

Assignment 17
Magnetic Field & AC Currents/Voltages

1. A proton travels around a loop of radius R with a speed of 10% of the speed of light. The proton motion is circular due to a magnetic field of 100 Gauss.
- a) What is the radius of the proton path?

$$\begin{array}{ccc} \text{Centripetal} & = & \text{Lorentz} \\ \text{Force} & & \text{Force} \end{array}$$

$$\frac{mv^2}{R} = qvB$$

$$R = \frac{mv}{qB}$$

$$= \frac{1.67 \times 10^{-27} \text{ kg} \times 3 \times 10^7 \text{ m/sec}}{1.6 \times 10^{-19} \text{ C} \times 10^{-2} \text{ Tesla}}$$

$$= 31 \text{ meters}$$

- b) What is the ~~maximum~~ ^{the} kinetic energy of a proton?

$$\text{Kinetic Energy} = \frac{mv^2}{2}$$

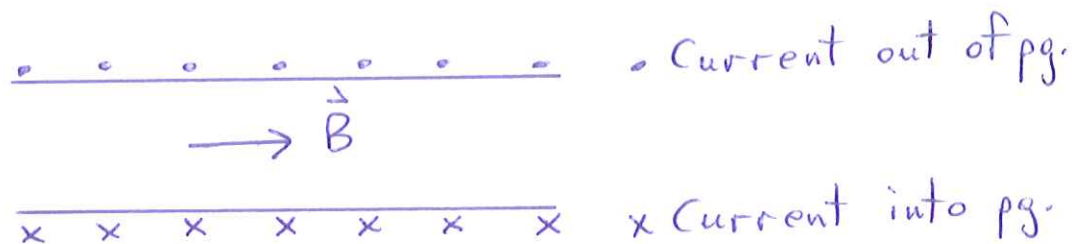
$$= \frac{1.67 \times 10^{-27} \times (3 \times 10^7)^2}{2}$$

$$= 7.5 \times 10^{-13} \text{ J}$$

$$= 4.7 \times 10^6 \text{ eV}$$

$$= 4.7 \text{ MeV.}$$

2. A solenoid consists of a hollow tube of length 5 meters around which 1000 turns of wire are wrapped as shown below. A current I flowing in the wire generates a magnetic field whose direction is given by the right hand rule. (Point fingers of right hand in direction of current and right hand thumb points along magnetic field.) It can be shown the magnetic field in a long narrow tube is uniform and given by $B(\text{tesla}) = 1.26 \times 10^{-6} N I(\text{amp})$ where N is number of wire loops/meter. Assuming solenoid is aligned along the Earth's magnetic field, how much current is needed to cancel the Earth's field?



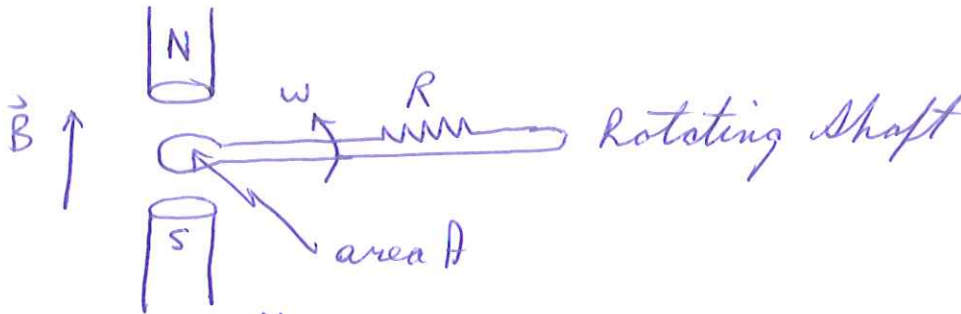
$$B_{\text{Solenoid}} = B_{\text{Earth}}$$

$$1.26 \times 10^{-6} N I = 10^{-4} \text{ tesla} \quad (B_{\text{Earth}} \approx 1 \text{ G})$$

$$I = \frac{10^{-4}}{1.26 \times 10^{-6} \times \frac{1000}{5}}$$

$$\therefore I = 4.0 \times 10^{-1} \text{ amps}$$

3. Electricity is generated by having a circular loop of wire enclosing area A spin through a uniform magnetic field B at an angular frequency ω as shown below.



