Assignment 19
Modern Physics

1. Radioactive iodine (^{131}I) has a half-life of 8 days. Iodine is absorbed especially strongly by the thyroid gland. Hence, radioactive iodine can cause thyroid cancer. How long does it take for the amount of ^{131}I to be reduced by a factor of 100?

$N(t) = N_0 e^{-kt}$ where $k = \frac{\ln 2}{t_{1/2}}$

$$\frac{N(t)}{N_0} = \frac{1}{100} \Rightarrow \frac{1}{100} = e^{-kt}$$

$$-\ln 100 = -kt$$

$$t = t_{1/2} \frac{\ln 100}{\ln 2}$$

$$= 8 \text{ days} \frac{\ln 100}{\ln 2}$$

$$\therefore t = 53 \text{ days}.$$

Hence, after Chernobyl people were told to avoid vegetables grown in their garden for about 2 months.

2. A metal has a work function of 3.5 eV.

a) What is the longest wavelength photon that can generate photoelectrons?

Work Function $W = h\nu$

$$= \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{W}$$

$$= \frac{6.64\times10^{-34}\,\text{J sec} \times 3\times10^{8}\,\text{m/sec}}{3.5\,\text{eV} \times 1.6\times10^{-19}\,\text{J/eV}}$$

$$= 356\,\text{nm}$$

b) Is this photon in the visible, infrared or ultraviolet portion of the spectrum?

Ultraviolet

3. The maximum shift in the wavelength of scattered X rays in the Compton effect is 0.0486 Angstrom and occurs when the X ray is scattered in the backwards direction.

a) What is the difference in momentum of the incident and scattered X rays?

$$\Delta p = \frac{h\nu}{c} - \frac{h\nu'}{c}$$

$$= \frac{h}{\lambda} - \frac{h}{\lambda'}$$

$$= \frac{h}{\lambda\lambda'}(\lambda' - \lambda)$$

$$\Delta p = \frac{6.64\times10^{-34} \times .0486\times10^{-10}}{(1\times10^{-10})^2}$$

$$= 3.22\times10^{-25}\,\text{kg m/sec}$$

b) What is the final kinetic energy of the electron that was initially at rest?

Conservation of Energy: $h\nu = h\nu' + T_{electron}$

$$\therefore T_{electron} = h\nu - h\nu'$$

$$= c\,\Delta p$$

$$= 9.68\times10^{-17}\,\text{J}$$

$$= 605\,\text{eV}.$$

4. Compute the de Broglie wavelength of the following.

a) A human walking briskly

$$P_{human} \simeq 100\, kg \times 2\, m/sec = 200\, kg\, m/sec$$

$$\lambda_{dB} = \frac{h}{p} = \frac{6.64 \times 10^{-34}}{200} = 3.32 \times 10^{-36}\, m.$$

b) A proton traveling at 3 x 10^5 m/sec

$$P_{prot} = 1.67 \times 10^{-27}\, kg \times 3 \times 10^{5}\, m/sec = 5 \times 10^{-22}\, kg\, m/sec$$

$$\lambda_{dB} = \frac{h}{p} = \frac{6.64 \times 10^{-34}}{5 \times 10^{-22}} = 1.33 \times 10^{-12}\, m$$

c) An alpha particle traveling at 3 x 10^5 m/sec

$$P_{alpha} = 4 \times 1.67 \times 10^{-27}\, kg \times 3 \times 10^{5}\, m/sec = 2 \times 10^{-21}\, kg\, m/sec$$

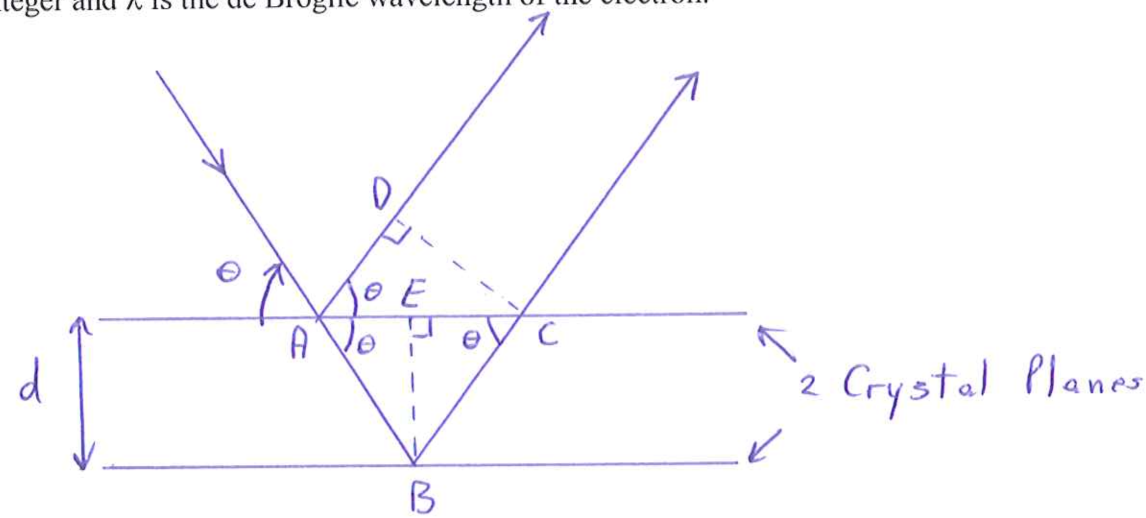
$$\lambda = \frac{6.64 \times 10^{-34}}{2 \times 10^{-21}} \simeq 3.3 \times 10^{-13}\, m$$

d) A beta particle having an energy of 50 KeV

$$p = \sqrt{2mE} = \sqrt{2 \times 9.11 \times 10^{-31}\, kg \times 50 \times 10^{3} \times 1.6 \times 10^{-19}\, J} = 1.21 \times 10^{-22}\, kg\, m/sec$$

$$\lambda = \frac{6.64 \times 10^{-34}}{1.21 \times 10^{-22}} = 5.5 \times 10^{-12}\, m$$

