# PHYS 2020 Assignment 4

9) Let $s$ be spacing between ions.

$$\leftarrow s \rightarrow$$

$$\cdots + \quad - \quad + \quad - \quad + \quad -$$

To add the next ion requires work

$$W = -\frac{e^2}{s} + \frac{e^2}{2s} - \frac{e^2}{3s} + \frac{e^2}{4s} - + \cdots$$

$$= -\frac{e^2}{s}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + - \cdots\right]$$

Note: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + - \cdots$

$$\therefore W = -\frac{e^2}{s}\ln 2$$

$\therefore$ potential energy per ion is $-\frac{e^2}{s}\ln 2$.

10) From class, energy of uniformly charged sphere having radius $r_0$ + charge $Q$ is $U = \frac{3Q^2}{5r_0}$.

If $U = mc^2$

$$\frac{3}{5}\frac{Q^2}{r_0} = mc^2$$

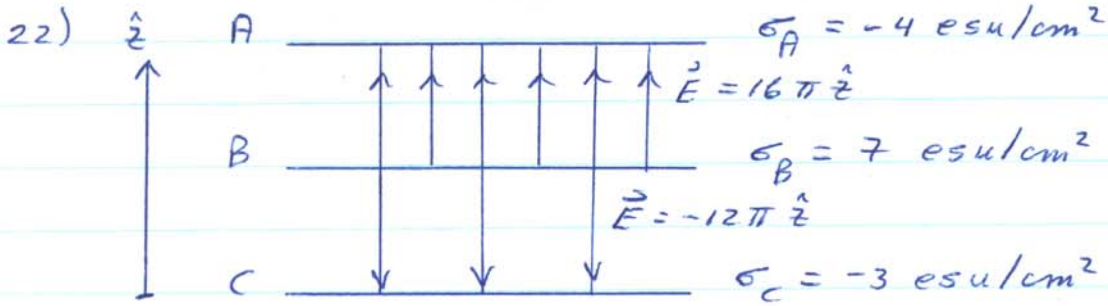
$$r_0 = \frac{3Q^2}{5mc^2}$$

$$= \frac{3}{5}\frac{(4.8\times10^{-10}\,\text{esu})^2}{9.11\times10^{-28}\,\text{gm}\times(3\times10^{10}\,\text{cm/sec})^2}$$

$$= 1.69\times10^{-13}\,\text{cm}.$$

$r_0$ is called the classical electron radius.

22)




$$\vec{E} = \hat{z}\, 2\pi \left\{ \text{charge}/cm^2 \text{ below } z - \text{charge}/cm^2 \text{ above } z \right\}$$

$z < z_C \qquad \vec{E} = 0.$

$z_C < z < z_B \qquad \vec{E} = \hat{z}\ 2\pi \left\{ -3 - (7 - 4) \right\}$

$$= -12\pi\ \hat{z}\ \ esu/cm^2$$

$z_B < z < z_A \qquad \vec{E} = \hat{z}\ 2\pi \left\{ 7 - 3 - (-4) \right\}$

$$= 16\pi\ \hat{z}\ \ esu/cm^2$$

$z > z_A \qquad \vec{E} = 0.$

Force per unit area on sheet A is $\vec{F}_A = \dfrac{\vec{E}_{above A} + \vec{E}_{below A}}{2}\sigma_A$

$$= \frac{0 + 16\pi\hat{z}}{2}(-4)$$

$$= -32\pi\ \hat{z}\ \ dynes/cm^2$$

Similarly $\vec{F}_B = \dfrac{16\pi\hat{z} + (-12\pi\hat{z})}{2}\,7 = 14\pi\ \hat{z}\ \ dynes/cm^2$

$\vec{F}_C = \dfrac{-12\pi\hat{z} + 0}{2}(-3) = 18\pi\ \hat{z}\ \ dynes/cm^2$

23)


$r < R \quad \vec{E} = 0$

$r > R \quad \vec{E} = \dfrac{Q}{r^2}\,\hat{r}.$

Energy density is $\dfrac{E^2}{8\pi}$.

$\therefore$ energy stored by sphere of radius $r$ is

$$U(r) = \int_{\text{sphere of radius } r} \frac{E^2}{8\pi}\, dV$$

$$= \int_R^r \frac{Q^2}{8\pi r^4}\, 4\pi r^2\, dr$$

$$= \frac{Q^2}{2} \int_R^r \frac{dr}{r^2}$$

$$= \frac{Q^2}{2} \left[ \frac{-1}{r} \right]_R^r$$

$$U(r). = \frac{Q^2}{2} \left[ \frac{1}{R} - \frac{1}{r} \right]$$

Total energy stored in electrostatic field is $U(\infty) = \dfrac{Q^2}{2R.}$

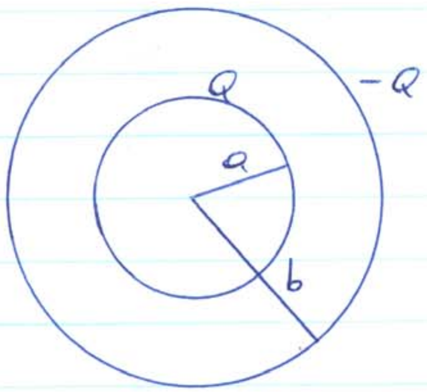
$$U = .9\,U(\infty) \Rightarrow \quad .9\,\frac{Q^2}{2R} = \frac{Q^2}{2}\left[\frac{1}{R} - \frac{1}{r}\right]$$

$$\frac{.9}{R} = \frac{1}{R} - \frac{1}{r}$$

$$\frac{1}{r} = \frac{.1}{R} \quad \text{or } r = 10R$$

$\therefore$ a sphere of radius $10R$ contains 90% of electrostatic energy.

30)




From Gauss Law $\vec{E} = 0$ except between spheres where $\vec{E} = \frac{Q}{r^2}\hat{r}$.

Electric field energy density is $\frac{E^2}{8\pi}$.

∴ total energy stored in electric field is

$$\int_a^b \frac{E^2}{8\pi} \underbrace{4\pi r^2 dr}_{=dV}$$

$$= \int_a^b \frac{1}{8\pi} \frac{Q^2}{r^4} 4\pi r^2 dr$$

$$= \frac{Q^2}{2} \int_a^b \frac{dr}{r^2}$$

$$= \frac{Q^2}{2} \left[\frac{-1}{r}\right]_a^b$$

$$= \frac{Q^2}{2} \left[\frac{1}{a} - \frac{1}{b}\right]$$