The magnetic ^{flux} going through the loop is given by $\Phi(t) = B A \cos \omega t$. The voltage across the resistor is then found to be $V = -d\Phi / dt$.

a) Evaluate the voltage if $B = 1$ tesla, $A = 1 \text{ m}^2$ and $\omega = 377$ rad/sec.

$$\begin{aligned} V &= -\frac{d\Phi}{dt} \\ &= -\frac{d}{dt} (BA \cos \omega t) \\ &= BA \omega \sin \omega t \end{aligned}$$

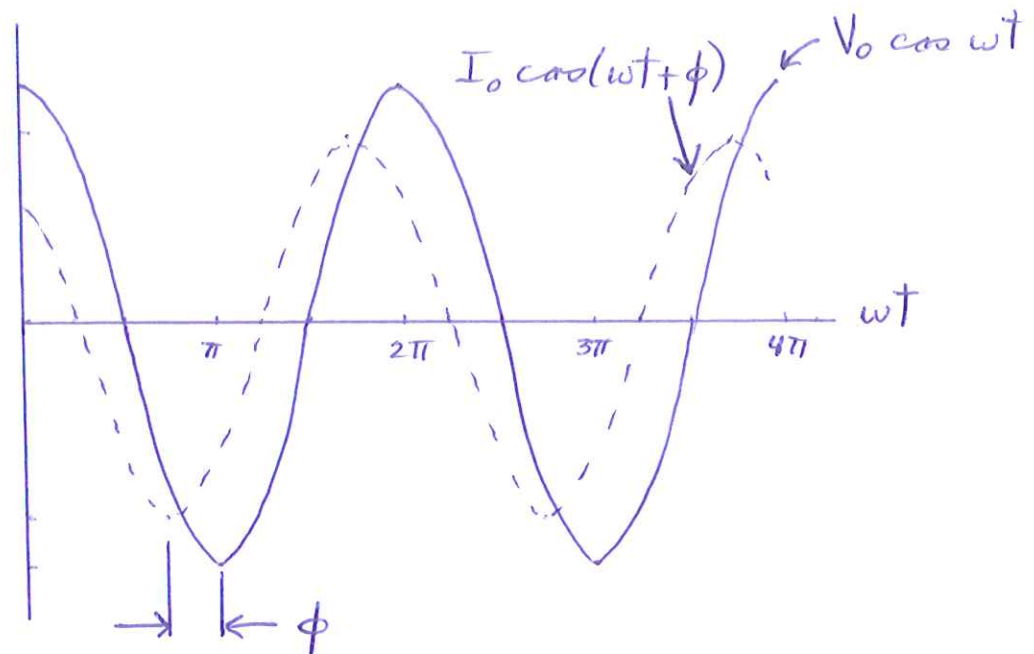
$$\begin{aligned} \therefore V_{\text{peak}} &= BA \omega \\ &= 1 \text{ tesla} \times 1 \text{ m}^2 \times 377 \text{ rad/sec} \\ &= 377 \text{ volts} \end{aligned}$$

- b) How is the answer in a affected if there are 5 loops of wire?

$$\Phi = NBA \cos \omega t \text{ where } N=5$$

$$\begin{aligned} \Rightarrow V_{\text{peak}} &= NBA \omega \\ &= 1.89 \text{ kV} \end{aligned}$$

4. Consider a current and voltage across a resistor are given by
 $I(t) = I_0 \cos(\omega t + \phi)$ and $V(t) = V_0 \cos(\omega t)$.
 a) Sketch the current and voltage.



- b) Find the time averaged power dissipated.

$$\begin{aligned}
 P_{ave} &= \frac{1}{T} \int_0^T I(t) V(t) dt \\
 &= \frac{I_0 V_0}{T} \int_0^T \cos(\omega t + \phi) \cos \omega t dt \\
 &= \frac{I_0 V_0}{T} \left\{ \cos \phi \underbrace{\int_0^T \cos^2 \omega t dt}_{= \frac{T}{2}} - \sin \phi \underbrace{\int_0^T \cos \omega t \sin \omega t dt}_{= 0} \right\} \\
 &= \frac{I_0 V_0}{2} \cos \phi
 \end{aligned}$$

$$\therefore P_{ave} = I_{RMS} V_{RMS} \cos \phi$$

Assignment 18 Optics

1. Consider an object of height 1 cm in front of a convex lens having a focal length of 5 cm. Describe the image size, type and position if the object is at the following positions from the lens.

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f} \quad \text{and} \quad \frac{y_i}{y_o} = \frac{f}{s_o - f}$$

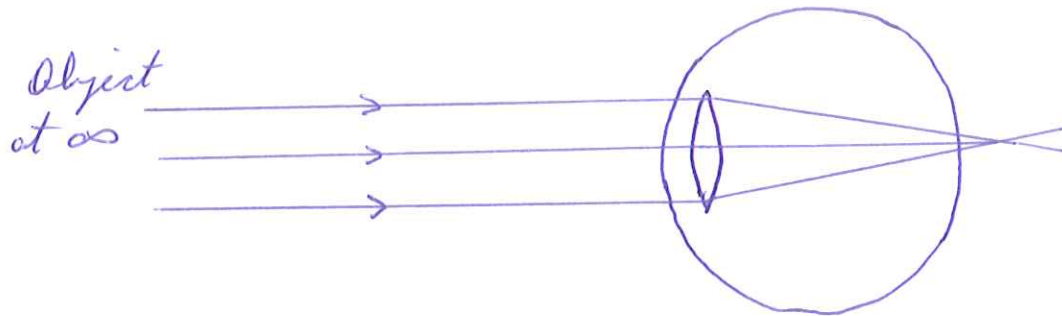
Object Position (cm)	Image Size (cm)	Image Type	Image Position (cm)
20	0.33 (inverted)	Real	6.67
10	1 (inverted)	Real	10
5			No Image
2	1.6 (erect)	Virtual	-3.33

