PHYS 2020 Assignment 11

1) $$\int_S \vec{E} \cdot d\vec{a} = 4\pi \int_V \rho \, dV$$

Flux of electric field leaving a surface equals $4\pi \times$ charge enclosed by the surface.

2) $$\int_S \vec{B} \cdot d\vec{a} = 0$$

Flux of magnetic field leaving a closed surface is always zero. i.e. there are no magnetic monopoles.

3) $$\oint \vec{E} \cdot d\vec{l} = -\frac{1}{c} \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a}$$

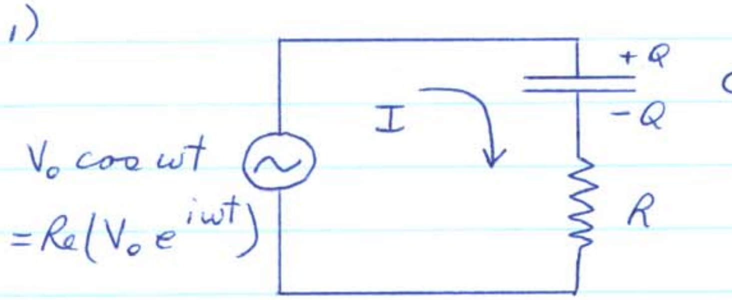
A changing magnetic field flux through surface S induces a rotating electric field around S. (Faraday Law)

4) $$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} \int_S \vec{J} \cdot d\vec{a} + \frac{1}{c} \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{a}$$

A changing electric field flux through surface S (displacement current) or a current flowing through S (Ampere) creates a rotating magnetic field around S.

PHYS 2020 Assignment 12

1)



$$V_0 e^{i\omega t} = \frac{Q}{C} + IR$$

Differentiating this equation and using $I = \frac{dQ}{dt}$ and $I = I_0 e^{i(\omega t + \varphi)}$ we get:

$$i\omega V_0 e^{i\omega t} = \frac{I}{C} + i\omega I R$$

$$I\left(\frac{1}{C} + i\omega R\right) = i\omega V_0 e^{i\omega t}$$

$$I(1 + i\omega RC) = i\omega C V_0 e^{i\omega t}$$

$$I_0 e^{i\varphi} = \frac{i\omega C}{1 + i\omega RC} V_0$$

$$I_0 e^{i\varphi} = \frac{V_0}{R - \frac{i}{\omega C}}$$

$$\therefore I_0 e^{i\varphi} = \frac{V_0}{R^2 + \frac{1}{\omega^2 C^2}} \left(R + \frac{i}{\omega C}\right)$$

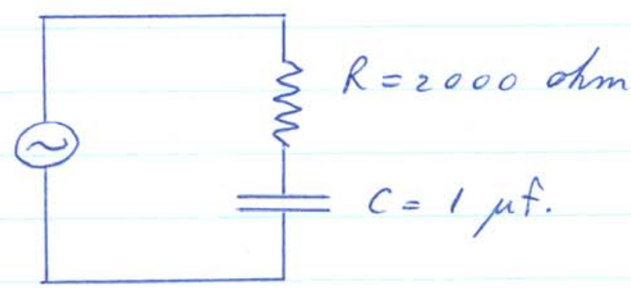
$$I_0 = \frac{V_0}{\left(R^2 + \frac{1}{\omega^2 C^2}\right)^{1/2}} \qquad \tan\varphi = \frac{1}{\omega RC}$$

$$\varphi = \arctan(\omega RC)^{-1}$$

$\therefore$ current in circuit $I = Re\left(I_0 e^{i(\omega t+\varphi)}\right)$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos\left[\omega t + \arctan(\omega RC)^{-1}\right]$$

8.2)



a) $$Z = R - \frac{i}{\omega C}$$

$$= 2000 - \frac{i}{2\pi \times 60 \times 10^{-6}}$$

$$= 2000 - i\,2653 \text{ ohm.}$$

b) $$I_{rms} = \frac{V_{rms}}{|Z|}$$

$$= \frac{120 \text{ volt}}{\left[(2000)^2 + (2653)^2\right]^{1/2} \text{ ohm}}$$