5. Consider an electron incident at angle θ as shown below on a crystal consisting of planes of atoms separated by a distance d. Derive the Bragg scattering criteria for constructive interference $n\lambda = 2\, d \sin\theta$ where n is an integer and λ is the de Broglie wavelength of the electron.



Scattered Electrons interfere constructively if

$n\lambda = AB + BC - AD$ (path difference of 2 rays)

$= AB + BC - AC\cos\theta$

$= 2AB - 2AE\cos\theta$

$= 2\left[\frac{d}{\sin\theta} - \frac{d}{\tan\theta}\cos\theta\right]$

$= 2d\left[\frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= 2d\,\frac{1-\cos^2\theta}{\sin\theta}$

$\therefore n\lambda = 2d\sin\theta$

Assignment 20
Bohr Atom

1. What is the value of the principal quantum number for a hydrogen atom to have a size of one micron?

$$r = n^2 a_0$$

$$n = \sqrt{\frac{r}{a_0}}$$

$$= \left(\frac{10^{-6}\ \text{m}}{5.29\times10^{-11}\ \text{m}}\right)^{1/2}$$

$$= 137$$

2. Determine an expression for the speed of an electron in the ground (lowest energy) state of hydrogen.

For ground state, angular momentum $mvr = \hbar$.

But $r = \dfrac{\hbar^2}{mke^2}$ (Bohr radius)

$$\therefore v = \frac{\hbar}{mr}$$

$$= \frac{ke^2}{\hbar c}\, c$$

$$= \frac{9\times10^9 \times (1.6\times10^{-19})^2}{\frac{6.64\times10^{-34}}{2\pi} \times 3\times10^8}\, c$$

$$= \frac{1}{137}\, c$$

$\alpha \equiv \dfrac{ke^2}{\hbar c} = \dfrac{1}{137}$

Fine Structure Const.

$$v = 2.2\times10^6\ \text{m/sec}$$

3. Determine the shortest wavelength for each of the Balmer and Passchen series. Are these wavelengths in the visible, infrared or ultraviolet parts of the spectrum?

Balmer: $\frac{hc}{\lambda} = E_R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

$$\Rightarrow \lambda_{max} = \frac{hc}{E_R}\frac{36}{5} = 659 \text{ nm.}$$

Passchen: $\frac{hc}{\lambda} = E_R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$

$$\Rightarrow \lambda_{max} = \frac{hc}{E_R}\frac{144}{7} = 1.88\,\mu m$$

4. A He^+ ion consists of a nucleus which is an alpha particle plus one orbiting electron. Hence it has a net positive charge.

a) Derive an expression for the electron state energies.

Repeat analysis for H but now nucleus has charge Ze. (i.e. review notes)

$$\Rightarrow E = -\frac{k^2 Z^2 e^4 m}{2n^2\hbar^2}$$

$$E = -Z^2\frac{E_{Ryd}}{n^2}$$

b) What is the wavelength associated with a transition between the lowest two energy states?

$$E_2 - E_1 = 4E_{Ryd}\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$= 4 \times 13.6\,eV \cdot \frac{3}{4}$$

$$= 40.8\,eV.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.64\times10^{-34} \times 3\times10^{8}}{40.8 \times 1.6\times10^{-19}} = 3.05\times10^{-8}\,m$$

$$\therefore \lambda = 30.5\,nm \text{ (far ultraviolet)}$$

5. A muon is a particle that has a negative charge equal to an electron but is 207 times heavier than an electron. Atoms have been created where the electron in a hydrogen atom has been replaced by a muon. Find the following.

a) The allowed radii of the muon atom.

Muon Bohr radius $a_0 = \dfrac{\hbar^2}{m_{muon} k q^2}$

$$= \frac{m_{elect}}{m_{muon}} a_{oH}$$

$$= \frac{1}{207} \times 5.29 \times 10^{-11} \text{ m}$$

$$= 2.56 \times 10^{-13} \text{ m}$$ (almost inside nucleus!)

b) The energies of the allowed states.

Muon Rydberg $E_{Ryd} = \dfrac{k^2 q^4 m_{muon}}{2\hbar^2}$

$$= \frac{m_{muon}}{m_{elect}} E_{Ryd\,H}$$

$$= 207 \times 13.6 \text{ eV}$$

$$= 2.82 \times 10^3 \text{ eV}$$

c) What is the wavelength corresponding to a transition between the ground and first excited state?

$$\Delta E_{if} = E_{Ryd}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\Delta E_{21} = 2.82 \times 10^3 \text{ eV}\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 2.12 \times 10^3 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E_{21}} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{2.12 \times 10^3 \times 1.6 \times 10^{-19}} = 5.89 \times 10^{-10} \text{ m}$$

$\therefore \lambda = 5.89$ Å