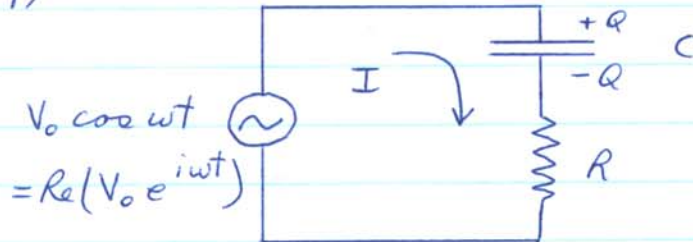


PHYS 2020 Assignment 12

1)



$$V_0 e^{i\omega t} = \frac{Q}{C} + IR$$

Differentiating this equation and using $I = \frac{dQ}{dt}$ and

$$I = I_0 e^{i(\omega t + \varphi)} \text{ we get:}$$

$$i\omega V_0 e^{i\omega t} = \frac{I}{C} + i\omega IR$$

$$I \left(\frac{1}{C} + i\omega R \right) = i\omega V_0 e^{i\omega t}$$

$$I (1 + i\omega RC) = i\omega C V_0 e^{i\omega t}$$

$$I_0 e^{i\varphi} = \frac{i\omega C}{1 + i\omega RC} V_0$$

$$I_0 e^{i\varphi} = \frac{V_0}{R - \frac{i}{\omega C}}$$

$$\therefore I_0 e^{i\varphi} = \frac{V_0}{R^2 + \frac{1}{\omega^2 C^2}} \left(R + \frac{i}{\omega C} \right)$$

$$I_0 = \frac{V_0}{\left(R^2 + \frac{1}{\omega^2 C^2} \right)^{1/2}}$$

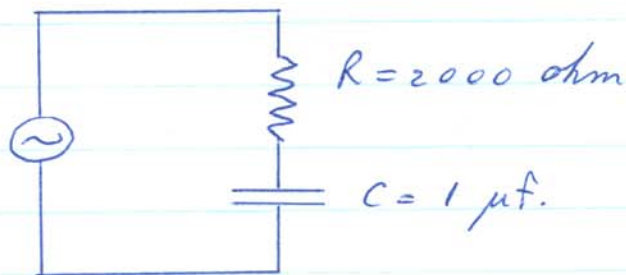
$$\tan \varphi = \frac{1}{\omega RC}$$

$$\varphi = \arctan(\omega RC)^{-1}$$

$$\therefore \text{current in circuit } I = \operatorname{Re}(I_0 e^{i(\omega t + \phi)})$$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos[\omega t + \arctan(\omega RC)]$$

8.2)



$$a) \quad Z = R - \frac{i}{\omega C}$$

$$= 2000 - \frac{i}{2\pi \times 60 \times 10^{-6}}$$

$$= 2000 - i 2653 \text{ ohm.}$$

$$b) \quad I_{rms} = \frac{V_{rms}}{|Z|}$$

$$= \frac{120 \text{ volt}}{[(2000)^2 + (2653)^2]^{1/2} \text{ ohm}}$$

$$= .036 \text{ amp.}$$

$$c) \quad \text{Complex current } I_0 e^{i\phi} = \frac{V_0}{Z}$$

$$\therefore = \frac{V_0}{2000 - i 2653}$$

$$\therefore \tan \phi = \frac{2653}{2000} \Rightarrow \phi = 53^\circ$$

$$\text{Average Power dissipated } \bar{P} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$= 120 \text{ volts} \times .036 \text{ amp} \cos 53^\circ$$

$$= 2.60 \text{ watts}$$

d) Voltage across resistor is

$$V_R (\text{rms}) = I_{\text{rms}} R$$

$$= .036 \text{ amp} \times 2000 \text{ ohm}$$

$$= 72 \text{ volts}$$

Voltage across capacitor is

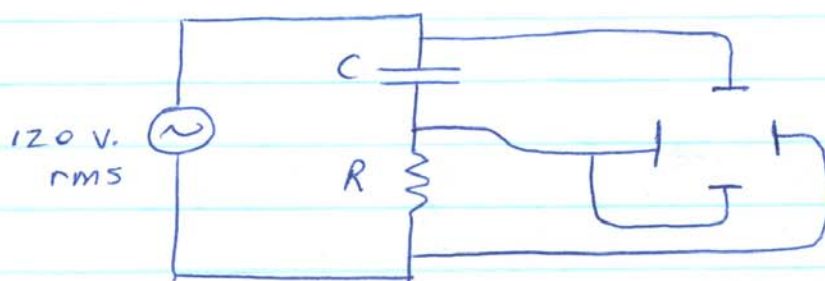
$$V_C (\text{rms}) = I_{\text{rms}} |Z_C|$$