$$= .036 \text{ amp.}$$

c) Complex current $I_0 e^{i\varphi} = \frac{V_0}{Z}.$

$$\therefore \quad = \frac{V_0}{2000 - i2653}$$

$$\therefore \tan\varphi = \frac{2653}{2000} \Rightarrow \varphi = 53^\circ$$

Average Power dissipated $\bar{P} = V_{rms} I_{rms} \cos\varphi$

$= 120 \text{ volts} \times .036 \text{ amp} \cos 53°$

$= 2.60 \text{ watts}$

d) Voltage across resistor is

$V_R(rms) = I_{rms} R$

$= .036 \text{ amp} \times 2000 \text{ ohm}$

$= 72 \text{ volts}$

Voltage across capacitor is

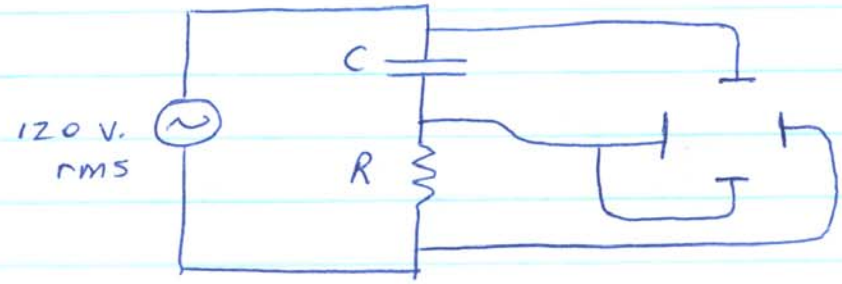
$V_C(rms) = I_{rms} |Z_C|$

$= \dfrac{I_{rms}}{\omega C}$

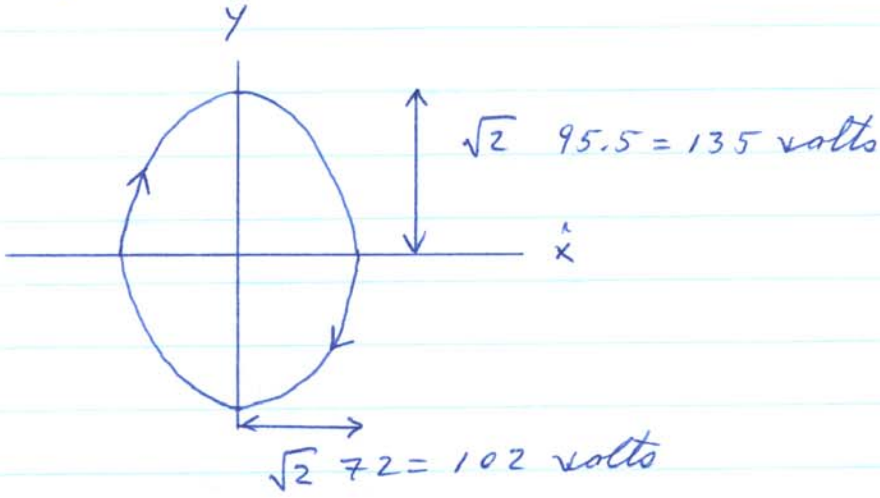
$= \dfrac{.036 \text{ amp}}{2\pi \times 60 \times 10^{-6} \text{ ohm}}$

$= 95.5 \text{ volts}$

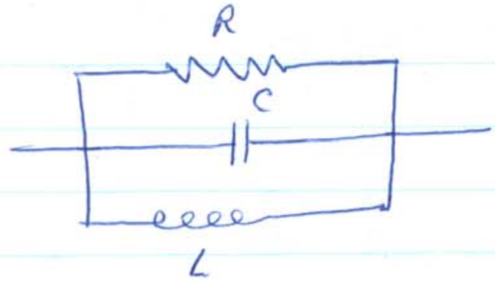
e) Suppose oscilloscope is connected as follows.



Oscilloscope Trace



When the capacitor is fully charged $I = 0$ and $V_R = 0$. When the capacitor discharges I increases and V_R increases. $\therefore$ beam follows clockwise direction.

8.3) 

$R = 1000$ ohm

$C = 500$ pf

$L = 1$ millihenry

$$z_{TOT} = \frac{1}{\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)}$$

$\underline{\omega = 2\pi \times 10^4 \text{ rad/sec}}$

$$z_{TOT} = \frac{1}{10^{-3} + i\left(2\pi \times 10^4 \times 5\times 10^{-10} - \frac{1}{2\pi \times 10^4 \times 10^{-3}}\right)}$$

$$= \frac{1}{10^{-3} - i\, 7.93 \times 10^{-3}}$$

$$= 10^3 \frac{(1 + 7.93 i)}{1 + (7.93)^2}$$

$$= 15.7 + 125 i \text{ ohm}$$

Similarly $\omega = 2\pi \times 10^7$ rad/sec $\Rightarrow z_{TOT} = 1.01 - 31.8 i$ ohm

z_{TOT} is largest when $\omega C - \frac{1}{\omega L} = 0$.

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{(10^{-3} \times 5 \times 10^{-10})^{1/2}}$$

$$= 1.41 \times 10^6 \text{ rad/sec.}$$

8.5 a) Oscillation period $T \simeq \frac{1}{4} \times 10^{-3}$ sec.

Angular oscillation frequency $\frac{2\pi}{T} = 8\pi \times 10^{3}$ rad/sec

But oscillation freq. $\omega = (LC)^{-1/2}$

$$\therefore (LC)^{-1/2} = 8\pi \times 10^{3}$$

$$\frac{1}{LC} = (8\pi \times 10^{3})^{2}$$

$$C = \frac{1}{L\ (8\pi \times 10^{3})^{2}}$$

$$= \frac{1}{.01 \times (8\pi \times 10^{3})^{2}}$$

$$C = 1.6 \times 10^{-7} \text{ farad}$$

b) Damping time $\tau = \frac{2L}{R} = .5 \times 10^{-3}$ sec.

$$\therefore R = \frac{2L}{.5 \times 10^{-3}}$$

$$= \frac{2 \times .01 \text{ henry}}{.5 \times 10^{-3} \text{ sec}}$$

$$= 40 \text{ ohm.}$$

c) Voltage across scope after long time is

$$\frac{R}{R + 10^{5}} \times 20 \text{ volts}$$

$$= \frac{40}{40 + 10^{5}} \times 20$$

$$= 8 \times 10^{-3} \text{ volts}$$