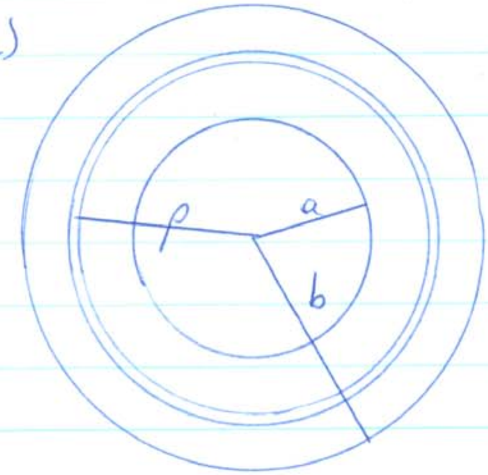
$$= \sigma \int_0^b \int_0^\theta \sec\theta' \, d\theta' \, dx$$

$$= \sigma \int_0^b \ln|\sec\theta' + \tan\theta'| \Big|_0^\theta dx.$$

$$= \sigma \int_0^b \ln|\sec\theta + \tan\theta| \, dx$$

$$\therefore \varphi(0) = \sigma b \ln|\sec\theta + \tan\theta|$$

2.28 a)



$\sigma = -4\ esu/cm^2$
$a = 1\ cm$
$b = 3\ cm.$

Consider a ring of radius ρ and thickness $d\rho$.

Area of ring is $2\pi\rho\, d\rho$.

Charge on ring is $\sigma\, 2\pi\rho\, d\rho$.

Distance of pt. on ring to ring center is ρ.

∴ contribution of ring to potential at disk center

$$d\varphi(o) = \frac{\sigma\, 2\pi\rho\, d\rho}{\rho}.$$

∴ potential at center due to entire disk is:

$$\varphi(o) = \int_a^b \frac{\sigma\, 2\pi\rho}{\rho}\, d\rho$$

$$= 2\pi\sigma\,(b-a)$$
$$= 2\pi\,(-4\ esu/cm^2)\,(3-1)\,cm$$
$$= -16\pi \text{ statvolts}$$

b) If electron moves from disk center to infinity it acquires energy:

$$W = q\,\varphi(0) \qquad q = \text{electron charge.}$$

$$= (-4.8 \times 10^{-10}\,\text{esu})\,(-16\pi\ \text{statvolts})$$

$$= 2.4 \times 10^{-8}\ \text{erg.}$$

For final velocity v, $W = \frac{m}{2} v^2$.

$$v = \sqrt{\frac{2W}{m}}$$

$$= \left(\frac{2 \times 2.4 \times 10^{-8}}{9.11 \times 10^{-28}}\right)^{1/2}$$

$$= 7.3 \times 10^{9}\ \text{cm/sec.}$$

Note $v \ll c$ otherwise relativistic formula would be needed.