$$= \frac{I_{\text{rms}}}{\omega C}$$

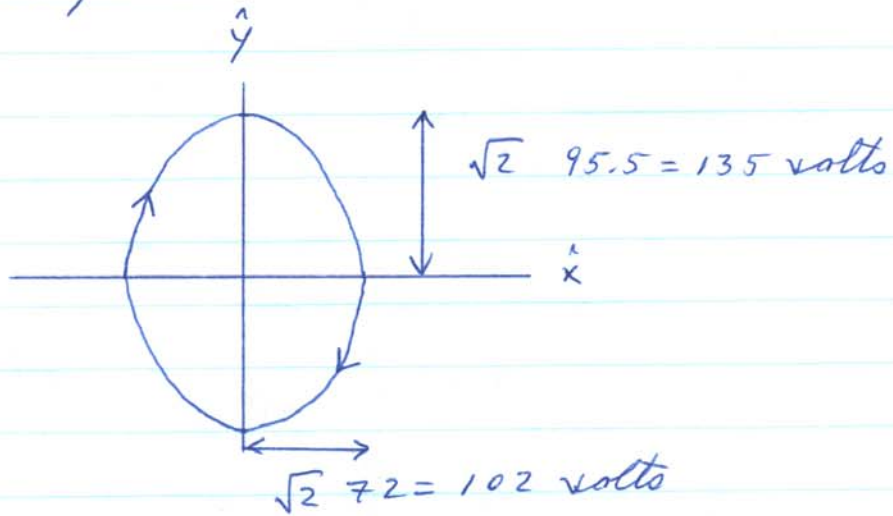
$$= \frac{.036 \text{ amp}}{2\pi \times 60 \times 10^{-6} \text{ ohm}}$$

$$= 95.5 \text{ volts}$$

e) Suppose oscilloscope is connected as follows.

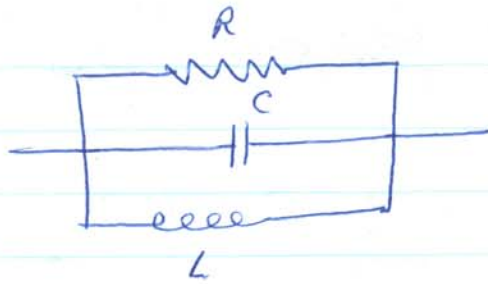


Oscilloscope Trace



When the capacitor is fully charged $I=0$ and $V_R=0$.
When the capacitor discharges I increases and V_R
increases. \therefore beam follows clockwise direction.

8.3)



$$R = 1000 \text{ ohm}$$

$$C = 500 \text{ pF}$$

$$L = 1 \text{ millihenry}$$

$$Z_{TOT} = \frac{1}{\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)}$$

$$\omega = 2\pi \times 10^4 \text{ rad/sec}$$

$$Z_{TOT} = \frac{1}{10^{-3} + i\left(2\pi \times 10^4 \times 5 \times 10^{-10} - \frac{1}{2\pi \times 10^4 \times 10^{-3}}\right)}$$

$$= \frac{1}{10^{-3} - i 7.93 \times 10^{-3}}$$

$$= 10^3 \frac{(1 + 7.93i)}{1 + (7.93)^2}$$

$$= 15.7 + 125i \text{ ohm}$$

Similarly $\omega = 2\pi \times 10^7 \text{ rad/sec} \Rightarrow Z_{TOT} = 1.01 - 31.8i \text{ ohm}$

Z_{TOT} is largest when $\omega C - \frac{1}{\omega L} = 0$.

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{(10^{-3} \times 5 \times 10^{-10})^{1/2}} = 1.41 \times 10^6 \text{ rad/sec.}$$

8.5 a) Oscillation period $T \approx \frac{1}{4} \times 10^{-3}$ sec.

Angular oscillation frequency $\frac{2\pi}{T} = 8\pi \times 10^3$ rad/sec

But oscillation freq. $\omega = (LC)^{-1/2}$

$$\therefore (LC)^{-1/2} = 8\pi \times 10^3$$

$$\frac{1}{LC} = (8\pi \times 10^3)^2$$

$$C = \frac{1}{L (8\pi \times 10^3)^2}$$

$$= \frac{1}{.01 \times (8\pi \times 10^3)^2}$$

$$C = 1.6 \times 10^{-7} \text{ farad}$$

b) Damping time $\tau = \frac{2L}{R} = .5 \times 10^{-3}$ sec.

$$\therefore R = \frac{2L}{.5 \times 10^{-3}}$$

$$= \frac{2 \times .01 \text{ henry}}{.5 \times 10^{-3} \text{ sec}}$$

$$= 40 \text{ ohm.}$$

c) Voltage across scope after long time is

$$\frac{R}{R + 10^5} \times 20 \text{ volts}$$

$$= \frac{40}{40 + 10^5} \times 20$$

$$= 8 \times 10^{-3} \text{ volts}$$