2. What are the approximate wavelengths and frequencies of the following photons?

	Wavelength (nm)	Frequency (Hz)
Red Light	650	4.6×10^{14}
Yellow Light	590	5.1×10^{14}
Blue Light	500	6.0×10^{14}
X ray	0.1	3×10^{18}
Infrared Light	1000	3×10^{14}
Ultraviolet Light	300	1×10^{15}
Gamma Ray	0.001	3×10^{20}

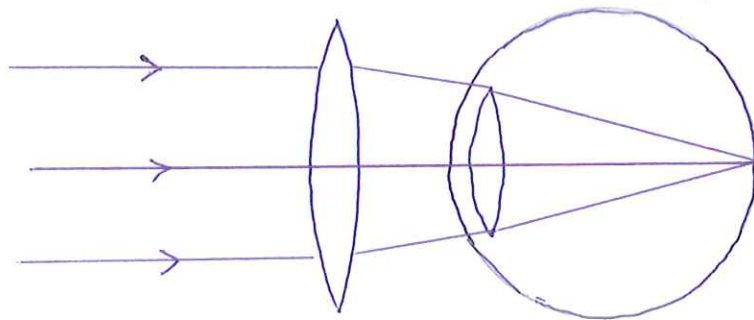
3. a) Explain using diagrams of light entering the eye, what farsightedness is.

Eye focuses objects behind retina.



- b) How can glasses correct this problem?

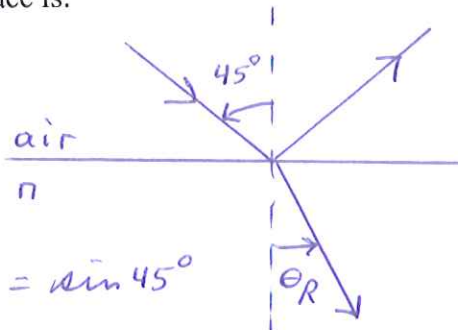
Add ^{convex} concave lens as shown below.



4. Explain the origin of the rainbow.

See textbook

5. A light ray is incident at 45° on a surface. Find the angle of refraction if the reflecting surface is:
- water
 - glass



$$n \sin \theta_R = \sin 45^\circ$$

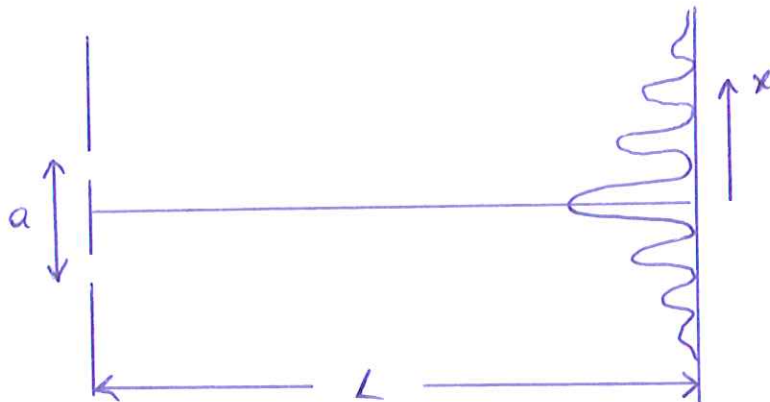
$$\sin \theta_R = \frac{1}{n\sqrt{2}}$$

$$n_{\text{water}} = 1.33 \Rightarrow \sin \theta_R = 0.53 \quad n_{\text{glass}} = 1.5 \Rightarrow \sin \theta_R = 0.47$$

$$\theta_R = 32.1^\circ$$

$$\theta_R = 28.1^\circ$$

6. Young's Double slit experiment is done using blue light. How does it differ from using red light?



$$\text{Dark lines on screen at } x = \frac{L}{a} \left(n - \frac{1}{2} \right) \lambda.$$

Blue light $\lambda \ll$ Red light λ .

\therefore entire pattern for blue light is slightly closer to central axis than for red light.

7. Sketch the electric and magnetic fields for a light wave propagating in the z direction that is linearly polarized in the y direction.

