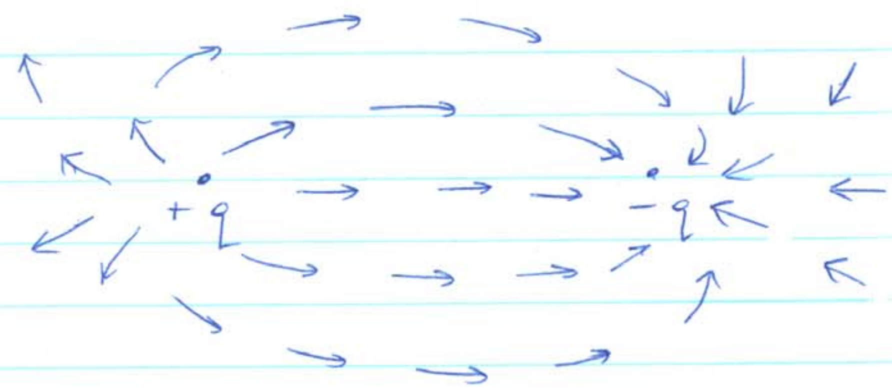
PHYS 2020 Assignment 2

1.



2. Electric field of +1esu at -2esu is

$$\vec{E}_1 = \frac{1\,esu}{(2\,cm)^2}(1,0,0) = \frac{1}{4}(1,0,0)$$

Electric field of -3esu at -2esu is

$$\vec{E}_2 = \frac{-3\,esu}{(2\sqrt{2}\,cm)^2}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = -\frac{3}{8}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

Electric field of +2esu at -2esu is

$$\vec{E}_3 = \frac{+2\,esu}{(2\,cm)^2}(0,1,0) = \frac{1}{2}(0,1,0)$$

∴ Total electric field felt by -2esu is

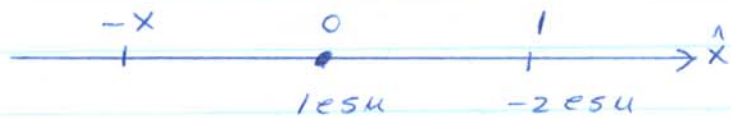
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \left(\frac{1}{4} - \frac{3}{8\sqrt{2}}, \frac{1}{2} - \frac{3}{8\sqrt{2}}, 0\right) \text{ esu/cm}^2$$

Force on $-2\,esu$ is $\vec{F} = (-2\,esu) \times \vec{E}$

$$= \left(-\frac{1}{2} + \frac{3}{4\sqrt{2}},\ -1 + \frac{3}{4\sqrt{2}},\ 0 \right) \text{dyne}$$

1.11a)



The only possible place where $\vec{E} = 0$ is when repulsion of unit charge by 1esu is cancelled by the attraction of unit charge by −2esu. This only occurs for some $-x < 0$.

$$E(-x) = \frac{1\,esu}{(x\,cm)^2} - \frac{2\,esu}{(1+x)^2\,cm^2}$$

$$0 = \frac{1}{x^2} - \frac{2}{(1+x)^2}$$

$$\frac{1}{x^2} = \frac{2}{(1+x)^2}$$

$$\frac{1}{x} = \pm \frac{\sqrt{2}}{1+x}.$$

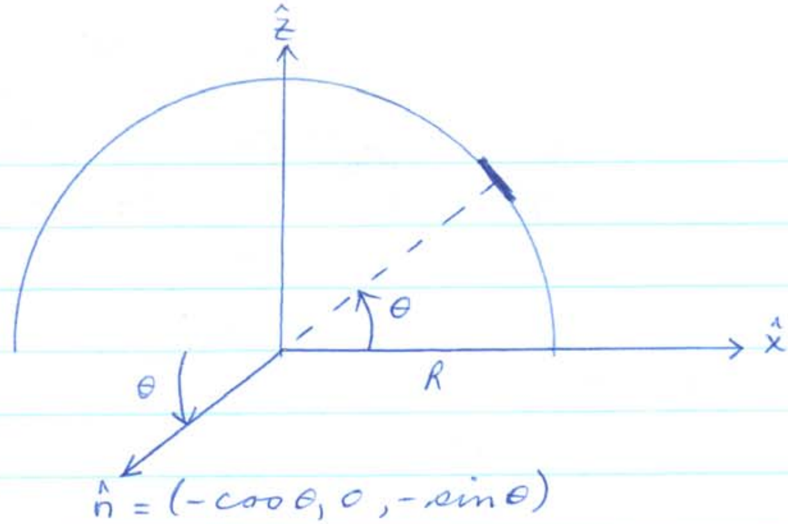
$$1 + x = \pm \sqrt{2}\, x.$$

$$(1 \mp \sqrt{2})\, x = -1.$$

$$x = 2.41,\ -.414.$$

But $-x < 0$ or $x > 0$. Hence electric field is zero at -2.41 cm.

1.5)



Charge per unit length of arc $\lambda = \dfrac{Q}{\pi R}$.

Consider an infinitesimal arc making angle θ with the horizontal and having length $R\,d\theta$.

Charge on arc is $\lambda R\,d\theta$

Electric field generated by $\lambda R\,d\theta$ at origin is

$$d\vec{E}(0) = \frac{\lambda R\,d\theta}{R^2}\,\hat{n}$$

$$= \frac{\lambda}{R}\,d\theta\,(-\cos\theta, 0, -\sin\theta)$$

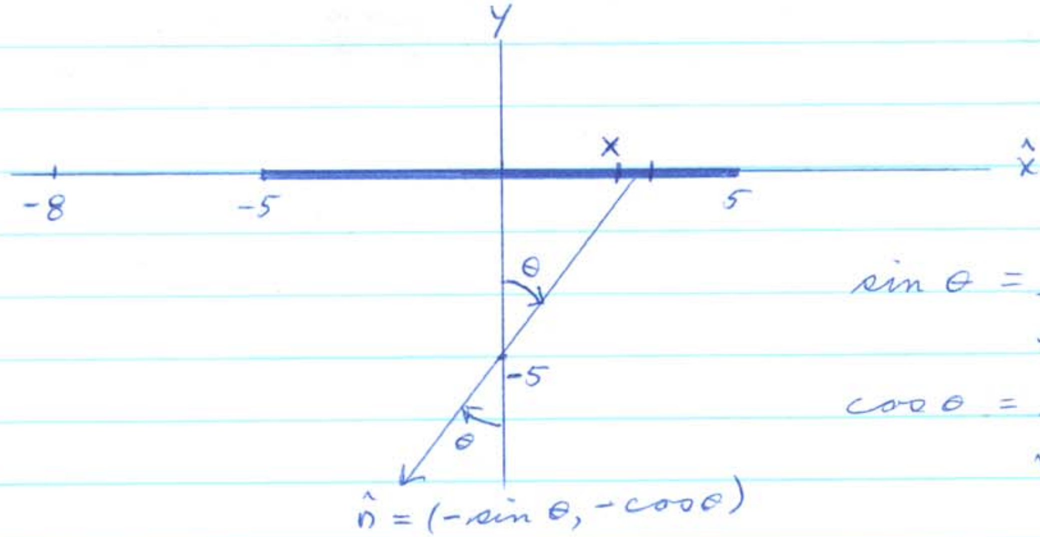
$\therefore$ total electric field due to entire semicircle is

$$\vec{E}(0) = \int_0^{\pi} \frac{\lambda}{R}\,d\theta\,(-\cos\theta, 0, -\sin\theta)$$

$$= \frac{\lambda}{R}\left(-\sin\theta\Big|_0^{\pi},\ 0,\ \cos\theta\Big|_0^{\pi}\right)$$

$$\therefore \vec{E}(0) = -\frac{2\lambda}{R}\,\hat{z} = -\frac{2Q}{\pi R^2}\,\hat{z}$$

1,24)



$$\sin\theta = \frac{x}{\sqrt{x^2+5^2}}$$

$$\cos\theta = \frac{5}{\sqrt{x^2+5^2}}$$

Charge per unit length $\lambda = \frac{8\,esu}{10\,cm} = .8$ esu/cm.

Consider an infinitesimal part of rod at x of length dx.

Charge on dx is $\lambda\,dx$.

Distance from $(-8,0)$ is $x+8$.

$\therefore$ electric field at $(-8,0)$ due to $\lambda\,dx$ is

$$d\vec{E}(-8,0) = \frac{\lambda\,dx}{(x+8)^2}\,(-\hat{x})$$

$$\therefore \vec{E}(-8,0) = -\hat{x}\int_{-5}^{5}\frac{\lambda\,dx}{(x+8)^2}$$

$$= -\hat{x}\,\lambda\left[\frac{-1}{x+8}\right]_{-5}^{5}$$

$$= -\hat{x}\,\lambda\left(\frac{-1}{13} + \frac{1}{3}\right)$$

$$\vec{E}(-8,0) = -.205\,\hat{x}\ \ esu/cm^2$$

Distance of $\lambda\, dx$ from $(0,-5)$ is $\sqrt{x^2+5^2}$.

$\therefore$ electric field at $(0,-5)$ due to $\lambda\, dx$ is

$$d\vec{E}(0,-5) = \frac{\lambda\, dx}{x^2+5^2}\,\hat{n}$$

$$= \frac{\lambda\, dx}{x^2+5^2}\left(-\sin\theta, -\cos\theta\right)$$

$$= \frac{-\lambda\, dx}{(x^2+25)^{3/2}}\,(x,\ 5)$$

$$\therefore\ \vec{E}(0,-5) = \int_{-5}^{5} \frac{-\lambda\, dx}{(x^2+25)^{3/2}}\,(x,5).$$

$$= -\lambda\left(\underbrace{\int_{-5}^{5} \frac{x\, dx}{(x^2+25)^{3/2}}}_{=0.},\ 5\int_{-5}^{5}\frac{dx}{(x^2+25)^{3/2}}\right)$$

$$= -\lambda\,\hat{y}\ 5\int_{-5}^{5}\frac{dx}{(x^2+25)^{3/2}}$$

$$= -10\lambda\,\hat{y}\int_{0}^{5}\frac{dx}{(x^2+25)^{3/2}}.$$

Let $x = 5\tan\theta$.

$dx = 5\sec^2\theta\, d\theta$.

$x=0 \Rightarrow \theta = 0$

$x=5 \Rightarrow \theta = \frac{\pi}{4}$.

$$\therefore\ \vec{E}(0,-5) = -10\lambda\,\hat{y} \int_0^{\pi/4} \frac{5\sec^2\theta\,d\theta}{5^3\sec^3\theta}$$

$$= -\frac{2}{5}\lambda\,\hat{y} \int_0^{\pi/4} \cos\theta\,d\theta$$

$$= \frac{-2}{5}\lambda\,\hat{y}\ \sin\theta \Big|_0^{\pi/4}$$

$$= -\frac{\sqrt{2}}{5}\lambda\,\hat{y}$$

$$\therefore\ \vec{E}(0,-5) = -.226\,\hat{y}\ \ esu/cm^2$$